DEPT. OF MECHANICAL ENGINEERING

TRIDENT ACADEMY OF TECHNOLOGY, BHUBANESWAR

LECTURE NOTES

ON

KINEMATICS & DYNAMICS OF MACHINES (3-0-0)

B. Tech

3rd Semester

By

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MEPC2004 KINEMATICS & DYNAMICS OF MACHINES (3-0-0)

Module I (6hrs)
Kinematic Fundamental: Mechanisms: Basic kinematic concepts & definitions, mechanisms, link, kinematic pair, degrees of freedom, Kinematic chain, degrees of freedom for plane mechanism, Gruebler's equation, Inversion of mechanism, Four bar chain & their inversions, Single slider crank chain, Double slider crank chain & their inversion.
Kinematic Analysis: Graphical analysis of position, Velocity and acceleration of four barand Slider crank mechanisms. Instantaneous centre method, Aronhold-Kennedy Theorem, Rubbing velocity at a Pin-joint.Coriolis component of acceleration.

Module II (6 hrs)
Gear and Gear Trains: Gear terminology, Types of Gear, Tooth properties and methods of generation of standard tooth profiles, Force analysis. Types of gear trains: Simple, Compound, Reverted and Epicyclic gear trains, Train value, Methods of finding train value/velocity ratio: Tabular method and analytical method for Epicyclic gear trains.

Module III (6hrs)
Turning Moment Diagram and Flywheel: Turning moment diagram, Turning moment diagrams for different types of engines, Fluctuation of energy and fluctuation of speed,Theory of Flywheel.
Mechanism for Control (Governors): Governors-Watt, Porter, Proell, Hartnell. Performance parameters: Sensitiveness, Stability, Hunting, Isochronism.Governor Effort and Power.

Module IV (6hrs)
Friction Effects: Screwjack, Friction between pivot and collars, Single, Multi-plate and cone clutches, anti-friction bearing.
Flexible Mechanical Elements: Belt, Rope and chain drives, Initial tension, Effect of centrifugal tension on power transmission, Maximum power transmission capacity, Beltcreepandslip.
Brakes: Classification of brakes, Types of brakes, Analysis of different brakes, Braking of a vehicle.

Module V (6hrs)
Balancing of rotating components and linkages: Static and Dynamic Balancing, Balancing of Single Rotating Mass by Balancing Masses in Same plane and in Different planes. Balancing of Several Rotating Masses rotating in same plane and in Different planes.

TEXT BOOKS:
1. Theory of Machines by S.S.Rattan, Tata MacGraw Hill
2. Theory of Machines by Thomas Bevan, CBS Publications
3. Kinematics and Dynamics of Machinery by Charles E. Wilson and J.Peter Saddler, Pearson Education.

REFERENCE BOOKS:
1. Theory of Machines and Mechanisms: By Joseph Edward Shigley
2. Mechanism & Machine Theory by J. S. Rao and R. V. Dukipatti, New Age International.
3. Theory of Mechanisms and Machines by A. Ghosh & A. K. Mallick, East West Press.
4. Kinematics and Dynamics of Machines by G.H. Martin, Mc Graw-Hill.
5. Theory of Machines and Mechanisms by P.L.Ballaney, Khanna Publishers .

1.1 Concepts of Kinematics and Dynamics

Kinematics: It is that branch of Theory of Machines which deals with the relative motion between the various parts of the machines without forces applying to it.

Dynamics: It is that branch of Theory of Machines which deals with the forces and their effects while acting upon the machine parts in motion.

Kinetics: It is that branch of Theory of Machines which deals with the inertia forces which arise from the combined effect of the mass and motion of the machine parts.

Statics: It is that branch of Theory of Machines which deals with the forces and their effects while the machine parts are at rest. The mass of the parts is assumed to be negligible.

Mechanism: If a number of bodies are assembled in such a way that the motion of one causes constrained and predictable motion to the others, it is known as a mechanism.

Machine: A machine is a mechanism or a combination of mechanisms which, apart from imparting definite motions to the parts, also transmits and modifies the available mechanical energy into some kind of desired work.

Structure: It is an assemblage of a number of resistant bodies (known as members) having no relative motion between them and meant for carrying loads having straining action. A railway bridge, a roof truss, machine frames etc., are the examples of a structure.

*Table **1.1** - Difference Between a Machine and a Structure*

Machine	Structure
Parts of a machine move relative to one another	Members of a structure do not move relative to one another
The machine transforms the available energy into some useful work	In a structure, no energy is transformed into useful work
Links of a machine may transmit both power and motion	Members of a structure transmit forces only

1.2 Kinematic Links

Each part of a machine, which moves relative to some other part, is known as a ***kinematic link*** (or simply link). A link may consist of several parts, which are rigidly fastened together so that they do not move relative to one another.

A link or element need not be a rigid body, but it must be a resistant body. A body is said to be a ***resistant body*** if it is capable of transmitting the required forces with negligible deformation.

1.2.1 Types of Kinematic Links

a) Rigid Link

A rigid link is one which does not undergo any deformation while transmitting motion. Strictly speaking, rigid links do not exist. However, as the deformation of a connecting rod, crank etc. of a reciprocating steam engine is not appreciable, they can be considered as rigid links.

Fig.1.1 - Connecting Rod

b) Flexible Link

A flexible link is one which is partly deformed in a manner not to affect the transmission of motion. For example, belts, ropes, chains and wires are flexible links and transmit tensile forces only.

Fig.1.2 - Chain Drive

c) Fluid Link

A fluid link is one which is formed by having fluid in a receptacle and the motion is transmitted through the fluid by pressure or compression only, as in the case of hydraulic presses, jacks and brakes.

Fig.1.3 - Hydraulic jack

1.3 Types of Constrained Motions

a) Completely Constrained Motion

When the motion between a pair is limited to a definite direction irrespective of the direction of force applied, then the motion is said to be a completely constrained motion.

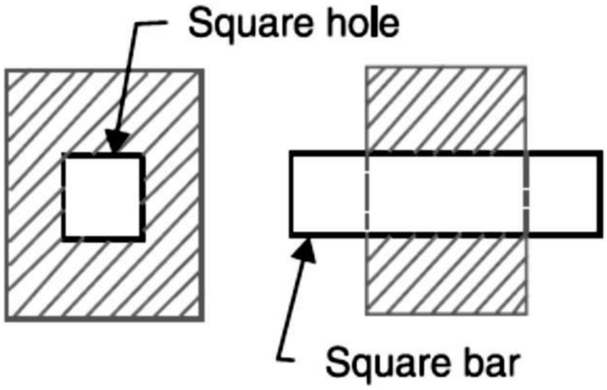


Fig.1.4 - The motion of a square bar in a square hole

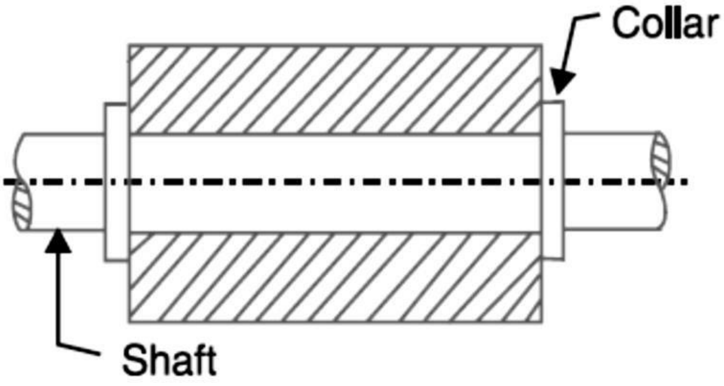


Fig.1.5 - The motion of a shaft with collars at each end in a circular hole

b) Incompletely Constrained Motion

When the motion between a pair can take place in more than one direction, then the motion is called an incompletely constrained motion. The change in the direction of the impressed force may alter the direction of relative motion between the pair.

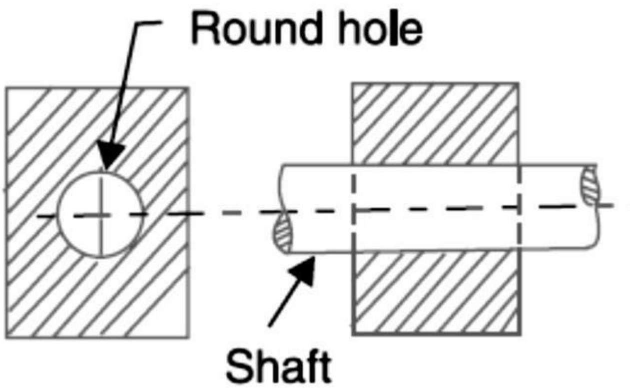


Fig.1.6 - A circular bar or shaft in a circular hole

c) Successfully Constrained Motion

When the motion between the elements, forming a pair, is such that the constrained motion is not completed by itself, but by some other means, then the motion is said to be successfully constrained motion.

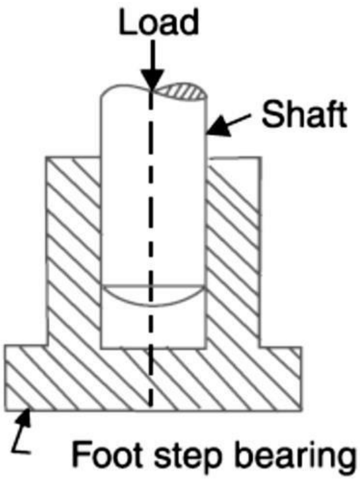


Fig.1.7 - Shaft in a foot-step bearing

1.4 Kinematic Pair

The two links or elements of a machine, when in contact with each other, are said to form a pair.

If the relative motion between them is completely or successfully constrained (i.e. in a definite direction), the pair is known as kinematic pair.

Classification of Kinematic Pairs:

1.4.1 According to the type of relative motion between the elements

a) Sliding pair

When the two elements of a pair are connected in such a way that one can only slide relative to the other, the pair is known as a sliding pair. The piston and cylinder, cross-head and guides of a reciprocating steam engine, ram and its guides in shaper, tail stock on the lathe bed etc. are the examples of a sliding pair. A little consideration will show, that a sliding pair has a completely constrained motion.

- Example: Piston and Cylinder

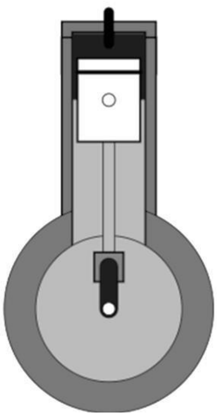


Fig.1.8 - Piston and Cylinder

b) Turning pair

When the two elements of a pair are connected in such a way that one can only turn or revolve about a fixed axis of another link, the pair is known as turning pair. A shaft with collars at both ends fitted into a circular hole, the crankshaft in a journal bearing in an engine, lathe spindle supported in head stock, cycle wheels turning over their axles etc. are the examples of a turning pair. A turning pair also has a completely constrained motion.

- Example: Circular shaft revolving inside a bearing

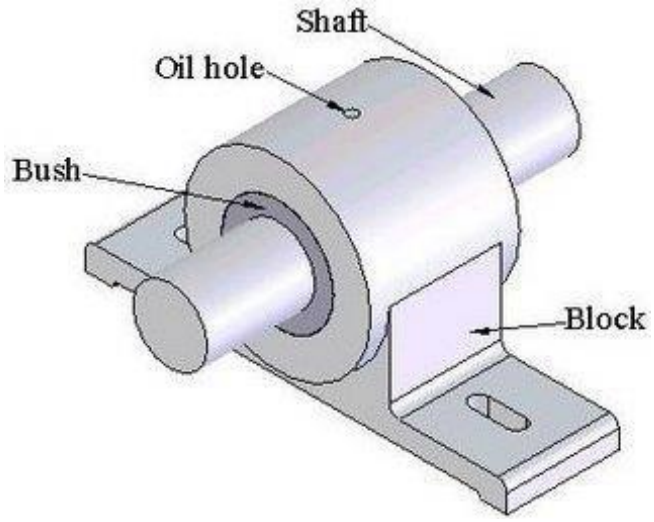


Fig.1.9 Circular shaft revolving inside a bearing

c) Rolling pair

When the two elements of a pair are connected in such a way that one rolls over another fixed link, the pair is known as rolling pair. Ball and roller bearings are examples of rolling pair.

- Example: Ball bearing

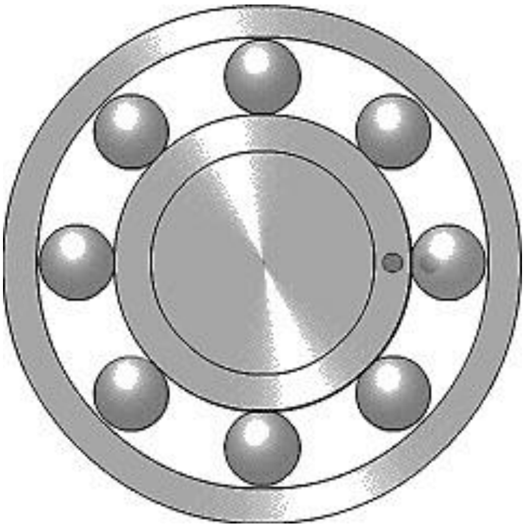


Fig.1.10 - Ball bearing

d) Screw pair

When the two elements of a pair are connected in such a way that one element can turn about the other by screw threads, the pair is known as screw pair. The lead screw of a lathe with nut and bolt with a nut are examples of a screw pair.

- Example: Bolt with a nut

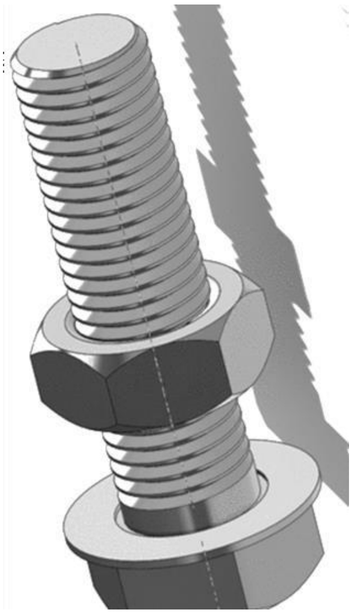


Fig.1.11 - Bolt with a nut

e) Spherical pair

When the two elements of a pair are connected in such a way that one element (with the spherical shape) turns or swivels about the other fixed element, the pair formed is called a spherical pair. The ball and socket joint, attachment of a car mirror, pen stand etc., are the examples of a spherical pair.

- Example: Ball and socket joint

Fig.1.12 - Ball and socket joint

1.4.2 According to the type of contact between the elements

a) Lower pair

When the two elements of a pair have a **surface contact** when relative motion takes place and the surface of one element slides over the surface of the other, the pair formed is known as lower pair.

It will be seen that sliding pairs, turning pairs and screw pairs form lower pairs.

- Example: A square bar in a square hole

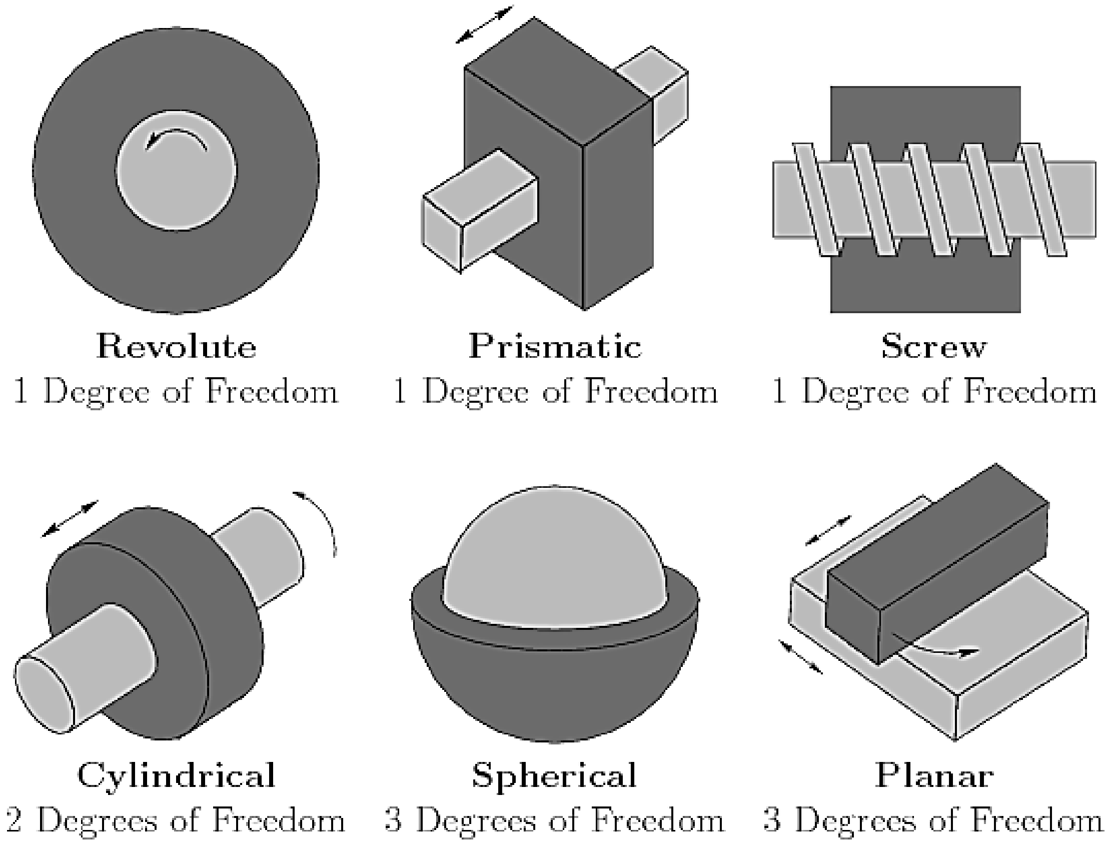


Fig.1.13 - Types of lower pair

b) Higher pair

When the two elements of a pair have a **line or point contact** when relative motion takes place and the motion between the two elements is partly turning and partly sliding, then the pair is known as higher pair.

A pair of friction discs toothed gearing, belt and rope drives, ball and roller bearings and cam and follower are the examples of higher pairs.

- Example: Cam and Follower

Fig.1.14 - Cam and Follower

1.4.3 According to the type of closure

a) Self closed pair

When the two elements of a pair are connected together mechanically in such a way that only required kind of relative motion occurs, it is then known as self-closed pair. The lower are self-closed pair.

- Example: Bolt with a nut

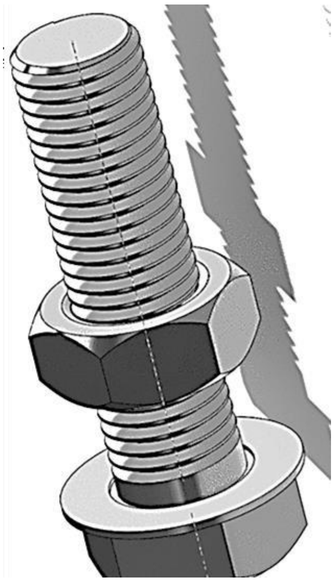


Fig.1.15 - Bolt with a nut

b) Force - closed pair

When the two elements of a pair are not connected mechanically but are kept in contact by the action of external forces, the pair is said to be a force-closed pair. The cam and follower is an example of force closed pair, as it is kept in contact by the forces exerted by spring and gravity.

- Example: Cam and follower with spring

Fig.1.16 - Cam and follower with spring

1.5 Kinematic Chain

When the kinematic pairs are coupled in such a way that the last link is joined to the first link to transmit definite motion (i.e. completely or successfully constrained motion), it is called a ***kinematic chain***.

In other words, a kinematic chain may be defined as a combination of kinematic pairs, joined in such a way that each link forms a part of two pairs and the relative motion between the links or elements is completely or successfully constrained.

For example, the crankshaft of an engine forms a kinematic pair with the bearings which are fixed in a pair, the connecting rod with the crank forms a second kinematic pair, the piston with the connecting rod forms a third pair and the piston with the cylinder forms the fourth pair. The total combination of these links is a kinematic chain.

Table 1.2 - Difference Between Kinematic Pair and Kinematic Chain

Kinematic Pair	Kinematic Chain
Two elements connected together	Four or more than four elements connected together
Have constrained relative motion with respect to each other	The motion of any point on the link with respect to any other point follows a definite direction
Each pair has two links	The chain has minimum four links. The first link and last link are connected to form a closed chain
Kinematic pair is a part of the chain.	Kinematic Chain is not a part of the Kinematic pair.
Examples: 1. Cylinder and Piston 2. Crank and connecting rod 3. Piston and connecting rod	Examples: 1. Four bar chain 2. Slider crank chain 3. Double crank chain

- When one of the links of a kinematic chain is fixed, the chain is known as a mechanism. It may be used for transmitting or transforming motion e.g. engine indicators, typewriter etc.
- A mechanism with four links is known as a simple mechanism.
- A mechanism with more than four links is known as a compound mechanism.
- When a mechanism is required to transmit power or to do some particular type of work, it then becomes a machine.

In such cases, the various links or elements have to be designed to withstand the forces (both static and kinetic) safely. A little consideration will show that a mechanism may be regarded as a machine in which each part is reduced to the simplest form to transmit the required motion.

1.6 Degrees of freedom/mobility of a mechanism

It is the number of inputs (number of independent coordinates) required to describe the configuration or position of all the links of the mechanism, with respect to the fixed link at any given instant.

1.6.1 Kutzbach Criterion to Plane Mechanisms

Kutzbach criterion is used for determining the number of degrees of freedom or movability (*n*) of a plane mechanism.

$$n = 3\,(l - 1) - 2\,j - h \qquad \textit{Eq. (1.1)}$$

where

,

n = Number of degrees of freedom;

j = Number of binary joints;

h = Number of higher pairs;

l = Number of links

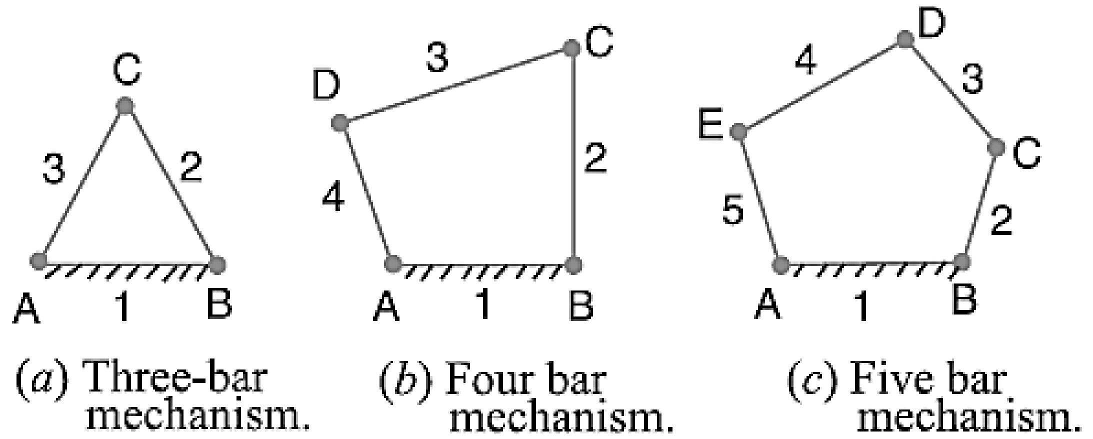


Fig.1.17 - Plane mechanisms

1. The mechanism, as shown in (a), has three links and three binary joints, i.e. l = 3 and j = 3.

$$n = 3\,(3 - 1) - 2 \times 3 = 0$$

2. The mechanism, as shown in (b), has four links and four binary joints, i.e. l = 4 and j = 4.

$$n = 3\,(4 - 1) - 2 \times 4 = 1$$

3. The mechanism, as shown in (c), has five links and five binary joints, i.e. l = 5, and j = 5.

$$n = 3\,(5 - 1) - 2 \times 5 = 2$$

1.6.2 Grubler's Criterion for Plane Mechanisms

The Grubler's criterion applies to mechanisms with an only single degree of freedom joints where the overall movability of the mechanism is unity. Substituting n = 1 and h = 0 in Kutzbach equation, we have

$$1 = 3\,(l - 1) - 2\,j$$

$$\therefore\ 3l - 2j - 4 = 0$$

This equation is known as the Grubler's criterion for plane mechanisms with constrained motion.

A little consideration will show that a plane mechanism with a movability of 1 and only a single degree of freedom joints can't have odd number of links. The simplest possible mechanisms of this type are a four-bar mechanism and a slider-crank mechanism in which l = 4 and j = 4.

1.7 Inversion of Mechanism

When one of the links is fixed in a kinematic chain, it is called a mechanism. So we can obtain as many mechanisms as the number of links in a kinematic chain by fixing, in turn, different links in a kinematic chain. This method of obtaining different mechanisms by fixing different links in a kinematic chain is known as inversion of the mechanism.

It may be noted that the relative motions between the various links are not changed in any manner through the process of inversion, but their absolute motions (those measured with respect to the fixed link) may be changed drastically.

1.7.1 Types of Kinematic Chains

The most important kinematic chains are those which consist of four lower pairs, each pair being a sliding pair or a turning pair. The following three types of kinematic chains with four lower pairs are important:

1. Four bar chain or quadric cyclic chain

2. Single slider crank chain

3. Double slider crank chain

1.8 Four Bar Chain or Quadric Cycle Chain

The simplest and the basic kinematic chain is a four-bar chain or quadric cycle chain. It consists of four links, each of them forms a turning pair at A, B, C and D. The four links may be of different lengths. According to Grashof 's law for a four-bar mechanism, the sum of the shortest and longest link lengths should not be greater than the sum of the remaining two link lengths if there is to be continuous relative motion between the two links.

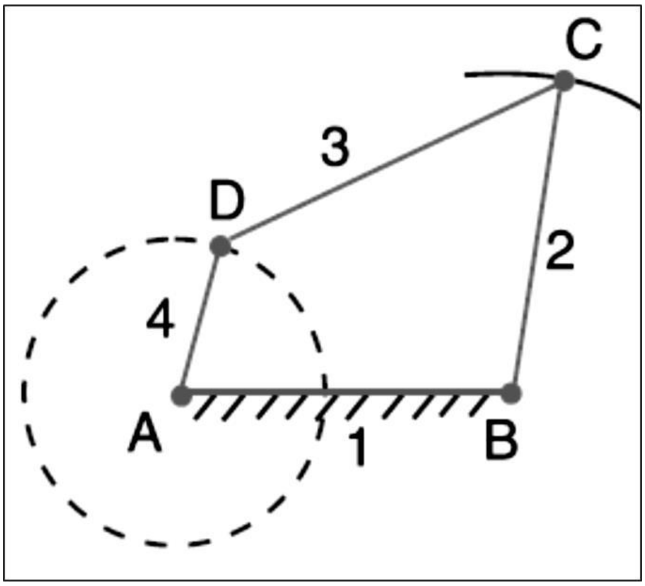


Fig.1.18 - Four Bar Chain

In a four-bar chain, one of the links, in particular the shortest link, will make a complete revolution relative to the other three links, if it satisfies the Grashof 's law. Such a link is known as crank or driver.

AD (link 4) is a crank. The link **BC** (link 2) which makes a partial rotation or oscillates is known as lever or rocker or follower and the link **CD** (link 3) which connects the crank and lever is called connecting rod or coupler. The fixed link **AB** (link 1) is known as the frame of the mechanism.

1.8.1 Inversions of Four Bar Chain

Though there are many inversions of the four-bar chain, yet the following are important:

1.8.1.1 Beam engine (crank and lever mechanism)

A part of the mechanism of a beam engine (also known as crank and lever mechanism) which consists of four links.

In this mechanism, when the crank rotates about the fixed centre A, the lever oscillates about a fixed centre D. The end E of the lever CDE is connected to a piston rod which reciprocates due to the rotation of the crank. In other words, the purpose of this mechanism is to convert rotary motion into reciprocating motion.

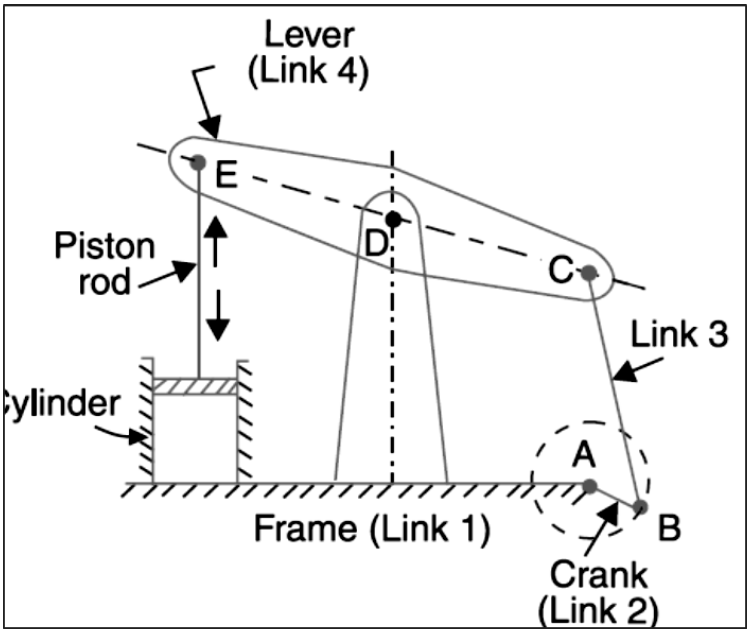


Fig.1.19 - Beam engine

1.8.1.2 Coupling rod of a locomotive (Double crank mechanism)

The mechanism of a coupling rod of a locomotive (also known as a double crank mechanism) which consists of four links.

In this mechanism, the links AD and BC (having equal length) act as cranks and are connected to the respective wheels. The link CD acts as a coupling rod and the link AB is fixed in order to maintain a constant centre to centre distance between them.

This mechanism is meant for transmitting rotary motion from one wheel to the other wheel.

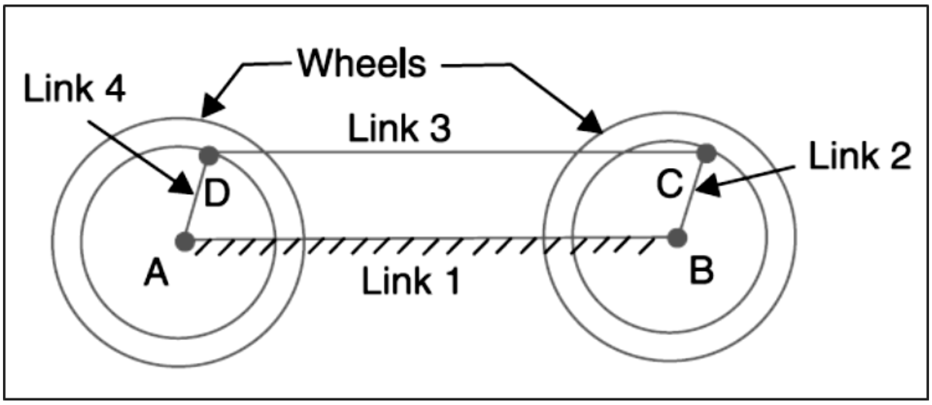


Fig.1.20 - Coupling rod of a locomotive

1.8.1.3 Watt's indicator mechanism (Double lever mechanism)

A Watt's indicator mechanism (also known as Watt's straight-line mechanism or double lever mechanism) which consists of four links.

The four links are fixed link at A, link AC, link CE and link BFD. It may be noted that BF and FD form one link because these two parts have no relative motion between them. The links CE and BFD act as levers. The displacement of the link BFD is directly proportional to the pressure of gas or steam which acts on the indicator plunger.

On any small displacement of the mechanism, the tracing point E at the end of the link CE traces out approximately a straight line.

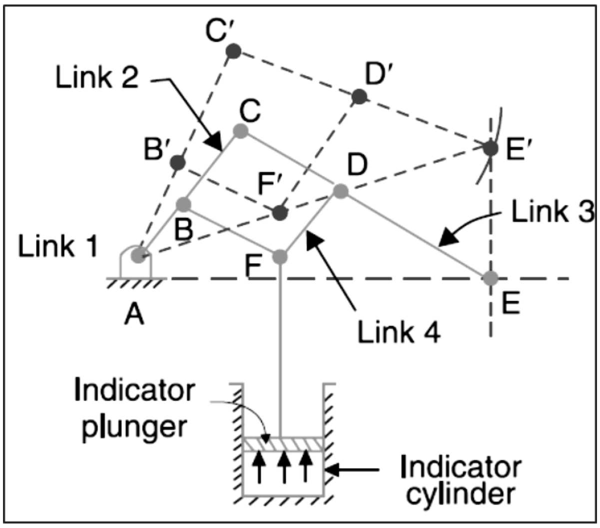


Fig.1.21 - Watt's straight-line mechanism

1.9 Single Slider Crank Chain

A single slider crank chain is a modification of the basic four-bar chain. It consists of one sliding pair and three turning pairs. It is usually, found in reciprocating steam engine mechanism. This type of mechanism converts rotary motion into reciprocating motion and vice versa.

In a single slider crank chain, the links 1 and 2, links 2 and 3, and links 3 and 4 form three turning pairs while the links 4 and 1 form a sliding pair. The **link 1** corresponds to the frame of the engine, which is fixed. The **link 2** corresponds to the crank; **link 3** corresponds to the connecting rod and **link 4** corresponds to cross-head. As the crank rotates, the cross-head reciprocates in the guides and thus the piston reciprocates in the cylinder.

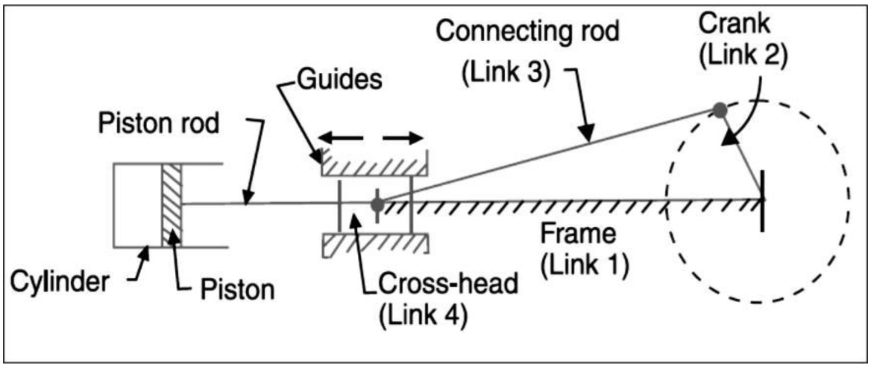


Fig.1.22 - Single slider crank chain

1.9.1 Inversions of Single Slider Crank Chain

Following are the Inversions of Single Slider Crank Chain:

1.9.1.1 Pendulum pump or Bull engine

In this mechanism, the inversion is **obtained by fixing the cylinder or link 4** (i.e. sliding pair).

In this case, when the crank (link 2) rotates, the connecting rod (link 3) oscillates about a pin pivoted to the fixed link 4 at A and the piston attached to the piston rod (link 1) reciprocates.

- **Application**: The duplex pump which is used to supply feed water to boilers have two pistons attached to link 1.

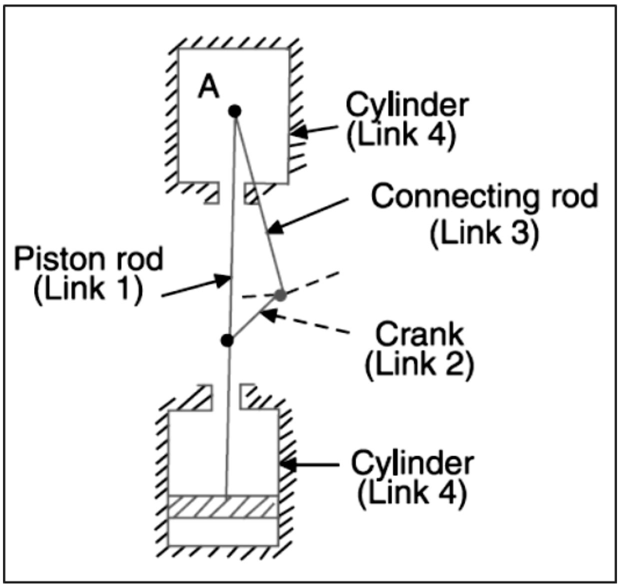


Fig.1.23 - Pendulum pump

1.9.1.2 Oscillating cylinder engine

The arrangement of the oscillating cylinder engine mechanism is <u>used to convert reciprocating motion into rotary motion</u>. In this mechanism, **the link 3 forming the turning pair is fixed**.

Link 3 corresponds to the connecting rod of a reciprocating steam engine mechanism. When the crank (link 2) rotates, the piston attached to piston rod (link 1) reciprocates and the cylinder (link 4) oscillates about a pin pivoted to the fixed link at A.

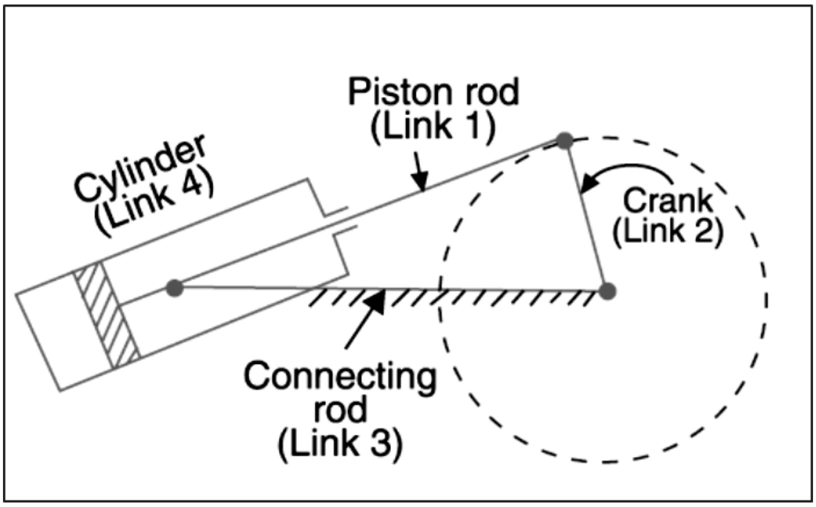


Fig.1.24 - Oscillating cylinder engine

1.9.1.3 Rotary internal combustion engine or Gnome engine

It consists of seven cylinders in one plane and all revolves about fixed centre D, while the **crank (link 2) is fixed**. In this mechanism, when the connecting rod (link 4) rotates, the piston (link 3) reciprocates inside the cylinders forming link 1.

- **Application**: Sometimes back, rotary internal combustion engines were used in aviation. But nowadays gas turbines are used in its place.

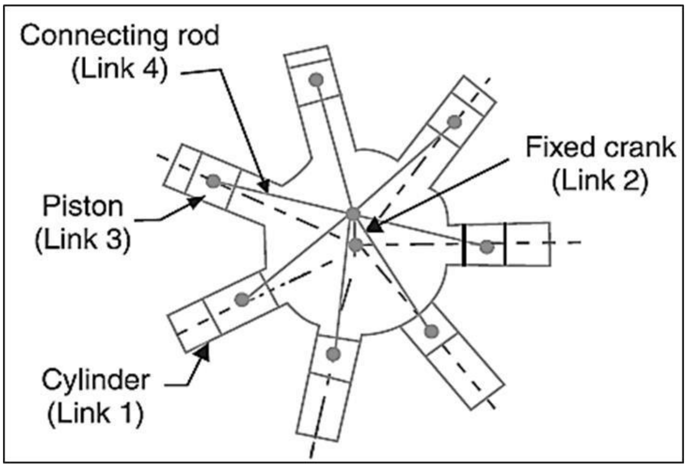


Fig.1.25 - Rotary internal combustion engine

1.9.1.4 Crank and slotted lever quick return motion mechanism

In this mechanism, the link AC (i.e. link 3) forming the turning pair is fixed. Link 3 corresponds to the connecting rod of a reciprocating steam engine.

The driving crank CB revolves with uniform angular speed about the fixed centre C. A sliding block attached to the crank pin at B slides along the slotted bar AP and thus causes AP to oscillate about the pivoted point A. A short link PR transmits the motion from AP to the ram which carries the tool and reciprocates along the line of stroke R1 R2. The line of stroke of the ram (i.e. R1 R2) is perpendicular to AC produced.

- **Application**: This mechanism is mostly used in shaping machines

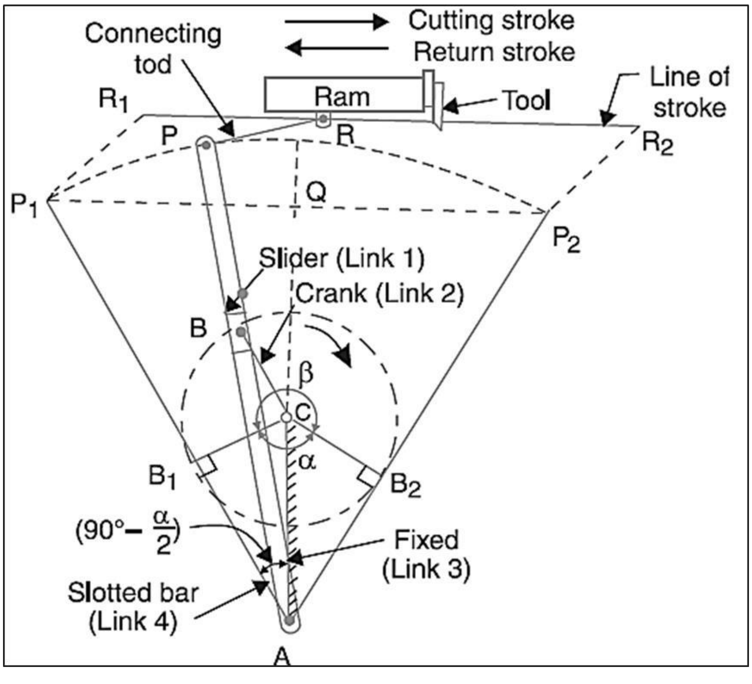


Fig.1.26 - Crank and slotted lever quick return motion mechanism

1.9.1.5 Whitworth quick return motion mechanism

In this mechanism, the link CD (link 2) forming the turning pair is fixed. Link 2 corresponds to a crank in a reciprocating steam engine.

The driving crank CA (link 3) rotates at a uniform angular speed. The slider (link 4) attached to the crank pin at A slides along the slotted bar PA (link 1) which oscillates at a pivoted point D. The connecting rod PR carries the ram at R to which a cutting tool is fixed. The motion of the tool is constrained along the line RD produced, i.e. along a line passing through D and perpendicular to CD.

- **Application**: This mechanism is mostly used in and slotting machines

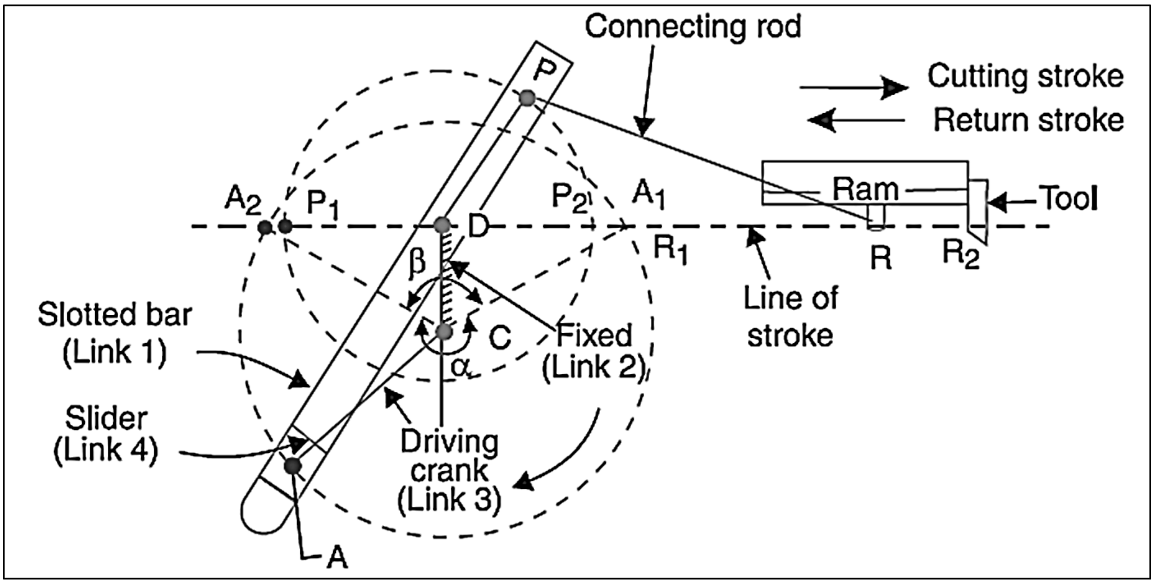


Fig.1.27 - Whitworth quick return motion mechanism

*Table **1.3** - Inversions of single slider crank chain and their application*

Mechanism	Application
Pendulum pump or Bull engine	Duplex pump - supply feed water to boilers
Oscillating cylinder engine	Reciprocating steam engine
Rotary internal combustion engine or Gnome engine	Used in aviation
Crank and slotted lever quick return motion mechanism	Mostly used in shaping machines
Whitworth quick return motion mechanism	Mostly used in slotting machines

1.10 Double Slider Crank Chain

A kinematic chain which consists of two turning pairs and two sliding pairs is known as a double slider crank chain.

1.10.1 Inversions of Double Slider Crank Chain

Following are the Inversions of Double Slider Crank Chain:

1.10.1.1 Elliptical trammels

This inversion is **obtained by fixing the slotted plate (link 4)**.

The fixed plate or link 4 has two straight grooves cut in it, at right angles to each other. Link 1 and link 3, are known as sliders and form sliding pairs with link 4.

The link AB (link 2) is a bar which forms turning pair with links 1 and 3. When the links 1 and 3 slides along their respective grooves, any point on link 2 such as P traces out an ellipse on the surface of link 4.

- **Application**: It is an instrument used for drawing ellipses

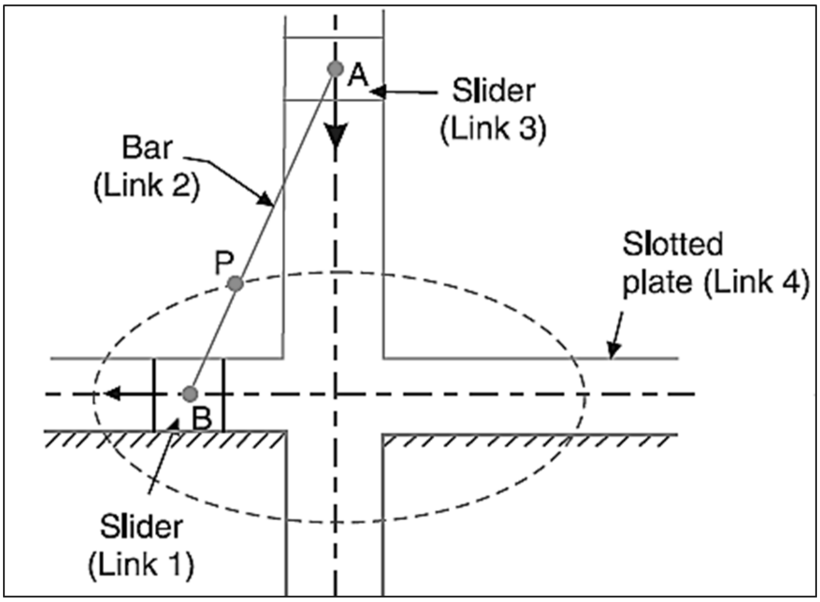


Fig.1.28 - Elliptical trammels

1.10.1.2 Scotch yoke mechanism

The inversion is obtained by fixing either the link 1 or link 3. **Link 1 is fixed**.

In this mechanism, when the link 2 (which corresponds to crank) rotates about B as centre, the link 4 (which corresponds to a frame) reciprocates. The fixed link 1 guides the frame.

- **Application**: This mechanism is used for converting rotary motion into a reciprocating motion

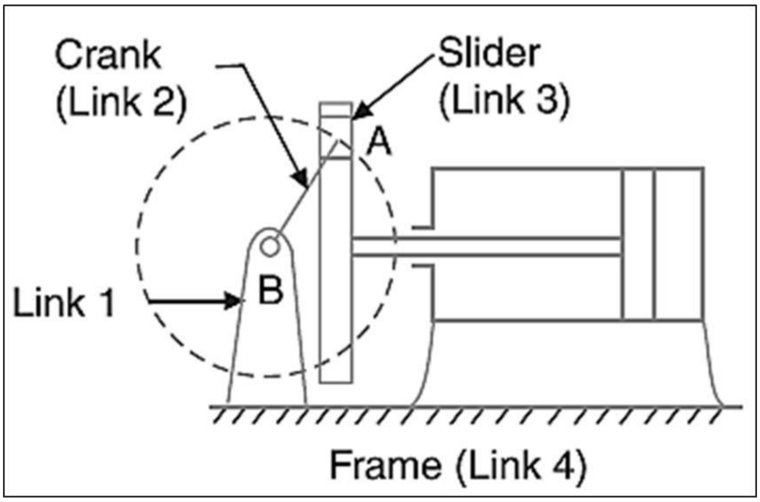


Fig.1.29 - Scotch yoke mechanism

1.10.1.3 Oldham's coupling

The shafts are coupled in such a way that if one shaft rotates, the other shaft also rotates at the same speed. This inversion is **obtained by fixing the link 2**. The shafts to be connected have two flanges (link 1 and link 3) rigidly fastened at their ends by forging.

- **Application**: It is used for connecting two parallel shafts whose axes are at a small distance apart

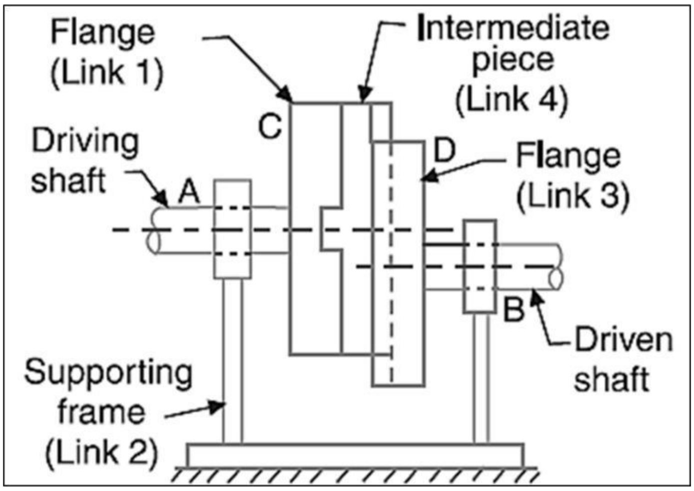


Fig.1.30 - Oldham's coupling

1.11 Straight Line Mechanisms

One of the most common forms of the constraint mechanisms is that it <u>permits only the relative motion of an oscillatory nature along a straight line</u>. The mechanisms used for this purpose are called straight- line mechanisms. These mechanisms are of the following two types:

1. in which only turning pairs are used, and

2. in which one sliding pair is used.

These two types of mechanisms may produce exact straight-line motion or approximate straight-line motion.

The principle adopted for a mathematically correct or exact straight-line motion is described in following figure. Let O be a point on the circumference of a circle of diameter OP.

Let OA be any chord and B is a point on OA produced, such that OA × OB = constant. Then the locus of a point B will be a straight line perpendicular to the diameter OP.

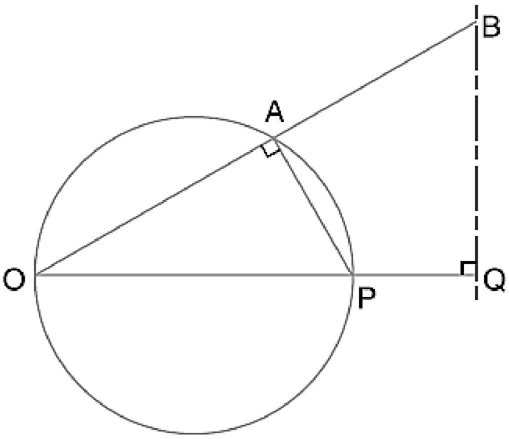


Fig.1.31 - Exact straight line motion mechanism

1.11.1 Exact Straight Line Motion Mechanisms

Following are some important Exact Straight Line Motion Mechanisms:

1.11.1.1 Peaucellier mechanism

It consists of a fixed link OO_1 and the other straight links O_1A, OC, OD, AD, DB, BC and CA are connected by turning pairs at their intersections. The pin at A is constrained to move along the circumference of a circle with the fixed diameter OP, by means of the link O_1A.

AC = CB = BD = DA; OC = OD; and $OO_1 = O_1A$

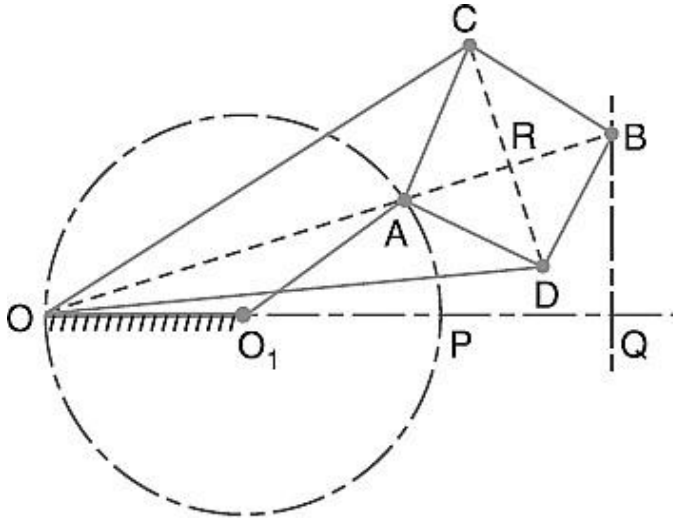


Fig.1.32 - Peaucellier mechanism

1.11.1.2 Hart's mechanism

This mechanism requires only six links as compared with the eight links required by the Peaucellier mechanism.

It consists of a fixed link OO_1 and other straight links O_1A, FC, CD, DE and EF are connected by turning pairs at their points of intersection.

FC = DE; CD = EF;

The points O, A and B divide the links FC, CD and EF in the same ratio. A little consideration will show that BOCE is a trapezium and OA and OB are respectively parallel to FD and CE. Hence OAB is a straight line.

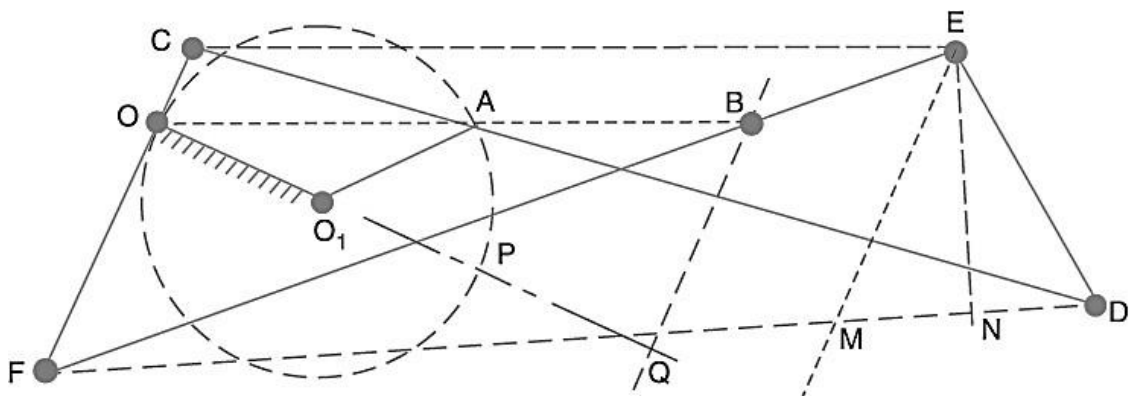


Fig.1.33 - Hart's mechanism

1.11.1.3 Scott Russell's Mechanism

It consists of a fixed member and moving member P of a sliding pair. The straight link PAQ is connected by turning pairs to the link OA and the link P. The link OA rotates about O.

A little consideration will show that the mechanism OAP is the same as that of the reciprocating engine mechanism in which OA is the crank and PA is the connecting rod.

In this mechanism, the straight-line motion is not generated but it is merely copied.

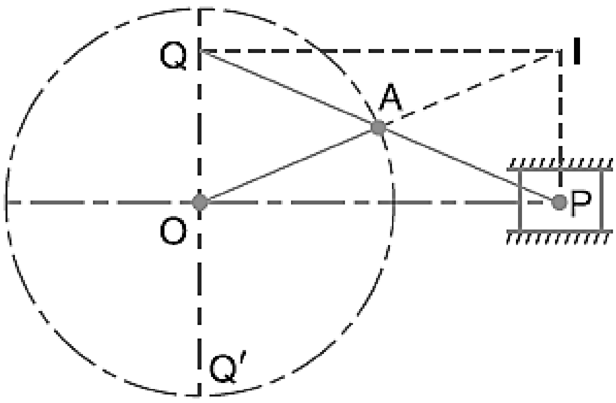


Fig.1.34 - Scott Russell's Mechanism

1.11.2 Approximate Straight Line Motion Mechanisms

The approximate straight-line motion mechanisms are the modifications of the four-bar chain mechanisms. Following are the important mechanisms to give approximate straight-line motion.

1.11.2.1 Watt's mechanism

It is a crossed four-bar chain mechanism and was used by Watt for his early steam engines to guide the piston rod in a cylinder to have an approximately straight-line motion.

$OBAO_1$ is a crossed four-bar chain in which O and O_1 are fixed. In the mean position of the mechanism, links OB and O_1A are parallel and the coupling rod AB is perpendicular to O_1A and OB.

The tracing point P traces out an approximately straight line over certain positions of its movement, if

$$PB/PA = O_1A/OB$$

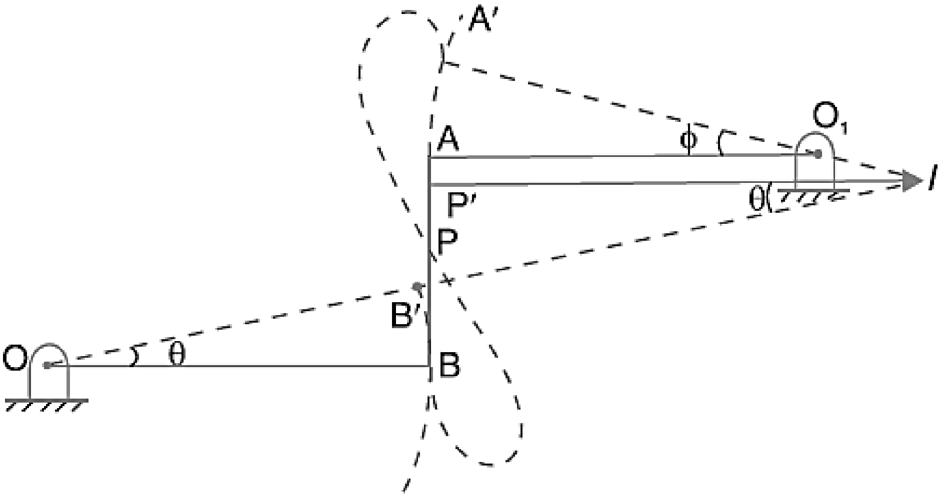


Fig.1.35 - Watt's mechanism

1.11.2.2 *Modified Scott-Russel mechanism*

This mechanism is similar to Scott-Russel mechanism, but in this case, AP is not equal to AQ and the points P and Q are constrained to move in the horizontal and vertical directions.

A little consideration will show that it forms an elliptical trammel so that any point A on PQ traces an ellipse with semi-major axis AQ and semi-minor axis AP.

If the point A moves in a circle, then for point Q to move along an approximate straight line, the length OA must be equal $(AP)^2 / AQ$

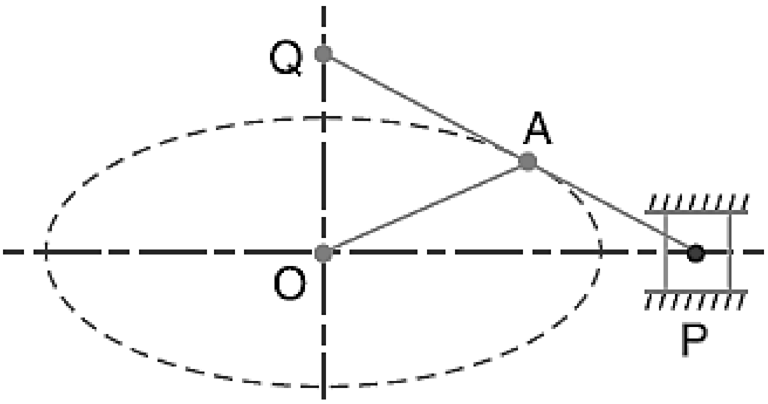


Fig.1.36 - Modified Scott-Russel mechanism

1.11.2.3 *Grasshopper mechanism*

This mechanism is a modification of modified Scott-Russel's mechanism with the difference that the point P does not slide along a straight line but moves in a circular arc with centre O. It is a four-bar mechanism and all the pairs are turning pairs

In this mechanism, the centres O and O_1 are fixed. The link OA oscillates about O through an angle AOA_1 which causes the pin P to move along a circular arc with O_1 as centre and O_1P as radius.

The Grasshopper mechanism was <u>used in early days as an engine mechanism which gave long stroke with a very short crank</u>.

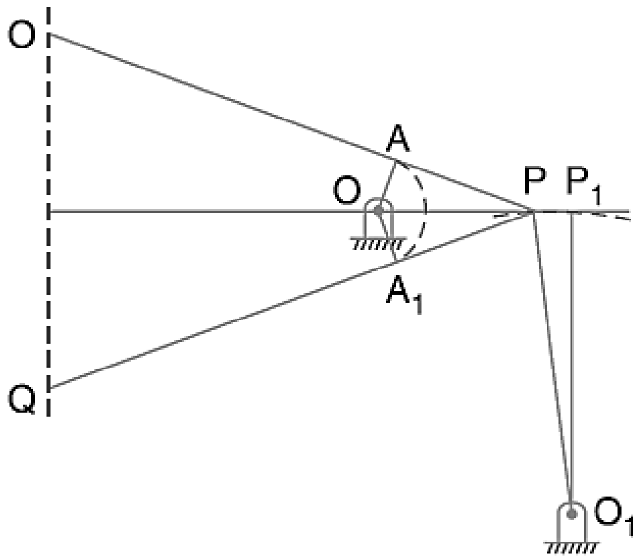


Fig.1.37 - Grasshopper mechanism

1.11.2.4 Tchebicheff's mechanism

It is a four-bar mechanism in which the crossed links OA and O_1B are of equal length. The point P, which is the mid-point of AB traces out an approximately straight line parallel to OO_1.

The proportions of the links are, usually, such that point P is exactly above O or O_1 in the extreme positions of the mechanism i.e. when BA lies along OA or when BA lies along BO_1.

It may be noted that the point P will lie on a straight line parallel to OO_1, in the two extreme positions and in the mid position, if the lengths of the links are in proportions **AB: OO_1: OA = 1: 2: 2.5**.

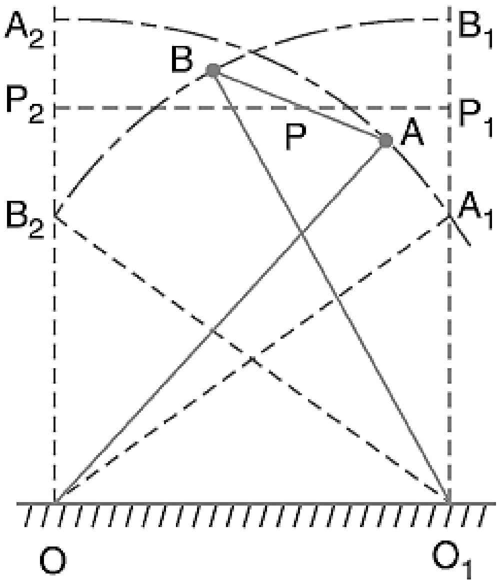


Fig.1.38 - Tchebicheff's mechanism

1.11.2.5 Roberts mechanism

It is also a four-bar chain mechanism, which, in its mean position, has the form of a trapezium. The links OA and O_1B are of equal length and OO1 is fixed.

A bar PQ is rigidly attached to the link AB at its middle point P. A little consideration will show that if the mechanism is displaced as shown by the dotted lines, the point Q will trace out an approximately straight line.

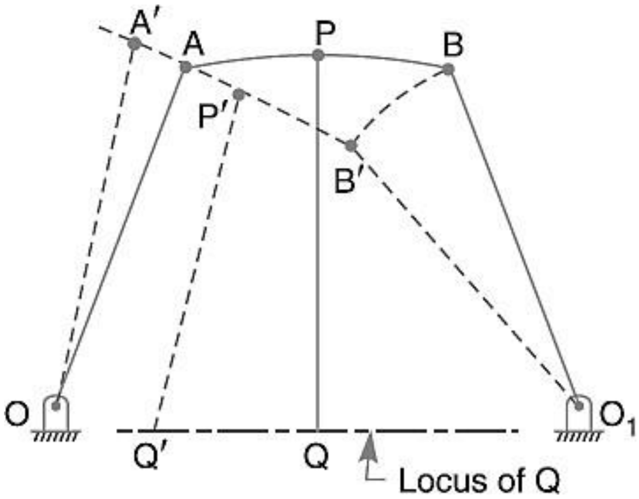


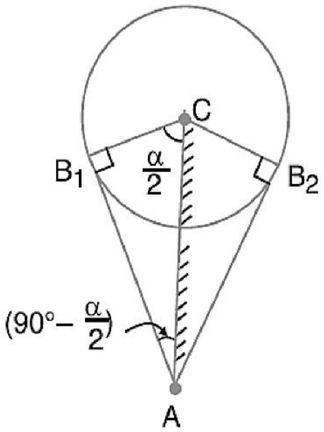
Fig.1.39 - Roberts mechanism

1.12 Problems

Ex. 1.1 **[GTU; June-2016; 7 Marks] [GTU; Jan.-2016; 4 Marks]**

A crank and slotted lever mechanism used in a shaper has a centre distance of 300 mm between the centre of oscillation of the slotted lever and the centre of rotation of the crank. The radius of the crank is 120 mm. Find the ratio of the time of cutting to the time of return stroke.

Solution:



Given Data:

$AC = 300\ mm$

$CB_1 = 120$ mm

To be Calculated:

The ratio of time of cutting to time of return stroke

- The extreme positions of the crank are shown in Figure. We know that,

$$Sin\ \angle CAB_1 = Sin\left(90° - \frac{\alpha}{2}\right) = \frac{CB_1}{AC} = \frac{120}{300} = 0.4$$

$$\therefore\ \angle CAB_1 = \left(90° - \frac{\alpha}{2}\right) = \sin^{-1}(0.4) = 23.6°$$

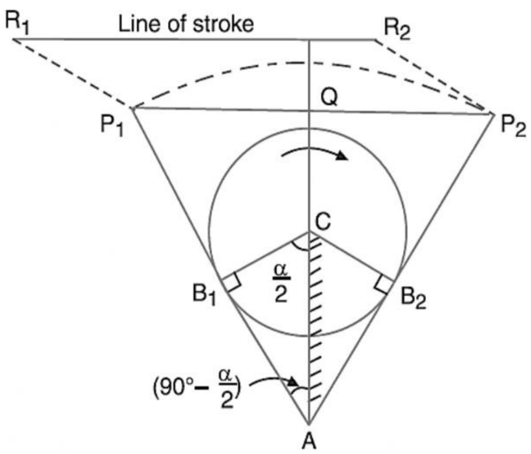
$$\therefore\ \frac{\alpha}{2} = (90° - 23.6°) = 66.4°$$

$$\therefore\ \alpha = 2 \times 66.4° = 132.8°$$

- $\dfrac{\textit{Time of cutting stroke}}{\textbf{Time of return stroke}} = \dfrac{360° - \alpha}{\alpha} = \dfrac{360° - 132.8°}{132.8°}$ = **1.72**

Ex. 1.2 In a crank and slotted lever quick return motion mechanism, the distance between the fixed centres is 240 mm and the length of the driving crank is 120 mm. Find the time ratio of cutting stroke to the return stroke. If the length of the slotted bar is 450 mm, find the length of the stroke if the line of stroke passes through the extreme positions of the free end of the lever.

Solution:



Given Data:

$AC = 240\ mm$

$CB_1 = 120$ mm

$AP_1 = 450$ mm

To be Calculated:

a) The ratio of time of cutting stroke to time of return stroke
b) Length of stroke

- The extreme positions of the crank are shown in Figure. We know that,

$$Sin\ \angle CAB_1 = Sin\ \left(90° - \frac{\alpha}{2}\right) = \frac{CB_1}{AC} = \frac{120}{240} = 0.5$$

$$\therefore\ \angle CAB_1 = \left(90° - \frac{\alpha}{2}\right) = \sin^{-1}(0.5) = 30°$$

$$\therefore\ \frac{\alpha}{2} = (90° - 30°) = 60°$$

$$\therefore\ \alpha = 2 \times 60° = 120°$$

- $\dfrac{\boldsymbol{Time\ of\ cutting\ stroke}}{\textbf{Time of return stroke}} = \dfrac{360° - \alpha}{\alpha} = \dfrac{360° - 120°}{120°}$ = **2**
- Length of the stroke,

$$R_1R_2 = P_1P_2 = 2P_1Q = 2 \times AP_1 \times Sin\ \left(90° - \frac{\alpha}{2}\right)$$

$$\therefore\ \textbf{Length of Stroke} = 2 \times 450 \times Sin\ (90° - 60°) = 900 \times 0.5 = \boldsymbol{450\ mm}$$

2.1 Introduction

- The turning moment diagram (also known as *crank-effort diagram*) is the graphical representation of the turning moment or crank-effort for various positions of the crank (A plot of T vs θ). It is plotted on cartesian co-ordinates, in which the turning moment is taken as the ordinate and crank angle as abscissa. The inertia effect of the connecting rod is usually ignored while drawing these diagrams, but can be taken into account if desired.
- During one revolution of the crankshaft of a steam engine or I.C. engine, the torque on it varies and is given by,

$$T = F_t \times r = F_P \times r\left(\sin\theta + \frac{\sin 2\theta}{2\sqrt{n^2 - \sin^2\theta}}\right)$$

Where, F_t = Tangential force,

r = Radius of crank,

θ = Angle turned by the crank from inner dead centre.

F_P = Axial force acting on piston or Piston effort,

n = Ratio of the connecting rod length and radius of crank

2.2 Turning Moment Diagrams

(1) Single Cylinder Double Acting Steam Engine

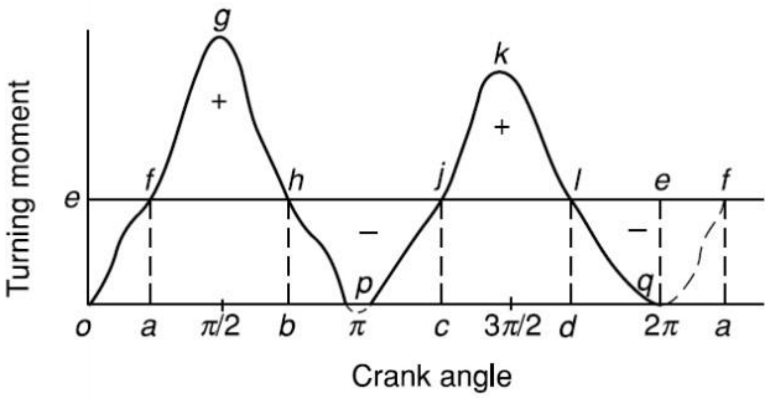


Figure 2.1 Turning moment diagram for a single cylinder, double acting steam engine.

- Figure 2.1 shows a turning-moment diagram for a single-cylinder double-acting steam engine. The crank angle θ is represented along x-axis & the turning-moment along y-axis. It can be observed that during the outstroke (ogp) the turning moment is maximum when the crank angle is a little less than 90^0 and zero with the crank angle is zero and 180^0. A somewhat similar turning moment diagram is obtained during the instroke (pkg).
- Note that the area of the turning-moment diagram is proportional to the work done per revolution as the work is the product of the turning-moment diagram and the angle turned.
- The mean torque against which the engine works is given by

$$oe = \frac{Area\ ogpkp}{2\pi}$$

- Where oe is the mean torque & is the mean height of the turning-moment diagram.
- When the crank turns from the angle oa to ob, the work done bt the engine is represented by the area afghb. But the work done against the resisting torque is represented by afhb. Thus, the engine has work done more work than what has been taken from it. The excess work is represented by the area fgh. This excess work increases the speed of the engine & is stored in the flywheel.
- During the crank travel from the ob or oc, the work needed for the external resistance is proportional to bhjc whereas the work produced by the engine is represented by the area under hpj. Thus, during this period, more work has been taken from the engine that is produced. The loss is made up by the flywheel which gives up some of its energy& the speed decreases during this period.
- Similarly, during the period of crank travel from oc to od, excess work is again developed and is stored in the flywheel & the speed of the engine increases. During the crank travel from od to oa, the loss of work is made up by the flywheel and the speed again decreases.
- The areas fgh, hpj, jkl&lqf represent fluctuations of energy of the flywheel. When the crank is at b, the flywheel has absorbed energy while the crank has moved from a to b & thereby, the speed of the engine is maximum. At c, the flywheel has given out energy while the crank has moved from b to c & thus the engine has a maximum speed. Similarly, the engine speed is again maximum at d & minimum at a. Thus, there are two maximum & two minimum speeds for the turning-moment diagram.
- The greatest speed is the greater of the two maximum speeds & the least speed is the lesser of the two minimum speeds. The difference between the greatest & the least speeds of the engine over one revolution is known as fluctuation of speed.

(2) Single cylinder four stroke engine

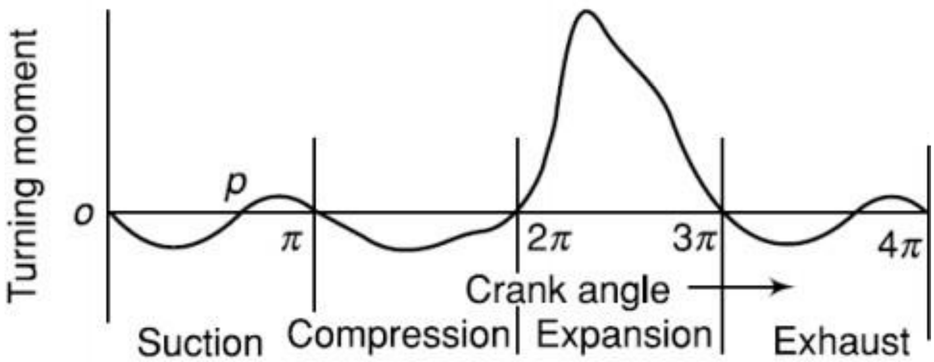


Figure 2.2 Turning moment diagram for a four stroke cycle internal combustion engine.

- A turning moment diagram for a four stroke cycle internal combustion engine is shown in Figure 2.2.
- We know that in a four stroke cycle internal combustion engine, there is one working stroke after the crank has turned through two revolutions, i.e. 720° (or 4π radians).
- Since the pressure inside the engine cylinder is less than the atmospheric pressure during the suction stroke, therefore a negative loop is formed as shown in Figure 2.2.
- During the compression stroke, the work is done on the gases, therefore a higher negative loop is obtained.
- During the expansion or working stroke, the fuel burns and the gases expand, therefore a large positive loop is obtained. In this stroke, the work is done by the gases.
- During exhaust stroke, the work is done on the gases, therefore a negative loop is formed. It may be noted that the effect of the inertia forces on the piston is taken into account in Figure 2.2.

(3) Multi-Cylinder Engines

- The turning moment diagram for a single-cylinder engine varies considerably and a greater variation of the same is observed in case of a four-stroke, single-cylinder engine.
- For engines with more than one cylinder, the total crankshaft torque at any instant is given by the sum of the torque developed by each cylinder at the instant. For example, if an engine has two cylinders with cranks at 90^0, the resultant turning moment diagram has a less variation than that for a single cylinder.
- In a three-cylinder engine having its crank at 120^0, the variation is still less.
- Figure 2.3 shows the turning moment diagram for a multi-cylinder engine. The mean torque line ab intersects the turning moment curve at c, d, e, f, g and h.
- The area under the wavy curve is equal to the area oabk. The speed of the engine will be maximum when the crank positions correspond to d, f and h, and minimum corresponding to c, e and g.

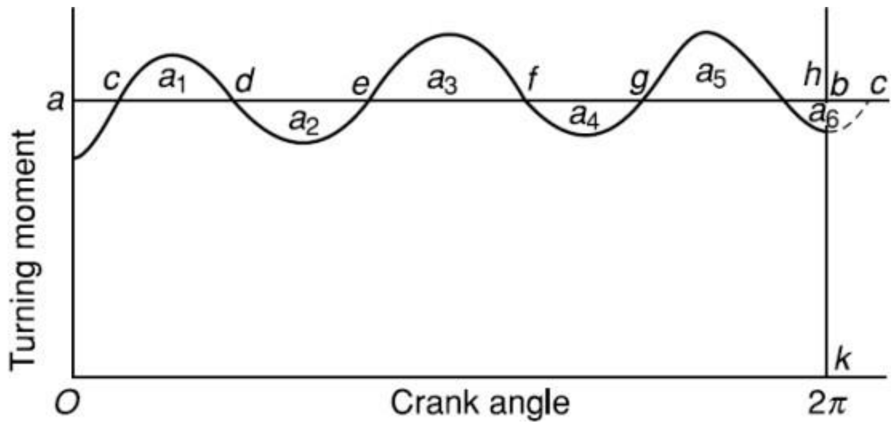


Figure 2.3. Turning moment diagram for a multi-cylinder engine.

2.3 Fluctuation of Energy

– Let a_1, a_3, and a_5 be the areas in work units of the portions above the mean torque ab of the turning moment diagram (Figure 2.4). These areas represent quantities of energies added to flywheel. Similarly, areas a_2, a_4 and a_6 below ab represent quantities of energies taken from the flywheel.

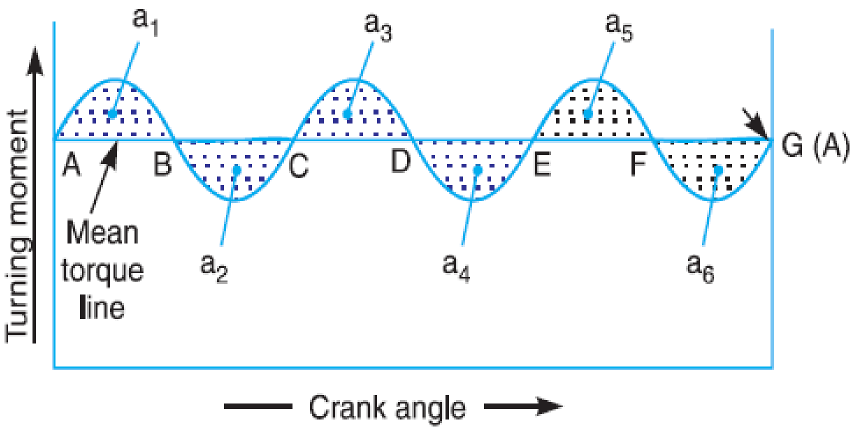


Figure 2.4. Determination of maximum fluctuation of energy.

– The energies of the flywheel corresponding to positions of the crank are as follows:
Let the energy in the flywheel at A = E,
then from Figure 2.4, we have
Energy at B = $E + a_1$
Energy at C = $E + a_1 - a_2$
Energy at D = $E + a_1 - a_2 + a_3$
Energy at E = $E + a_1 - a_2 + a_3 - a_4$
Energy at F = $E + a_1 - a_2 + a_3 - a_4 + a_5$
Energy at G = $E + a_1 - a_2 + a_3 - a_4 + a_5 - a_6$
= Energy at A (i.e. cycle repeats after G)

– Let us now suppose that the greatest of these energies is at B and least at E.
Therefore,
Maximum energy in flywheel = $E + a_1$
Minimum energy in the flywheel = $E + a_1 - a_2 + a_3 - a_4$

– The greatest of these energies is the maximum kinetic energy of the flywheel and for the corresponding crank position, the speed is maximum.

– The least of these energies is the least kinetic energy of the flywheel and for the corresponding crank position, the speed is minimum.

– The different between the maximum and minimum kinetic energies of the flywheel is known as the maximum fluctuation of energy

Maximum fluctuation of energy,
ΔE = Maximum energy – Minimum energy
$= (E + a_1) - (E + a_1 - a_2 + a_3 - a_4) = a_2 - a_3 + a_4$

2.4 Coefficient of Fluctuation of Energy

- It may be defined as the ratio of the maximum fluctuation of energy to the work done per cycle. Mathematically, coefficient of fluctuation of energy,

$$C_E = \frac{\text{Maximum fluctuation of energy}}{\text{Workdone per cycle}}$$

- The work done per cycle (in N-m or joules) may be obtained by using the following two relations:

1. Work done per cycle = Tmean × θ

   where, Tmean = Mean torque, and

   θ = Angle turned (in radians), in one revolution.

   = 2π, in case of steam engine and two stroke internal combustion engines

   = 4π, in case of four stroke internal combustion engines.

- The mean torque (Tmean) in N-m may be obtained by using the following relation:

$$T_{mean} = \frac{P \times 60}{2\pi N} = \frac{P}{\omega}$$

where, P = Power transmitted in watts,

N = Speed in r.p.m., and

ω = Angular speed in rad/s = 2πN/60

2. The work done per cycle may also be obtained by using the following relation :

$$Workdone\ per\ cycle = \frac{P \times 60}{n}$$

where, n = Number of working strokes per minute,

= N, in case of steam engines and two stroke internal combustion engines,

= N /2, in case of four stroke internal combustion engines.

- The following table shows the values of coefficient of fluctuation of energy for steam engines and internal combustion engines.

2.5 Coefficient of Fluctuation of Speed

- The difference between the maximum and minimum speeds during a cycle is called the maximum fluctuation of speed. The ratio of the maximum fluctuation of speed to the mean speed is called the coefficient of fluctuation of speed.

  Let N_1 and N_2 = Maximum and minimum speeds in r.p.m. during the cycle, and

$$N = \text{Mean speed in r.p.m.} = \frac{N_1 + N_2}{2}$$

Coefficient of fluctuation of speed,

$$C_S = \frac{N_1 - N_2}{2} = \frac{2(N_1 - N_2)}{N_1 + N_2}$$

$$= \frac{\omega_1 - \omega_2}{\omega} = \frac{2(\omega_1 - \omega_2)}{\omega_1 + \omega_2} \qquad \text{(In terms of angular speeds)}$$

$$= \frac{v_1 - v_2}{v} = \frac{2(v_1 - v_2)}{v_1 + v_2} \qquad \text{(In terms of linear speeds)}$$

- The coefficient of fluctuation of speed is a limiting factor in the design of flywheel. It varies depending upon the nature of service to which the flywheel is employed.
- The reciprocal of the coefficient of fluctuation of speed is known as *coefficient of steadiness* and is denoted by *m*.

$$m = \frac{1}{C_S} = \frac{N}{N_1 - N_2}$$

2.6 Flywheel

- A flywheel used in machines serves as a reservoir, which stores energy during the period when the supply of energy is more than the requirement, and releases it during the period when the requirement of energy is more than the supply.
- In case of steam engines, internal combustion engines, reciprocating compressors and pumps, the energy is developed during one stroke and the engine is to run for the whole cycle on the energy produced during this one stroke.
- For example, in internal combustion engines, the energy is developed only during expansion or power stroke which is much more than the engine load and no energy is being developed during suction, compression and exhaust strokes in case of four stroke engines and during compression in case of two stroke engines.
- The excess energy developed during power stroke is absorbed by the flywheel and releases it to the crankshaft during other strokes in which no energy is developed, thus rotating the crankshaft at a uniform speed.
- A little consideration will show that when the flywheel absorbs energy, its speed increases and when it releases energy, the speed decreases. Hence a flywheel does not maintain a constant speed, it simply reduces the fluctuation of speed.
- In other words, a flywheel controls the speed variations caused by the fluctuation of the engine turning moment during each cycle of operation.
- In machines where the operation is intermittent like punching machines, shearing machines, rivetting machines, crushers, etc., the flywheel stores energy from the power source during the greater portion of the operating cycle and gives it up during a small period of the cycle. Thus, the energy from the power source to the machines is supplied practically at a constant rate throughout the operation.

2.7 Energy stored in Flywheel

- When a flywheel absorbs energy, its speed increases and when it gives up energy, its speed decreases.

Let *m* = Mass of the flywheel in kg,
k = Radius of gyration of the flywheel in metres,

I = Mass moment of inertia of the flywheel about its axis of rotation in kg-m^2 = $m.k^2$,

N_1 and N_2 = Maximum and minimum speeds during the cycle in r.p.m.,

ω_1 and ω_2 = Maximum and minimum angular speeds during the cycle in rad/s,

$N =$ Mean speed in r.p.m. $= \dfrac{N_1 + N_2}{2}$

$\omega =$ Mean angular speed during the cycle in rad/s $= \dfrac{\omega_1 + \omega_2}{2}$

$C_S =$ Coefficient of fluctuation of speed $= \dfrac{N_1 - N_2}{N}$ or $\dfrac{\omega_1 + \omega_2}{\omega}$

We know that the mean kinetic energy of the flywheel,

$$E = \frac{1}{2} I.\omega^2 = \frac{1}{2} m.k^2.\omega^2$$

As the speed of the flywheel changes from ω_1 to ω_2, the maximum fluctuation of energy,

$$\Delta E = \text{Maximum K.E.} - \text{Minimum K.E.}$$

$$= \frac{1}{2} I.(\omega_1)^2 - \frac{1}{2} I.(\omega_2)^2 = \frac{1}{2} I \left[(\omega_1)^2 - (\omega_2)^2 \right]$$

$$= \frac{1}{2} I (\omega_1 + \omega_2)(\omega_1 - \omega_2) = I.\omega(\omega_1 - \omega_2) \qquad \text{... ....(i)}$$

$$= I.\omega^2 \left(\frac{\omega_1 - \omega_2}{\omega} \right)$$

$$= I.\omega^2 C_S = m.k^2.\omega^2 C_S \qquad \text{.... ....(ii)}$$

$$= 2.E.C_S \qquad \text{…........... (iii)}$$

– The radius of gyration (k) may be taken equal to the mean radius of the rim (R), because the thickness of rim is very small as compared to the diameter of rim. Therefore, substituting $k = R$, in equation (*ii*), we have

$$\Delta E = m.R^2.\omega^2.C_S = m.v^2.C_S$$

v = Mean linear velocity (*i.e.* at the mean radius) in m/s

2.8 Dimensions of Flywheel Rims

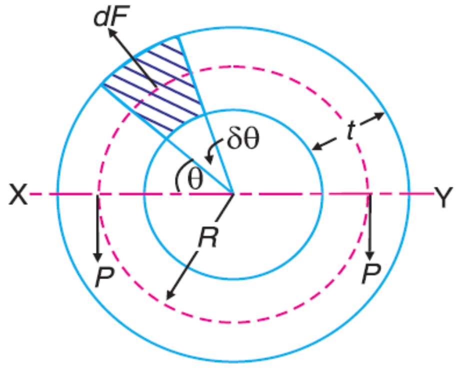


Figure 2.5 Rim of Flywheel

– Consider a rim of the flywheel as shown in Figure 2.5.

Let D = Mean diameter of rim in metres,

R = Mean radius of rim in metres,

A = Cross-sectional area of rim in m^2,

ρ = Density of rim material in kg/m^3,

N = Speed of the flywheel in r.p.m.,

ω = Angular velocity of the flywheel in rad/s,

v = Linear velocity at the mean radius in m/s

= ω .R = π D.N/60, and

σ = Tensile stress or hoop stress in N/m^2 due to the centrifugal force.

– Consider a small element of the rim as shown shaded in Figure 2.5. Let it subtends an angle δθ at the centre of the flywheel.

Volume of the small element = A × R.δθ

∴ Mass of the small element, dm = Density × volume = ρ.A.R.δθ

and centrifugal force on the element, acting radially outwards,

$$dF = dm.\omega 2.R = \rho.A.R^2.\omega^2.\delta\theta$$

Vertical component of dF = dF.sin θ = ρ.A.R2.ω2.δθ.sin θ

Total vertical upward force tending to burst the rim across the diameter X Y.

$$= \rho.A.R^2.\omega^2 \int_0^\pi \sin\theta \; d\theta = \rho.A.R^2.\omega^2 \left[-\cos\theta\right]_0^\pi$$

$$= 2\rho.A.R^2.\omega^2$$

– This vertical upward force will produce tensile stress or hoop stress (also called centrifugal stress or circumferential stress), and it is resisted by 2P, such that

$$2P = 2\,\sigma.A \; . . . \text{(ii)}$$

Equating equations (i) and (ii),

$$2.\rho.A.R^2.\omega^2 = 2\sigma.A$$

or $\sigma = \rho.R^2.\omega^2 = \rho.v^2$ ……. (∵ v = ω.R)

$$v = \sqrt{\frac{\sigma}{\rho}}$$

We know that mass of the rim,

$$m = \text{Volume} \times \text{density} = \pi\, D.A.\rho$$

$$A = \frac{m}{\pi D \rho}$$

– From equations (iii) and (iv), we may find the value of the mean radius and cross-sectional area of the rim.

If the cross-section of the rim is a rectangular, then

$$A = b \times t$$

Where, b = Width of the rim, and

t = Thickness of the rim.

2.9 Flywheel in Punching Press

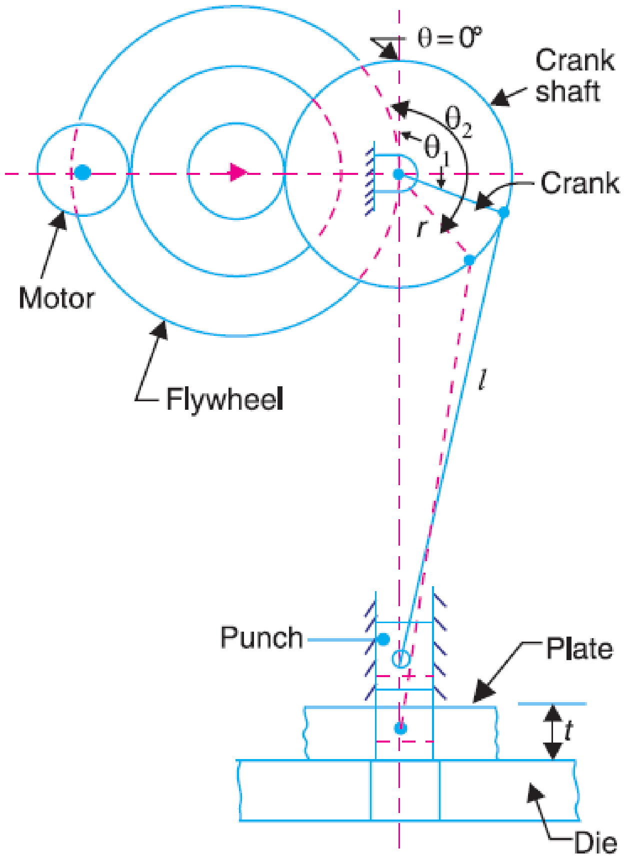


Figure 2.6 Operation of flywheel in a punching press

- The function of a flywheel in an engine is to reduce the fluctuations of speed, when the load on the crankshaft is constant and the input torque varies during the cycle.
- The flywheel can also be used to perform the same function when the torque is constant and the load varies during the cycle. Such an application is found in punching press or in a riveting machine.
- A punching press is shown diagrammatically in Figure 2.6. The crank is driven by a motor which supplies constant torque and the punch is at the position of the slider in a slider-crank mechanism.
- From Figure 2.6, we see that the load acts only during the rotation of the crank from $\theta = \theta_1$ to $\theta = \theta_2$, when the actual punching takes place and the load is zero for the rest of the cycle.
- Unless a flywheel is used, the speed of the crankshaft will increase too much during the rotation of crankshaft will increase too much during the rotation of crank from $\theta = \theta_2$ to $\theta = 2\pi$ or $\theta = 0$ and again from $\theta = 0$ to $\theta = \theta_1$, because there is no load while input energy continues to be supplied.
- On the other hand, the drop in speed of the crankshaft is very large during the rotation of crank from $\theta = \theta_1$ to $\theta = \theta_2$ due to much more load than the energy

supplied. Thus the flywheel has to absorb excess energy available at one stage and has to makeup the deficient energy at the other stage to keep to fluctuations of speed within permissible limits. This is done by choosing the suitable moment of inertia of the flywheel.

– Let E_1 be the energy required for punching a hole. This energy is determined by the size of the hole punched, the thickness of the material and the physical properties of the material.

– Let d_1 = Diameter of the hole punched,
  t_1 = Thickness of the plate, and
  τ_u = Ultimate shear stress for the plate material.

– Maximum shear force required for punching,
  FS = Area sheared × Ultimate shear stress = $\pi\, d_1\, t_1 \tau_u$

– It is assumed that as the hole is punched, the shear force decreases uniformly from maximum value to zero.
  ∴ Work done or energy required for punching a hole,

$$E_1 = \frac{1}{2} \times F_s \times t$$

– Assuming one punching operation per revolution, the energy supplied to the shaft per revolution should also be equal to E_1. The energy supplied by the motor to the crankshaft during actual punching operation,

$$E_2 = E_1 \left(\frac{\theta_2 - \theta_1}{2\pi} \right)$$

– Balance energy required for punching

$$E_1 - E_2 = E_1 - E_1 \left(\frac{\theta_2 - \theta_1}{2\pi} \right) = E_1 \left(1 - \frac{\theta_2 - \theta_1}{2\pi} \right)$$

– This energy is to be supplied by the flywheel by the decrease in its kinetic energy when its speed falls from maximum to minimum. Thus maximum fluctuation of energy,

$$\Delta E = E_1 - E_2 = E_1 \left(1 - \frac{\theta_2 - \theta_1}{2\pi} \right)$$

– The values of θ_1 and θ_2 may be determined only if the crank radius (r), length of connecting rod (l) and the relative position of the job with respect to the crankshaft axis are known. In the absence of relevant data, we assume that

$$\frac{\theta_2 - \theta_1}{2\pi} = \frac{t}{2s} = \frac{t}{4r}$$

where, t = Thickness of the material to be punched,
  s = Stroke of the punch = 2 × Crank radius = 2r.

– By using the suitable relation for the maximum fluctuation of energy (ΔE) as discussed in the previous articles, we can find the mass and size of the flywheel.

Example 2.1: The mass of flywheel of an engine is 6.5 tonnes and the radius of gyration is 1.8 metres. It is found from the turning moment diagram that the fluctuation of energy is 56 kN-m. If the mean speed of the engine is 120 r.p.m., find maximum and minimum speeds.

m = 6.5 tonnes = 6500 kg　　　　k = 1.8 m

ΔE = 56 KN.m　　　　N = 120 r.p.m

$$\Delta E = I.\omega(\omega_1 - \omega_2)$$
$$= I \times \frac{2\pi N}{60} \times \left[\frac{2\pi N_1}{60} - \frac{2\pi N_2}{60}\right]$$
$$= m \times k^2 \times \frac{\pi^2}{900} \times N \times (N_1 - N_2)$$
$$56 \times 10^3 = 6500 \times (1.8)^2 \times \frac{\pi^2}{900} \times 120 \times (N_1 - N_2)$$

N1 – N2 = 2 rpm ....(i)

$$N = \frac{N_1 + N_2}{2}$$
$$120 = \frac{N_1 + N_2}{2}$$
$$N_1 + N_2 = 240 \text{.. (ii)}$$

From equation (i) and (ii)

N_1 = 121 rpm　and　N_2 = 119 rpm

Example 2.2: The flywheel of a steam engine has a radius of gyration of 1 m and mass 2500 kg. The starting torque of the steam engine is 1500 N-m and may be assumed constant. Determine: 1. the angular acceleration of the flywheel, and 2. the kinetic energy of the flywheel after 10 seconds from the start.

k = 1 m　　　　m = 2500 kg ;

T = 1500 N-m　　　　t = 10 sec

Mass moment of inertia of the flywheel,

$I = m.k^2 = 2500 \times 12 = 2500$ kg-m²

∴ Starting torque of the engine (T),

$1500 = I.\alpha = 2500 \times \alpha$

Angular acceleration of the flywheel, $\alpha = 1500 / 2500 = 0.6$ rad /s²

Let ω_1 = Angular speed at rest = 0

ω_2 = Angular speed after 10 seconds, and

t = Time in seconds.

We know that $\omega_2 = \omega_1 + \alpha t = 0 + 0.6 \times 10 = 6$ rad /s

Kinetic energy of the flywheel

$$E = \frac{1}{2} I.(\omega_2)^2 = \frac{1}{2} \times 2500 \times (6)^2 = 45000\ N.m$$

Example 2.3. The turning moment diagram for a petrol engine is drawn to the following scales : Turning moment, 1 mm = 500 N-m ; crank angle, 1 mm = 3°. The turning moment diagramrepeats itself at every half revolution of the engine and the areas above and below the mean turningmoment line taken in order are 260, –580, 80, –380, 870, –250 mm^2. The rotating parts are equivalentto a mass of 55 kg at a radius of gyration of 2.1 m. Determine the coefficient of fluctuation of speed when the engine runs at 1600 r.p.m.

m = 55 kg ; k = 2.1 m ;

N = 1600 r.p.m. $\omega = 2\pi \times 1600/60 = 167.46$ rad /s

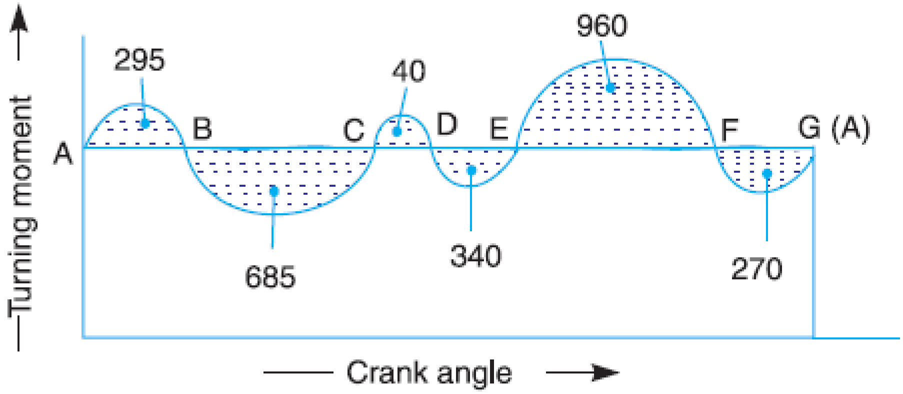


Figure 2.7

Since the turning moment scale is 1 mm = 5 N-m and crank angle scale is 1 mm = 3° = 3π /180 rad, therefore, 1 mm^2 on turning moment diagram

$$= 500 \times \frac{3\pi}{180} = 8.34\,\pi$$

Let the total energy at A = E, then referring to Figure 2.7,

Energy at B = E + 260 (Maximum energy)

Energy at C = E + 260 – 580 = E – 320

Energy at D = E – 320 + 80 = E – 240

Energy at E = E – 240 – 380 = E – 620 ...(Minimum energy)

Energy at F = E – 620 + 870 = E + 250

Energy at G = E + 250 – 250 = E = Energy at A

We know that maximum fluctuation of energy,

Δ E = Maximum energy – Minimum energy

= (E + 260) – (E – 620) = 880 mm^2

= 880 (8.34 π)

= 23038 N-m

Let CS = Coefficient of fluctuation of speed.

We know that maximum fluctuation of energy (Δ E),

$23038 = m.k^2\,\omega^2.C_S = 55 \times (2.1)^2 \times (167.46)^2\, C_S$

$\therefore C_S = 0.0034$ or 0.34%

Example 2.4: The turning moment diagram for a multi-cylinder engine has been drawn to a vertical scale of 1 mm = 650 N.m and a horizontal scale of 1mm = 4.5^0. The areas above and below the mean torque line are –28, +380, –260, +310, –300, +242, –380, +265 and –229 mm^2. The fluctuation of speed is limited to ±1.8% of the mean speed which is 400 rpm. The density of the rim material is 7000 kg/m^3 and width of the rim is 4.5 times its thickness. The centrifugal stress in the rim material is limited to 6 N/mm^2. Neglecting the effect of the boss and arms, determine the diameter and cross section of the flywheel rim.

N = 400 r.p.m.	$\omega = 2\pi \times 400 / 60 = 83.8$ rad/s
Stroke = 300 mm	$\sigma = 6$ MPa $= 6 \times 10^6$ N/m^2
$\rho = 7000$ kg/m^3	b = 4.5 t

Since the fluctuation of speed is ± 1.8% of mean speed, therefore total fluctuation of speed, Cs = 0.018 + 0.018 = 0.036

$$\sigma = \rho.v^2$$

$$6 \times 10^6 = 7000 \text{ x } v^2$$

$$v = 29.28 \text{ m/s}$$

$$v = \frac{\pi dN}{60}$$

$$29.28 = \frac{\pi d \times 400}{60}$$

$$d = 1.398 \text{ m}$$

Let the flywheel K.E. at a = E

at b = E – 28

at c = E – 28 + 380 = E + 352

at d = E + 352 – 260 = E + 92

at e = E + 92 + 310 = E + 402

at f = E + 402 – 300 = E + 102

at g = E + 102 + 242 = E + 344

at h = E + 344 – 380 = E – 36

at j = E – 36 + 265 = E + 229

at k = E +229 – 229 = E

Maximum energy = E + 402

Minimum energy = E – 36

Maximum fluctuation of energy

= Maximum energy – Minimum energy x Horizontal scale x vertical scale

$$= 438 \times \left(4.5 \times \frac{\pi}{180} \right) \times 650$$

= 22360 N.m

Maximum fluctuation of energy, $(\Delta E) = m.k^2\,\omega^2.C_S$

$$C_s = \frac{\Delta E}{mk^2\omega^2}$$

$$0.036 = \frac{22360}{m \times \left(\frac{1.398}{2}\right)^2 \times \left(\frac{2\pi \times 400}{60}\right)^2}$$

m = 724.5 kg

Mass = Density x Volume

724.5 = ρ x (πd) x t x 4.5t

724.5 = 7200 x (π x 1.398) x t x 4.5t

t = 0.0713 m = 71.3 mm

b = 4.5 x 71.3 = 320.85 mm

Example 2.5: Turning moment area for the revolution of a multi cylinder engine with reference to the mean turning moment in sq. cm are:

–0.32, 4.08, –2.67, 3.33, –3.1, 2.26, –3.74, 2.74, –2.58

The scale for the ordinate and abscissa are 1 cm = 14^0, 1 cm = 6000 N-m

The mean speed is 200 rpm with 1.5% fluctuation. If hoop stress in the rim material is not to exceed 56 bar, calculate the diameter and cross-section of rim of the flywheel. Neglect the effect of bars and arms. Density of material = 0.00672 kg/cm^3.

N = 200 r.p.m. ω = 2π × 200 / 60 = 20.94 rad/s;

σ = 56 bar = 56 × 10^5 N/m^2 ρ = 0.00672 kg/cm^3 = 6720 kg/m^3

Since the fluctuation of speed is 1.5% of mean speed, therefore total fluctuation of speed, Cs = 0.015

σ = ρ.v^2

56 × 105 = 6720 x v^2

v = 28.87 m/s

$$v = \frac{\pi dN}{60}$$

$$28.87 = \frac{\pi d \times 200}{60}$$

d = 2.756 m

Let the flywheel K.E. at a = E

at b = E – 0.32

at c = E – 0.32 + 4.08 = E + 3.76

at d = E + 3.76 – 2.67 = E + 1.09

at e = E + 1.09 + 3.33 = E + 4.42

at f = E + 4.42 – 3.1 = E + 1.32

at g = E + 1.32 + 2.26 = E + 3.58

at h = E + 3.58 – 3.74 = E – 0.16

at j = E – 0.16 + 2.74 = E + 2.58

at k = E +2.58 – 2.58 = E

Maximum energy = E + 4.42
Minimum energy = E – 0.32
Maximum fluctuation of energy

= Maximum energy – Minimum energy x Horizontal scale x vertical scale

$$= 4.74 \times \left(14 \times \frac{\pi}{180}\right) \times 6000$$

= 6949.2 N.m

Maximum fluctuation of energy, (ΔE) = m.k^2 ω^2.C$_S$

$$C_s = \frac{\Delta E}{mk^2 \omega^2}$$

$$0.015 = \frac{6949.2}{m \times \left(\frac{2.756}{2}\right)^2 \times \left(\frac{2\pi \times 200}{60}\right)^2}$$

m = 556.2 kg

Mass = Density x Volume
556.2 = ρ x (πd) x A
556.2 = 6720 x (π x 2.756) x A
A = 9.559 x 10^{-3} m^2
A = 9559 mm^2

Example 2.6 The T-θ diagram of an engine consists of intercepted areas which are +40, –85, +79, –68, +96 and –62 mm^2 in one cycle taken in the given order. The torque axis scale is 1 mm = 75 N-m and crank angle scale is 1 mm = 5^0. Mean speed of the engine is 500 rpm. Design the rim of the flywheel for the following data:
(a) Limiting rim speed at mean radius = 30 m/s.
(b) The fluctuation of speed = 2 % around mean speed.
(c) Width to thickness ratio for rectangular rim section is 1.5 which contributes 100% of MI of flywheel.
(d) Material density is 7200 kg/m^3. Neglect the flywheel effect of hub and arms.

N = 500 r.p.m. ω = 2π × 500 / 60 = 52.36 rad/s; v = 30 m/s;
ρ = 7200 kg/m^3 b = 1.5 t

Since the fluctuation of speed is 2% of mean speed, therefore total fluctuation of speed, Cs = 0.02

$$v = \frac{\pi d N}{60}$$

$$30 = \frac{\pi d \times 500}{60}$$

d = 1.146 m

Let the flywheel K.E. at a = E

at b = E + 40

at c = E + 40 – 85 = E – 45

at d = E – 45 + 79 = E + 34

at e = E + 34 – 68 = E – 34

at f = E – 34 + 96 = E + 62

at g = E + 62 – 62 = E

Maximum energy = E + 62

Minimum energy = E – 45

Maximum fluctuation of energy

= Maximum energy – Minimum energy x Horizontal scale x vertical scale

$$= 107 \times \left(5 \times \frac{\pi}{180}\right) \times 75$$

= 700.313 N.m

Maximum fluctuation of energy, (ΔE) = $m.k^2\,\omega^2.CS$

$$C_S = \frac{\Delta E}{mk^2\omega^2}$$

$$0.02 = \frac{700.313}{m \times \left(\frac{1.146}{2}\right)^2 \times \left(\frac{2\pi \times 500}{60}\right)^2}$$

m = 38.9 kg

Mass = Density x Volume

38.9 = ρ x (πd) x t x 1.5t

38.9 = 7200 x (π x 1.146) x t x 1.5t

t = 0.0316 m = 31.6 mm

b = 1.5 x 31.6 = 47.44 mm

Example 2.7: A punching machine carries out 6 holes per minute. Each hole of 40 mm diameter in 35 mm thick plate requires 8 Nm of energy/mm^2 of the sheared area. The punch has a stroke of 95 mm. Find the power of the motor required if the mean speed of the flywheel is 20 m/s. If total fluctuation of speed is not to exceed 3% of the mean speed, determine the mass of the flywheel.

d = 40 mm,	t = 35 mm,	E_1 = 8 Nm of energy/mm^2,
stroke = 95 mm	v = 20 m/s	Cs = 0.03

Sheared area per hole = π.d. t

= π x 40 x 35 =4398.23 mm^2

Energy required to punch a hole,

E_1 = 8 × 4398.23

= 35185.84 N-m

Energy required for punching work per second

= Energy required per hole × No. of holes per second

= 35185.84 x 6/60

= 3518.58 N-m/s

- The punch travels a distance of 190 mm (upstroke + downstroke) in 10 seconds (6 holes are punched in 1 minute)

  Actual time required to punch a hole in 35 mm thick plate $= \frac{10}{190} \times 35 = 1.842$ sec

  (Assuming uniform velocity of the punch throughout)

  Energy supplied by the motor in 1.842 second,

  E_2 = 3518.58 × 1.842

  = 6481.22 N-m

- Energy to be supplied by the flywheel during punching a hole **or**

  Maximum fluctuation of energy of the flywheel,

  $\Delta E = E_1 - E_2$

  = 35185.84 – 6481.22

  = 28704.6 N-m

  Maximum fluctuation of energy, (ΔE) = m.v^2.CS

$$C_S = \frac{\Delta E}{mv^2}$$

$$0.03 = \frac{28704.6}{m \times (20)^2}$$

m = 2392 kg

Example 2.8: A machine punching 38 mm holes in 32 mm thick plate requires 7 N-m of energy per sq. mm of sheared area, and punches one hole in every 10 seconds. Calculate the power of the motor required. The mean speed of the flywheel is 25 metres per second. The punch has a stroke of 100 mm. Find the mass of the flywheel required, if the total fluctuation of speed is not to exceed 3% of the mean speed. Assume that the motor supplies energy to the machine at uniform rate.

d = 38 mm,	t = 32 mm,	E_1 = 7 Nm of energy/mm^2,
stroke = 100 mm	v = 25 m/s	Cs = 0.03

Sheared area per hole = π.d. t

= π x 38 x 32

= 3820.17 mm^2

Energy required to punch a hole,

E_1 = 7 × 3820.17

= 26741.24 N-m

Time required to punch a hole is 10 second,

Energy required for punching work per second

= 26741.24/10 = 2674.12 N-m/s

Power of the motor = 2674.12 W

The punch travels a distance of 200 mm (upstroke + downstroke) in 10 seconds

Actual time required to punch a hole in 32 mm thick plate $= \frac{10}{200} \times 32 = 1.6$ sec

(Assuming uniform velocity of the punch throughout)

Energy supplied by the motor in 1.6second,

$E_2 = 2674.12 \times 1.6$

$= 4278.59$ N-m

– Energy to be supplied by the flywheel during punching a hole or maximum fluctuation ofenergy of the flywheel,

$\Delta E = E_1 - E_2$

$= 26741.24 – 4278.59$

$= 22462.65$ N-m

Maximum fluctuation of energy, $(\Delta E) = m.v^2.CS$

$$C_S = \frac{\Delta E}{mv^2}$$

$$0.03 = \frac{22462.65}{m \times (25)^2}$$

m = 1198 kg

Example 2.9: A punching press is required to punch 40 mm diameter holes in a plate of 15 mm thickness at a rate of 30 holes/min. It requires 6 N-m of energy per mm^2 of sheared area. If the punching takes 1/10 of a second and r.p.m. of the flywheel varies from 160 to 140. Then determine the mass flywheel having radius of gyration of 1 m.

d = 40 mm	t = 15 mm	No. of holes = 30 per min.
Energy required = 6 N-m/mm^2	Time = 1/10 s = 0.1 s	
N_1 = 160 r.p.m.	N_2 = 140 r.p.m.	k = 1m

Sheared area per hole = π d .t

$= \pi \times 40 \times 15$

$= 1885\ mm^2$

Energy required to punch a hole,

$E_1 = 6 \times 1885 = 11\,310$ N-m

Energy required for punching work per second

= Energy required per hole × No. of holes per second

= 11 310 × 30/60 = 5655 N-m/s

- Since the punching takes 1/10 of a second, therefore, energy supplied by the motor in 1/10 second,

$$E_2 = 5655 \times 1/10 = 565.5 \text{ N-m}$$

- Energy to be supplied by the flywheel during punching a hole or maximum fluctuation ofenergy of the flywheel,

$$\Delta E = E_1 - E_2$$
$$= 11\,310 - 565.5$$
$$= 10\,744.5 \text{ N-m}$$

Mean speed of the flywheel,

$$N = \frac{N_1 + N_2}{2} = \frac{160 + 140}{2} = 150\ rpm$$

Maximum fluctuation of energy (ΔE),

$$\Delta E = I.\omega\left(\omega_1 - \omega_2\right)$$
$$= I \times \frac{2\pi N}{60} \times \left[\frac{2\pi N_1}{60} - \frac{2\pi N_2}{60}\right]$$
$$= m \times k^2 \times \frac{\pi^2}{900} \times N \times \left(N_1 - N_2\right)$$
$$10744.5 = m \times (1)^2 \times \frac{\pi^2}{900} \times 150 \times \left(160 - 140\right)$$
$$m = 326.59 \text{ kg}$$

3.1 Introduction

– Often an unbalance of forces is produced in rotary or reciprocating machinery due to the inertia forces associated with the moving masses. Balancing is the process of designing or modifying machinery so that the unbalance is reduced to an acceptable level and if possible is eliminated entirely.

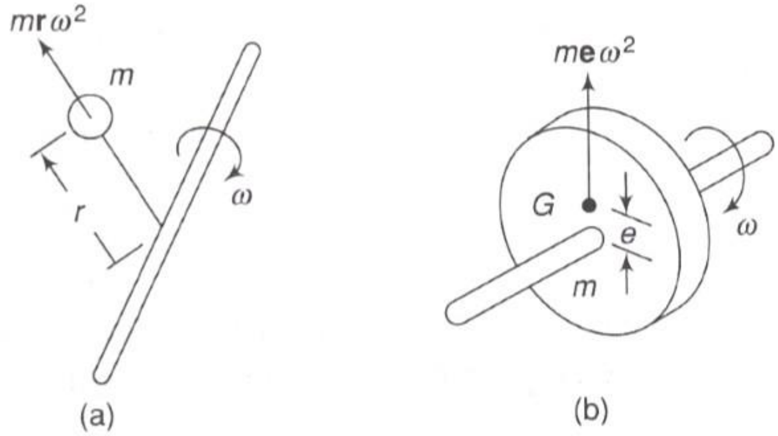


Fig. 3.1

– A particle or mass moving in a circular path experiences a centripetal acceleration and a force is required to produce it. An equal and opposite force acting radially outwards acts on the axis of rotation and is known as centrifugal force [Fig. 3.1(a)]. This is a disturbing force on the axis of rotation, the magnitude of which is constant but the direction changes with the rotation of the mass.

– In a revolving rotor, the centrifugal force remains balanced as long as the center of the mass of the rotor lies on the axis of the shaft. When the center of mass does not lie on the axis or there is an eccentricity, an unbalanced force is produced [Fig. 3.1(b)]. This type of unbalance is very common. For example, in steam turbine rotors, engine crankshafts, rotary compressors and centrifugal pumps.

– Most of the serious problems encountered in high-speed machinery are the direct result of unbalanced forces. These forces exerted on the frame by the moving machine members are time varying, impart vibratory motion to the frame and produce noise. Also, there are human discomfort and detrimental effects on the machine performance and the structural integrity of the machine foundation.

– The most common approach to balancing is by redistributing the mass which may be accomplished by addition or removal of mass from various machine members.

– There are two basic types of unbalance-rotating unbalance and reciprocating unbalance – which may occur separately or in combination.

3.2 Static Balancing:

– A system of rotating masses is said to be in static balance if the combined mass center of the system lies on the axis of rotation.

3.3 Types of Balancing:

There are main two types of balancing conditions

(i) Balancing of rotating masses

(ii) Balancing of reciprocating masses

(i) Balancing of Rotating Masses

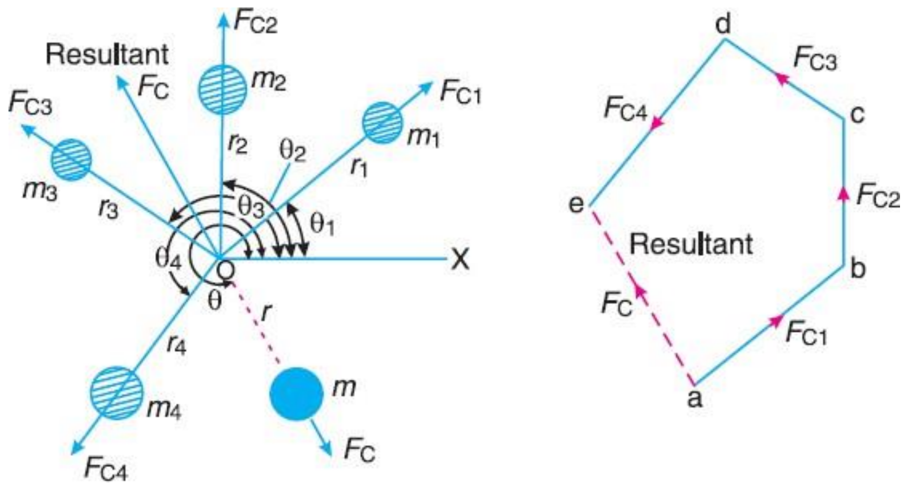
Whenever a certain mass is attached to a rotating shaft, it exerts some centrifugal force, whose effect is to bend the shaft and to produce vibrations in it. In order to prevent the effect of centrifugal force, another mass is attached to the opposite side of the shaft, at such a position so as to balance the effect of the centrifugal force of the first mass. This is done in such a way that the centrifugal forces of both the masses are made to be equal and opposite. The process of providing the second mass in order to counteract the effect of the centrifugal force of the first mass is called balancing of rotating masses.

The following cases are important from the subject point of view:

1. Balancing of a single rotating mass by a single mass rotating in the same plane.
2. Balancing of different masses rotating in the same plane.
3. Balancing of different masses rotating in different planes.

3.4 Balancing of Several Masses Rotating in the Same Plane

– Consider any number of masses (say four) of magnitude m_1, m_2, m_3 and m_4 at distances ofr_1, r_2, r_3 and r_4 from the axis of the rotating shaft. Let θ_1, θ_2, θ_3 and θ_4 be the angles of these masses with the horizontal line OX, as shown in Fig. 3.2 (a). Let these masses rotate about an axis through O and perpendicular to the plane of paper, with a constant angular velocity of ω rad/s.



(a) Space diagram. *(b) Vector diagram.*

Fig. 3.2 Balancing of several masses rotating in the same plane.

– The magnitude and position of the balancing mass may be found out analytically or graphically as discussed below:

1. *Analytical method*

- Each mass produces a centrifugal force acting radially outwards from the axis of rotation. Let F be the vector sum of these forces.

$$F = m_1r_1\omega^2 + m_2r_2\omega^2 + m_3r_3\omega^2 + m_4r_4\omega^2$$

- The rotor is said to be statically balanced if the vector sum F is zero.
- If F is not zero, i.e., the rotor is unbalanced, then produce a counterweight (balance weight) of mass m_c, at radius r_c to balance the rotor so that

$$m_1r_1\omega^2 + m_2r_2\omega^2 + m_3r_3\omega^2 + m_4r_4\omega^2 + m_cr_c\omega^2 = 0$$
$$m_1r_1 + m_2r_2 + m_3r_3 + m_4r_4 + m_cr_c = 0$$

- The magnitude of either m_c or r_c may be selected and of other can be calculated.
- In general, if $\Sigma\, mr$ is the vector sum of $m_1.r_1$, $m_2.r_2$, $m_3.r_3$, $m_4.r_4$, etc., then

$$\Sigma\, mr + m_cr_c = 0$$

- To solve these equation by mathematically, divide each force into its x and z components, $\Sigma\, mr\cos\theta + m_cr_c\cos\theta_c = 0$

  and $\Sigma\, mr\sin\theta + m_cr_c\sin\theta_c = 0$

$$m_cr_c\cos\theta_c = -\Sigma\, mr\cos\theta \qquad \text{… .... (i)}$$

and

$$m_cr_c\sin\theta_c = -\Sigma\, mr\sin\theta \qquad \text{............................(ii)}$$

- Squaring and adding (i) and (ii),

$$m_cr_c = \sqrt{(\Sigma\, mr\cos\theta)^2 + (\Sigma\, mr\sin\theta)^2}$$

- Dividing (ii) by (i),

$$\tan\theta c = \frac{-\Sigma\, mr\sin\theta}{-\Sigma\, mr\cos\theta}$$

- The signs of the numerator and denominator of this function identify the quadrant of the angle.

2. *Graphical method*

- First of all, draw the space diagram with the positions of the several masses, as shown in Fig. 3.2 (a).
- Find out the centrifugal force (or product of the mass and radius of rotation) exerted by each mass on the rotating shaft.
- Now draw the vector diagram with the obtained centrifugal forces (or the product of the masses and their radii of rotation), such that *ab* represents the centrifugal force exerted by the mass m_1 (or $m_1.r_1$) in magnitude and direction to some suitable scale. Similarly, draw *bc, cd* and *de* to represent centrifugal forces of other masses m_2, m_3 and m_4 (or $m_2.r_2$, $m_3.r_3$ and $m_4.r_4$).
- Now, as per polygon law of forces, the closing side *ae* represents the resultant force in magnitude and direction, as shown in Fig. 3.2 (b).

- The balancing force is, then, equal to resultant force, but in opposite direction.
- Now find out the magnitude of the balancing mass (m) at a given radius of rotation (r), such that

$$m.r.\omega^2 = \text{Resultant centrifugal force}$$

or
$$m.r = \text{Resultant of } m_1.r_1,\ m_2.r_2,\ m_3.r_3 \text{ and } m_4.r_4$$

- (In general for graphical solution, vectors $m_1.r_1$, $m_2.r_2$, $m_3.r_3$, $m_4.r_4$, etc., are added. If they close in a loop, the system is balanced. Otherwise, the closing vector will be giving $m_c.r_c$. Its direction identifies the angular position of the countermass relative to the other mass.)

Example 3.1 :A circular disc mounted on a shaft carries three attached masses of 4 kg, 3 kg and 2.5 kg at radial distances of 75 mm, 85 mm and 50 mm and at the angular positions of 45°, 135° and 240° respectively. The angular positions are measured counterclockwise from the reference line along the x-axis. Determine the amount of the countermass at a radial distance of 75 mm required for the static balance.

$m_1 = 4$ kg	$r_1 = 75$ mm	$\theta_1 = 45°$
$m_2 = 3$ kg	$r_2 = 85$ mm	$\theta_2 = 135°$
$m_3 = 2.5$ kg	$r_3 = 50$ mm	$\theta_3 = 240°$

$m_1r_1 = 4 \times 75 = 300$ kg.mm

$m_2r_2 = 3 \times 85 = 255$ kg.mm

$m_3r_3 = 2.5 \times 50 = 125$ kg.mm

Analytical Method:

$$\Sigma\, mr + m_cr_c = 0$$

$$300 \cos 45° + 255 \cos 135° + 125 \cos 240° + m_cr_c\cos\theta_c = 0 \quad \text{and}$$

$$300 \sin 45° + 255 \sin 135° + 125 \sin 240° + m_cr_c\sin\theta_c = 0$$

Squaring, adding and then solving,

$$m_C\, r_C = \sqrt{\begin{matrix}(300\cos 45 + 255\, cos135 + 125\, cos240)^2 + \\ (300\sin 45 + 255\, sin135 + 125\, sin240)^2\end{matrix}}$$

$$m_C \times 75 = \sqrt{(-30.68)^2 + (284.2)^2}$$

$= 285.8$ kg.mm

$\mathbf{m_c = 3.81\ kg}$

$$\tan\theta_c = \frac{-\sum mr\sin\theta}{-\sum mr\cos\theta} = \frac{-284.2}{-(-30.68)} = -9.26$$

$\theta_c = -83°50'$

θ_c lies in the fourth quadrant (numerator is negative and denominator is positive).

$\theta_c = 360 - 83°50'$

$\boldsymbol{\theta_c = 276°9'}$

Graphical Method:

- The magnitude and the position of the balancing mass may also be found graphically as discussed below :
- Now draw the vector diagram with the above values, to some suitable scale, as shown in Fig. 3.3. The closing side of the polygon *co* represents the resultant force. By measurement, we find that *co* = 285.84 kg-mm.

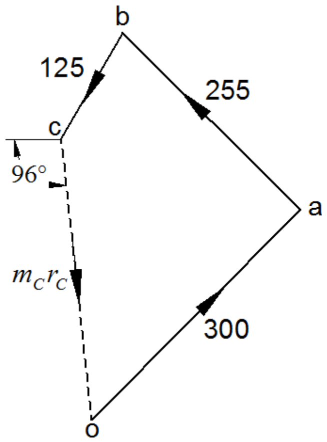


Fig. 3.3 Vector Diagram

- The balancing force is equal to the resultant force. Since the balancing force is proportional to *m.r*, therefore

$m_C \times 75$ = vector *co* = 285.84 kg-mm or m_C = 285.84/75

m_C **= 3.81 kg.**

- By measurement we also find that the angle of inclination of the balancing mass (m) from the horizontal or positive X-axis,

θ_C **= 276°.**

Example 3.2: Four masses m_1, m_2, m_3 and m_4 are 200 kg, 300 kg, 240 kg and 260 kg respectively. The corresponding radii of rotation are 0.2 m, 0.15 m, 0.25 m and 0.3 m respectively and the angles between successive masses are 45°, 75° and 135°. Find the position and magnitude of the balance mass required, if its radius of rotation is 0.2 m.

m_1 = 200 kg	r_1 = 0.2 m	θ_1 = 0°
m_2 = 300 kg	r_2 = 0.15 m	θ_2 = 45°
m_3 = 240 kg	r_3 = 0.25 m	θ_3 = 45° +75° = 120°
m_4 = 260 kg	r_4 = 0.3 m	θ_4 = 120° + 135° = 255°
m_1r_1 = 200 x 0.2 = 40	r_C = 0.2 m	
m_2r_2 = 300 x 0.15 = 45		
m_3r_3 = 240 x 0.25 = 60		
m_4r_4 = 260 x 0.3 = 78		

$\Sigma mr + m_c r_c = 0$

$40 \cos 0° + 45\cos 45° + 60\cos 120° + 78\cos 255° + m_c r_c \cos\theta_c = 0$ and

$40 \sin 0° + 45 \sin 45° + 60 \sin 120° + 78 \sin 255° + m_c r_c \sin\theta_c = 0$

Squaring, adding and then solving,

$$m_c r_c = \sqrt{\begin{aligned}&(40\cos 0° + 45\cos 45° + 60\cos 120° + 78\cos 255°)^2 + \\ &(40\sin 0° + 45\sin 45° + 60\sin 120° + 78\sin 255°)^2\end{aligned}}$$

$$m_c \times 0.2 = \sqrt{(21.6)^2 + (8.5)^2}$$

$$= 23.2 \text{ kg.mm}$$

$$\mathbf{m_c = 116 \text{ kg}}$$

$$\tan\theta_c = \frac{-\sum mr\sin\theta}{-\sum mr\cos\theta} = \frac{-8.5}{-21.6} = 0.3935$$

$$\theta_c = 21°28'$$

θ_c lies in the third quadrant (numerator is negative and denominator is negative).

$$\theta_c = 180 + 21°28'$$

$$\mathbf{\theta_c = 201°28'}$$

Graphical Method:

- For graphical method draw the vector diagram with the above values, to some suitable scale, as shown in Fig. 3.4. The closing side of the polygon *ae* represents the resultant force. By measurement, we find that *ae* = 23 kg-m.

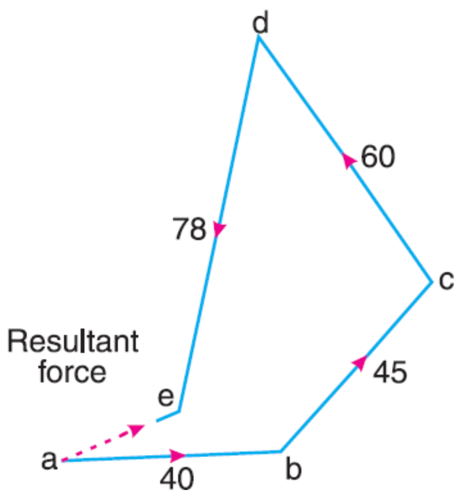


Fig. 3.4 Vector Diagram

- The balancing force is equal to the resultant force. Since the balancing force is proportional to *m.r*, therefore

$$m \times 0.2 = \text{vector } ea = 23 \text{ kg-m} \quad \text{or} \quad m_C = 23/0.2$$

$$\mathbf{m_C = 115 \text{ kg.}}$$

- By measurement we also find that the angle of inclination of the balancing mass (*m*) from the horizontal or positive X-axis,

$$\mathbf{\theta_C = 201°.}$$

3.5 Dynamic Balancing

- When several masses rotate in different planes, the centrifugal forces, in addition to being out of balance, also form couples. A system of rotating masses is in dynamic balance when there does not exist any resultant centrifugal force as well as resultant couple.
- In the work that follows, the products of mr and mrl (instead of $mr\omega^2$ and $mrl\omega^2$), usually, have been referred as force and couple respectively as it is more convenient to draw force and couple polygons with these quantities.

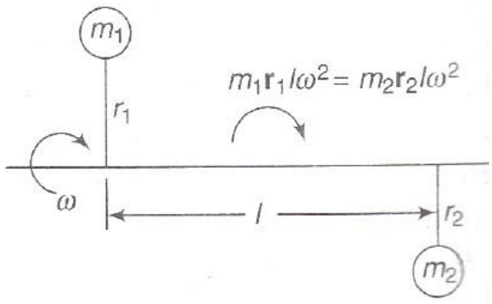


Fig. 3.5

- If m_1, and m_2 are two masses (Fig. 3.5) revolving diametrically opposite to each other in different planes such that $m_1r_1 = m_2r_2$, the centrifugal forces are balanced, but an unbalanced couple of magnitude m_1r_1l (= m_2r_2l) is introduced. The couple acts in a plane that contains the axis of rotation and the two masses. Thus, the couple is of constant magnitude but variable direction.

3.6 Balancing of Several Masses Rotating in the different Planes

- Let there be a rotor revolving with a uniform angular velocity ω. m_1, m_2 and m_3 are the masses attached to the rotor at radii r_1, r_2 and r_3 respectively. The masses m_1, m_2 and m_3 rotate in planes 1, 2 and 3 respectively. Choose a reference plane at O so that the distances of the planes 1, 2 and 3 from O are l_1, l_2 and l_3 respectively.
- Transference of each unbalanced force to the reference plane introduces the like number of forces and couples.
- The unbalanced forces in the reference plane are $m_1r_1\omega^2$, $m_2r_2\omega^2$ and $m_3r_3\omega^2$ acting radially outwards.
- The unbalanced couples in the reference plane are $m_1r_1\omega^2l_1$, $m_2r_2\omega^2l_2$ and $m_3r_3\omega^2l_3$ which may be represented by vectors parallel to the respective force vectors, i.e., parallel to the respective radii of m_1, m_2 and m_3.
- For complete balancing of the rotor, the resultant force and resultant couple both should be zero, i.e., $m_1r_1\omega^2 + m_2r_2\omega^2 + m_3r_3\omega^2 = 0$ .... (a)
  and $m_1r_1\omega^2l_1 + m_2r_2\omega^2l_2 + m_3r_3\omega^2l_3 = 0$.........................(b)
- If the Eqs (a) and (b) are not satisfied, then there are unbalanced forces and couples. A mass placed in the reference plane may satisfy the force equation but

the couple equation is satisfied only by two equal forces in different transverse planes.

- Thus in general, two planes are needed to balance a system of rotating masses.
- Therefore, in order to satisfy Eqs (a) and (b), introduce two counter-masses m_{C1} and m_{C2} at radii r_{C1} and r_{C2} respectively. Then Eq. (a) may be written as

$$m_1r_1\omega^2 + m_2r_2\omega^2 + m_3r_3\omega^2 + m_{C1}r_{C1}\omega^2 + m_{C2}r_{C2}\omega^2 = 0$$

$$m_1r_1 + m_2r_2 + m_3r_3 + m_{C1}r_{C1} + m_{C2}r_{C2} = 0$$

$$\Sigma\, mr + m_{C1}r_{C1} + m_{C2}r_{C2} = 0 \text{.................................... (c)}$$

- Let the two countermasses be placed in transverse planes at axial locations O and Q, i.e., the countermass m_{C1} be placed in the reference plane and the distance of the plane of m_{C2} be l_{C2} from the reference plane. Equation (b) modifies to (taking moments about O)

$$m_1r_1\omega^2l_1 + m_2r_2\omega^2l_2 + m_3r_3\omega^2l_3 + m_{C2}r_{C2}\omega^2l_{C2} = 0$$

$$m_1r_1l_1 + m_2r_2l_2 + m_3r_3l_3 + m_{C2}r_{C2}l_{C2} = 0$$

$$\Sigma\, mrl + m_{C2}r_{C2}l_{C2} = 0 \text{ .... (d)}$$

- Thus, Eqs (c) and (d) are the necessary conditions for dynamic balancing of rotor. Again the equations can be solved mathematically or graphically.

  Dividing Eq. (d) into component form

$$\Sigma\, mrl\cos\theta + m_{C2}r_{C2}l_{C2}\cos\theta_{C2} = 0$$

$$\Sigma\, mrl\sin\theta + m_{C2}r_{C2}l_{C2}\sin\theta_{C2} = 0$$

$$m_{C2}r_{C2}l_{C2}\cos\theta_{C2} = -\Sigma\, mrl\cos\theta \text{ .... (i)}$$

$$m_{C2}r_{C2}l_{C2}\sin\theta_{C2} = -\Sigma\, mrl\sin\theta \text{.................................(ii)}$$

- Squaring and adding (i) and (ii)

$$m_{C2}r_{C2}l_{C2} = \sqrt{(\Sigma\, mrl\cos\theta)^2 + (\Sigma\, mrl\sin\theta)^2}$$

- Dividing (ii) by (i),

$$\tan\theta_{C2} = \frac{-\Sigma\, mrl\sin\theta}{-\Sigma\, mrl\cos\theta}$$

- After obtaining the values of m_{C2} and θ_{C2} from the above equations, solve Eq. (c) by taking its components,

$$\Sigma\, mr\cos\theta + m_{C1}r_{C1}\cos\theta_{C1} + m_{C2}r_{C2}\cos\theta_{C2} = 0$$

$$\Sigma\, mr\sin\theta + m_{C1}r_{C1}\sin\theta_{C1} + m_{C2}r_{C2}\sin\theta_{C2} = 0$$

$$m_{C1}r_{C1}\cos\theta_{C1} = -(\Sigma\, mr\cos\theta + m_{C2}r_{C2}\cos\theta_{C2})$$

$$m_{C1}r_{C1}\sin\theta_{C1} = -(\Sigma\, mr\sin\theta + m_{C2}r_{C2}\sin\theta_{C2})$$

$$m_{C1}r_{C1} = \sqrt{(\Sigma\, mr\cos\theta + m_{C2}r_{C2}\cos\theta_{C2})^2 + (\Sigma\, mr\sin\theta + m_{C2}r_{C2}\sin\theta_{C2})^2}$$

$$\tan\theta_{C1} = \frac{-(\Sigma\, mr\sin\theta + m_{C2}r_{C2}\sin\theta_{C2})}{-(\Sigma\, mr\cos\theta + m_{C2}r_{C2}\cos\theta_{C2})}$$

Example 3.3 : A shaft carries four masses A, B, C and D of magnitude 200 kg, 300 kg, 400 kg and 200 kg respectively and revolving at radii 80 mm, 70 mm, 60 mm and 80 mm in planes measured from A at 300 mm, 400 mm and 700 mm. The angles between the cranks measured anticlockwise are A to B 45°, B to C 70° and C to D 120°. The balancing masses are to be placed in planes X and Y. The distance between the planes A and X is 100 mm, between X and Y is 400 mm and between Y and D is 200 mm. If the balancing masses revolve at a radius of 100 mm, find their magnitudes and angular positions.

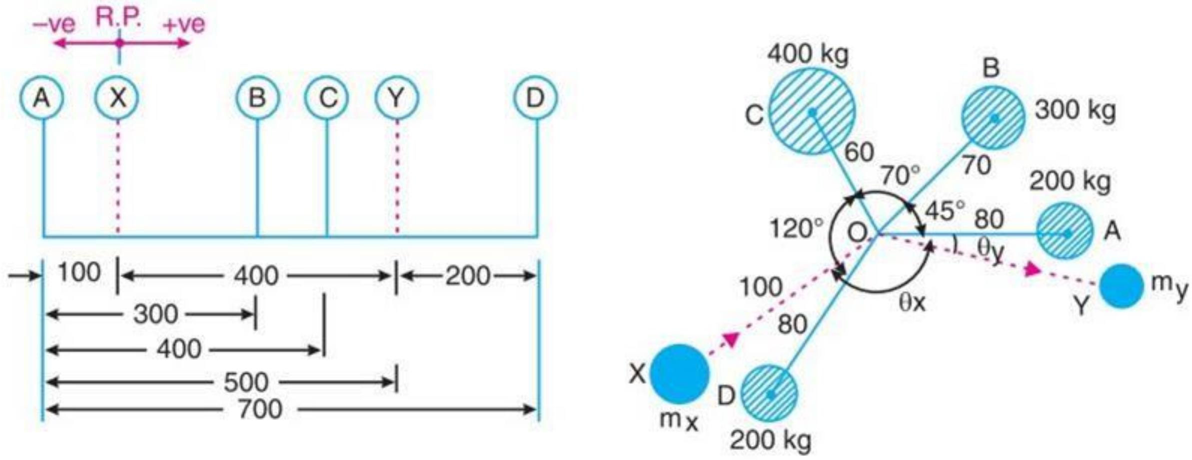
$m_A = 200$ kg	$r_A = 80$ mm	$\theta_A = 0°$	$l_A = -100$ mm
$m_B = 300$ kg	$r_B = 70$ mm	$\theta_B = 45°$	$l_B = 200$ mm
$m_C = 400$ kg	$r_C = 60$ mm	$\theta_C = 45° + 70° = 115°$	$l_C = 300$ mm
$m_D = 200$ kg	$r_D = 80$ mm	$\theta_D = 115° + 120° = 235°$	$l_D = 600$ mm
	$r_X = r_Y = 100$ mm		$l_Y = 400$ mm

Let m_X = Balancing mass placed in plane *X*, and

m_Y = Balancing mass placed in plane *Y*.

The position of planes and angular position of the masses (assuming the mass A as horizontal) are shown in Fig. 3.6 (*a*) and (*b*) respectively.

Assume the plane *X* as the reference plane (*R.P.*). The distances of the planes to the right of plane *X* are taken as + ve while the distances of the planes to the left of plane *X* are taken as –ve.



(a) Position of planes. *(b) Angular position of masses.*

Fig. 3.6

$m_A r_A l_A = 200 \times 0.08 \times (-0.1) = -1.6 \text{ kg.m}^2$ $m_A r_A = 200 \times 0.08 = 16 \text{ kg.m}$

$m_B r_B l_B = 300 \times 0.07 \times 0.2 = 4.2 \text{ kg.m}^2$ $m_B r_B = 300 \times 0.07 = 21 \text{ kg.m}$

$m_C r_C l_C = 400 \times 0.06 \times 0.3 = 7.2 \text{ kg.m}^2$ $m_C r_C = 400 \times 0.06 = 24 \text{ kg.m}$

$m_D r_D l_D = 200 \times 0.08 \times 0.6 = 9.6 \text{ kg.m}^2$ $m_D r_D = 200 \times 0.08 = 16 \text{ kg.m}$

Analytical Method:

For unbalanced couple

$\Sigma mrl + m_Y r_Y l_Y = 0$

$$m_Y r_Y l_Y = \sqrt{(\Sigma mrl\cos\theta)^2 + (\Sigma mrl\sin\theta)^2}$$

$$m_Y r_Y l_Y = \sqrt{\begin{aligned}&(-1.6\cos0° + 4.2\cos45° + 7.2\cos115° + 9.6\cos235°)^2 + \\ &(-1.6\sin0° + 4.2\sin45° + 7.2\sin115° + 9.6\sin235°)^2\end{aligned}}$$

$$m_Y r_Y l_Y = \sqrt{(-7.179)^2 + (1.63)^2}$$

$$m_Y \times 0.1 \times 0.4 = 7.36$$

m_Y = 184 kg.

$$\tan\theta_Y = \frac{-\sum mrl\sin\theta}{-\sum mrl\cos\theta} = \frac{-1.63}{-(-7.179)} = -0.227$$

$\theta_Y = -12°47'$

θ_Y lies in the fourth quadrant (numerator is negative and denominator is positive).

$\theta_Y = 360 - 12°47'$

$\theta_Y = 347°12'$

For unbalanced centrifugal force

$\Sigma mr + m_X r_X + m_Y r_Y = 0$

$$m_X r_X = \sqrt{(\Sigma mr\cos\theta + m_Y r_Y\cos\theta_Y)^2 + (\Sigma mr\sin\theta + m_Y r_Y\sin\theta_Y)^2}$$

$$m_X r_X = \sqrt{\begin{aligned}&(16\cos0° + 21\cos45° + 24\cos115° + 16\cos235° + 18.4\cos347°12')^2 + \\ &(16\sin0° + 21\sin45° + 24\sin115° + 16\sin235° + 18.4\sin347°12')^2\end{aligned}}$$

$$m_X r_X = \sqrt{(29.47)^2 + (19.42)^2}$$

$$m_X \times 0.1 = 35.29$$

m_X = 353 kg.

$$\tan\theta_X = \frac{-\Sigma mr\sin\theta}{-\Sigma mr\cos\theta} = \frac{-19.42}{-29.47} = 0.6589$$

$\theta_X = 33°22'$

θ_X lies in the third quadrant (numerator is negative and denominator is negative).

$\theta_X = 180 + 33°22'$

$\theta_X = 213°22'$

Graphical Method:

The balancing masses and their angular positions may be determined graphically as discussed below:

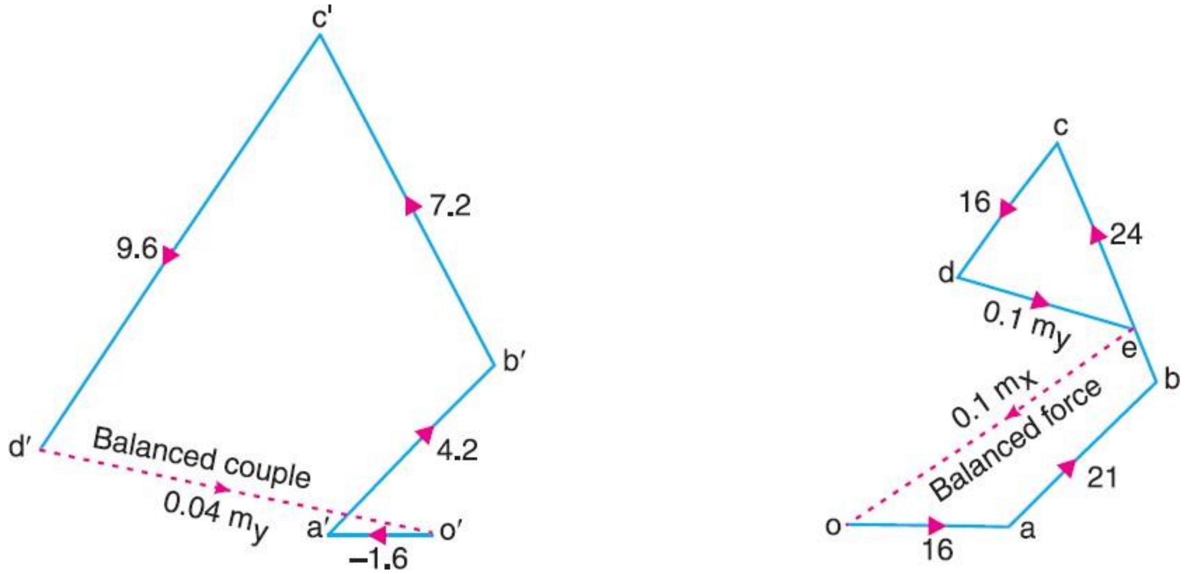
Table 3.1

Plane	Angle	Mass (m) kg	Radius (r)m	Cent.force ÷ ω² *(mr)* kg-m	Distance from Ref. Plane *(l)* m	Couple ÷ ω² *(mrl)* kg-m²
A	0°	200	0.08	16	– 0.1	–1.6
X (R.P.)	θ_X	m_X	0.1	0.1 m_X	0	0
B	45°	300	0.07	21	0.2	4.2
C	115°	400	0.06	24	0.3	7.2
Y	θ_Y	m_Y	0.1	0.1 m_Y	0.4	0.04 m_Y
D	235°	200	0.08	16	0.6	9.6

– First of all, draw the couple polygon from the data given in Table 3.1 (column 7) as shown in Fig. 3.7 (a) to some suitable scale. The vector *d′o′* represents the balanced couple. Since the balanced couple is proportional to $0.04m_Y$, therefore by measurement, $0.04m_Y$ = vector *d′o′* = 73 kg-m²

or **m_Y = 182.5 kg**

– The angular position of the mass m_Y is obtained by drawing Om_Y in Fig. 3.6 (*b*), parallel to vector *d′o′*. By measurement, the angular position of m_Y is θ_Y = 12° in the clockwise direction from mass m_A (*i.e*. 200 kg), so **θ_Y = 360° – 12° = 348°.**



(a) Couple Polygon *(b) Force Polygon*

Fig. 3.7

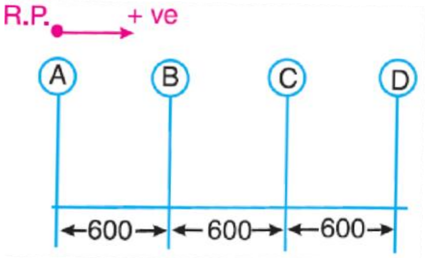
– Now draw the force polygon from the data given in Table 3.1 (column 5) as shown in Fig. 3.7 (b). The vector *eo* represents the balanced force. Since the balanced force is proportional to 0.1 m_X, therefore by measurement,

$0.1m_X$ = vector *eo* = 35.5 kg-m

or **m_X = 355 kg.**

– The angular position of the mass m_X is obtained by drawing Om_X in Fig. 3.6 (*b*), parallel to vector *eo*. By measurement, the angular position of m_X is θ_X = 145° in the clockwise direction from mass m_A (*i.e*. 200 kg), so **θ_X = 360° – 145° = 215°.**

Example 3.4: Four masses A, B, C and D carried by a rotating shaft are at radii 100, 140, 210 and 160 mm respectively. The planes in which the masses revolve are spaced 600 mm apart and the masses of B, C and D are 16 kg, 10 kg and 8 kg respectively. Find the required mass A and the relative angular positions of the four masses so that shaft is in complete balance.



m_A = ? $\quad$ r_A = 100 mm

m_B = 16 kg $\quad$ r_B = 140 mm $\quad$ l_B = 600 mm

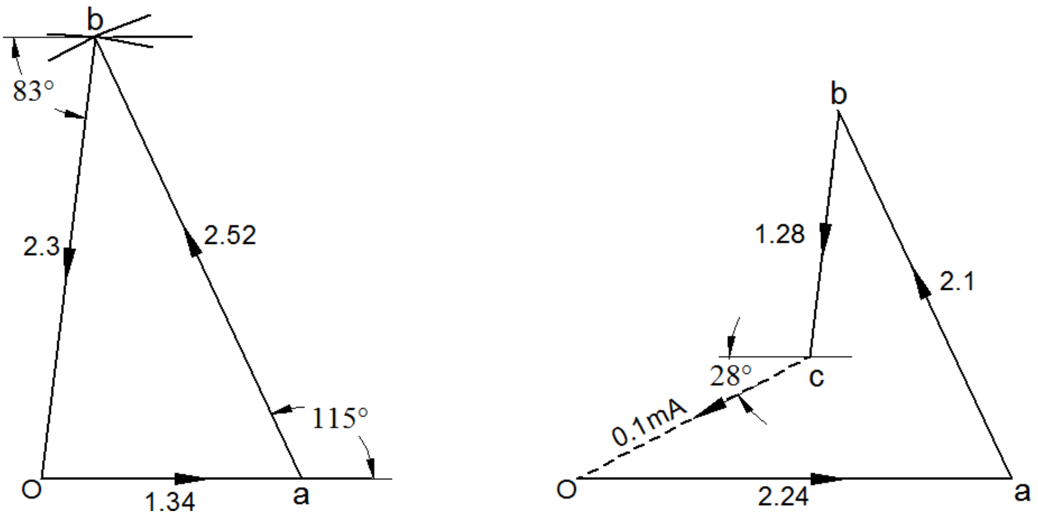
m_C = 10 kg $\quad$ r_C = 210 mm $\quad$ l_C = 1200 mm

m_D = 8 kg $\quad$ r_D = 160 mm $\quad$ l_D = 1800 mm

Table 3.2

Plane	Angle	Mass (m) kg	Radius (r) m	Cent.force ÷ ω^2 *(mr)* kg-m	Distance from Ref. Plane *(l)* m	Couple ÷ ω^2 *(mrl)* kg-m^2
A (R.P.)	θ_A	m_A	0.1	$0.1m_A$	0	0
B	0°	16	0.14	2.24	0.6	1.34
C	θ_C	10	0.21	2.1	1.2	2.52
D	θ_D	8	0.16	1.28	1.8	2.3

- First of all, draw the couple polygon from the data given in Table 3.2 (column 7) as shown in Fig. 3.8 (a) to some suitable scale. By measurement, the angular position of m_C is $\boldsymbol{\theta_C}$ **= 115°** in the anticlockwise direction from mass m_B and the angular position of m_D is $\boldsymbol{\theta_D}$ **= 263°** in the anticlockwise direction from mass m_B.



(a) Couple Polygon $\quad$ *(b) Force Polygon*

Fig. 3.8

- Now draw the force polygon from the data given in Table 3.2 (column 5) as shown in Fig. 3.8 (b). The vector *co* represents the balanced force. Since the balanced force is proportional to $0.1m_A$, therefore by measurement,

$$0.1m_A = \text{vector } co = 1.36 \text{ kg-m}$$

Or $\quad$ $\boldsymbol{m_A}$ **= 13.6 kg.**

- By measurement, the angular position of m_A is $\boldsymbol{\theta_A}$ **= 208°** in the anticlockwise direction from mass m_B (*i.e.* 16 kg).

Example 3.5: Four masses 150 kg, 200 kg, 100 kg and 250 kg are attached to a shaft revolving at radii 150 mm, 200 mm, 100 mm and 250 mm; in planes A, B, C and D respectively. The planes B, C and D are at distances 350 mm, 500 mm and 800 mm from plane A. The masses in planes B, C and D are at an angle 105°, 200° and 300° measured anticlockwise from mass in plane A. It is required to balance the system by placing the balancing masses in the planes P and Q which are midway between the planes A and B, and between C and D respectively. If the balancing masses revolve at radius 180 mm, find the magnitude and angular positions of the balance masses.

m_A = 150 kg	r_A = 150mm	θ_A = 0°
m_B = 200 kg	r_B= 200mm	θ_B = 105°
m_C = 100 kg	r_C = 100mm	θ_C = 200°
m_D = 250 kg	r_D = 250 mm	θ_D = 300°
	$r_X = r_Y$ = 180 mm	

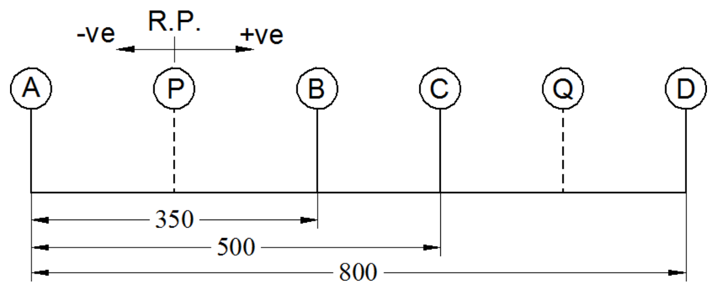


Fig. 3.9

Table 3.3

Plane	Angle	Mass (m) kg	Radius (r) m	Cent.force ÷ ω^2 *(mr)* kg-m	Distance from Ref. Plane *(l)* m	Couple ÷ ω^2 *(mrl)* kg-m^2
A (R.P.)	0°	150	0.15	22.5	–0.175	–3.94
P	θ_P	m_P	0.18	0.18 m_P	0	0
B	105°	200	0.2	40	0.175	7
C	200°	100	0.1	10	0.325	3.25
Q	θ_Q	m_Q	0.18	0.18 m_Q	0.475	0.0855 m_Q
D	300°	250	0.25	62.5	0.625	39.06

Analytical Method:

Table 3.4

*mrl*cosθ *(H_C)*	*mrl sin*θ *(V_C)*	*mr*cosθ *(H_F)*	*mr sin*θ *(V_F)*
–3.94	0	22.5	0
0	0	0.18 $m_P\cos\theta_P$	0.18 $m_P \sin\theta_P$
–1.81	6.76	–10.35	38.64
–3.05	–1.11	-9.4	–3.42
0.0855 $m_Q\cos\theta_Q$	0.0855 $m_Q \sin\theta_Q$	0.18 $m_Q\cos\theta_Q$	0.18 $m_Q \sin\theta_Q$
19.53	–33.83	31.25	–54.13

$\Sigma\ H_C = 0$

$-3.94 + 0 - 1.81 - 3.05 + 0.0855\ m_Q\cos\theta_Q + 19.53 = 0$

$0.0855\ m_Q\cos\theta_Q = -10.73$

$m_Q\cos\theta_Q = -125.497$ .... (i)

$\Sigma\ V_C = 0$

$0 + 0 + 6.76 - 1.11 + 0.0855\ m_Q \sin\theta_Q - 33.83 = 0$

$0.0855\ m_Q \sin\theta_Q = 28.18$

$m_Q \sin\theta_Q = 329.59$ .... (ii)

$$m_Q = \sqrt{(-125.497)^2 + (329.59)^2}$$

m_Q = 352.67 kg.

$$\frac{m_Q \sin\theta_Q}{m_Q \cos\theta_Q} = \frac{329.59}{-125.497}$$

$\tan\theta_Q = -2.626$

$\theta_Q = -69.15$

$\theta_Q = 180 - 69.15$

θ_Q = 110.84°

$\Sigma\ H_F = 0$

$22.5 + 0.18\ m_P\cos\theta_P - 10.35 - 9.4 + 0.18\ m_Q\cos\theta_Q + 31.25 = 0$

$22.5 + 0.18\ m_P\cos\theta_P - 10.35 - 9.4 + 0.18\ (352.67) \cos 110.84° + 31.25 = 0$

$0.18\ m_P\cos\theta_P = -11.416$

$m_P\cos\theta_P = -63.42$

$\Sigma\ V_F = 0$

$0 + 0.18\ m_P \sin\theta_P + 38.64 - 3.42 + 0.18\ m_Q \sin\theta_Q - 54.13 = 0$

$0 + 0.18\ m_P \sin\theta_P + 38.64 - 3.42 + 0.18\ (352.67) \sin 110.84° - 54.13 = 0$

$0.18\ m_P \sin\theta_P = -40.417$

$m_P \sin\theta_P = -224.54$

$$m_P = \sqrt{(-63.42)^2 + (-224.54)^2}$$

m_P = 233.32 kg.

$$\frac{m_P \sin\theta_P}{m_P \cos\theta_P} = \frac{-224.54}{-63.42}$$

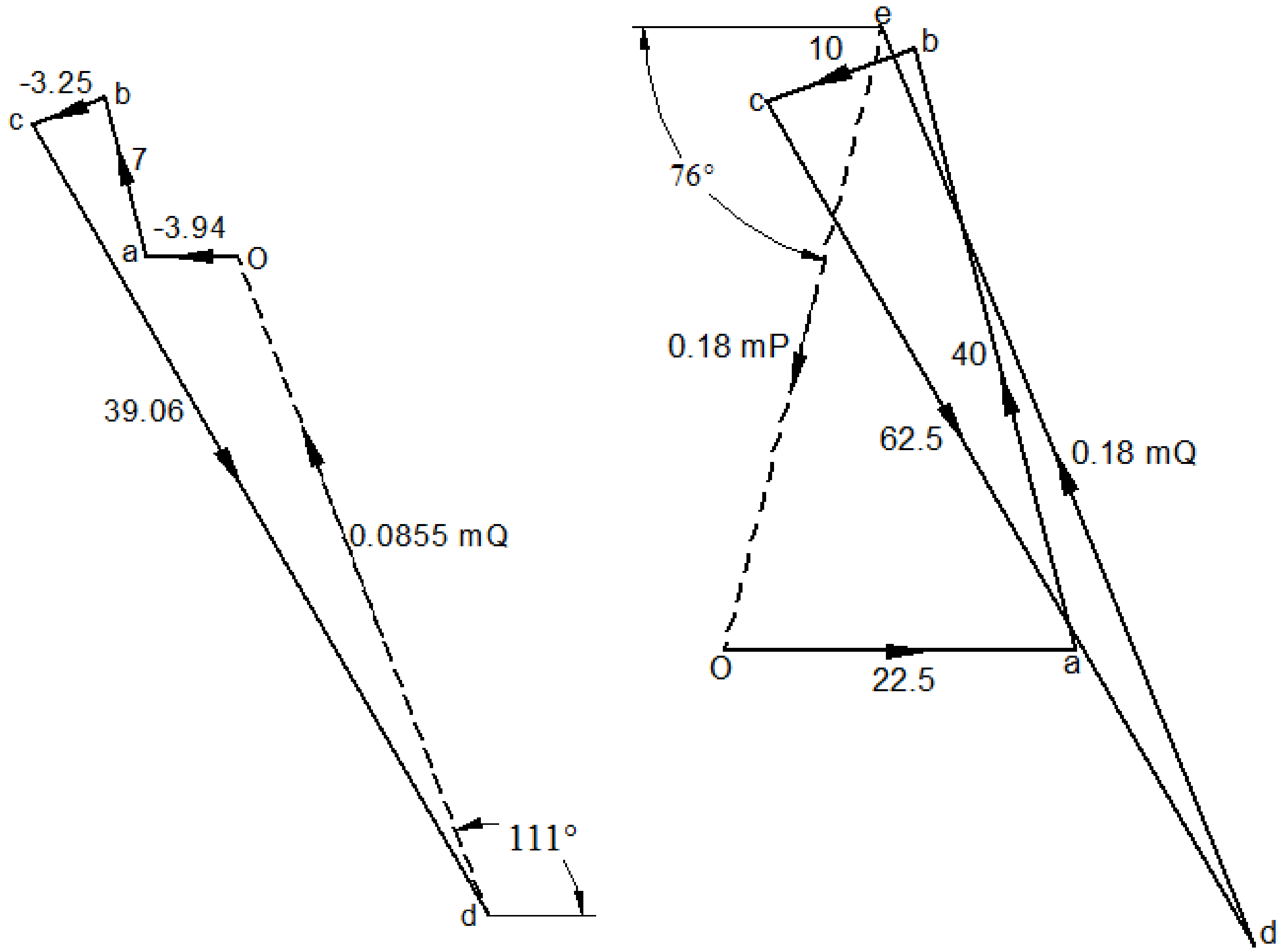
$\tan\theta_P = 3.54$

$\theta_P = 74.23$

$\theta_P = 180 + 74.23$

θ_P = 254.23°

Graphical Method :



(a) Couple Polygon *(b) Force Polygon*

Fig. 3.10

- First of all, draw the couple polygon from the data given in Table 3.4 (column 7) as shown in Fig. 3.10 (a) to some suitable scale. The vector *do* represents the balanced couple. Since the balanced couple is proportional to 0.0855 m_Q, therefore by measurement,

$$0.0855\ m_Q = \text{vector } do = 30.15 \text{ kg-m}^2$$

or $\qquad$ $\mathbf{m_Q = 352.63}$ **kg.**

- By measurement, the angular position of m_Q is θ_Q = **111°** in the anticlockwise direction from mass m_A (*i.e.* 150 kg).

- Now draw the force polygon from the data given in Table 3.4 (column 5) as shown in Fig. 3.10 (b). The vector *eo* represents the balanced force. Since the balanced force is proportional to 0.18 m_P, therefore by measurement,

$$0.18\ m_P = \text{vector } eo = 41.5 \text{ kg-m}$$

Or $\qquad$ $\boldsymbol{m_P}$ **= 230.5 kg.**

- By measurement, the angular position of m_P is $\boldsymbol{\theta_P}$ **= 256°** in the anticlockwise direction from mass m_A (*i.e.* 150kg).

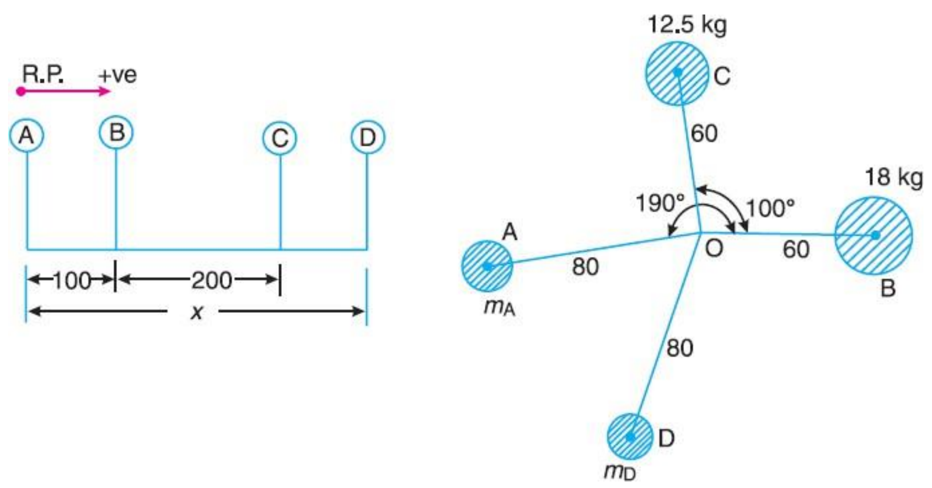
Example 3.6: A shaft carries four masses in parallel planes A, B, C and D in this order along its length. The masses at B and C are 18 kg and 12.5 kg respectively, and each has an eccentricity of 60 mm. The masses at A and D have an eccentricity of 80 mm. The angle between the masses at B and C is 100° and that between the masses at B and A is 190°, both being measured in the same direction. The axial distance between the planes A and B is 100 mm and that between B and C is 200 mm. If the shaft is in complete dynamic balance, determine: 1. The magnitude of the masses at A and D;

2. The distance between planes A and D; and

3. The angular position of the mass at D.

m_A = ?	r_A = 80 mm	θ_A = 190°
m_B = 18 kg	r_B= 60 mm	θ_B = 0°
m_C = 12.5 kg	r_C = 60 mm	θ_C = 100°
m_D = ?	r_D = 80 mm	θ_D = ?

X= Distance between planes *A* and *D*.



(a) Position of planes. *(b) Angular position of masses.*

Fig. 3.11

– The position of the planes and angular position of the masses is shown in Fig. 3.11 (a) and (b) respectively. The position of mass B is assumed in the horizontal direction, i.e. along OB. Taking the plane of mass A as the reference plane, the data may be tabulated as below:

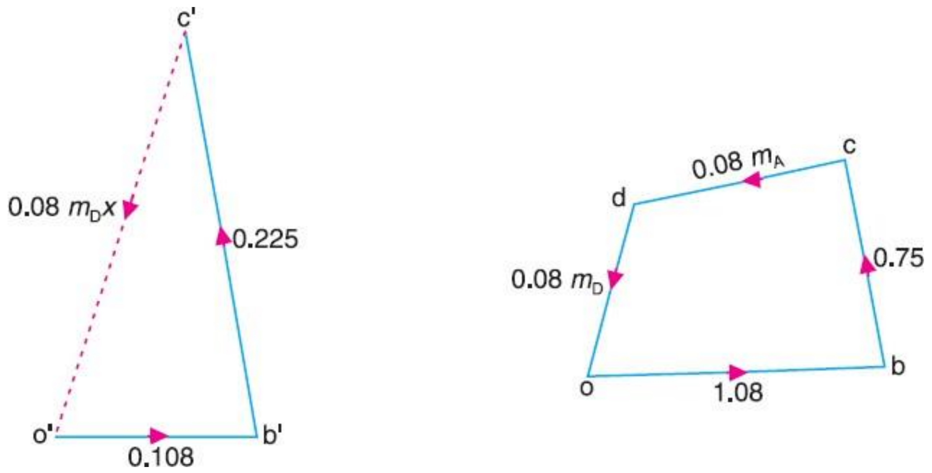
Table 3.5

Plane	Angle	Mass (m) kg	Radius (r) m	Cent.force ÷ ω^2 *(mr)* kg-m	Distance from Ref. Plane *(l)* m	Couple ÷ ω^2 *(mrl)* kg-m^2
A (R.P.)	190°	m_A	0.08	0.08 m_A	0	0
B	0°	18	0.06	1.08	0.1	0.108
C	100°	12.5	0.06	0.75	0.3	0.225
D	θ_D	m_D	0.08	0.08 m_D	*X*	0.08 m_DX

– First of all, draw the couple polygon from the data given in Table 3.5 (column 7) as shown in Fig. 3.12 (a) to some suitable scale. The closing side of the polygon (vector c'o') is proportional to 0.08 $m_D.X$, therefore by measurement,

$$0.08\ m_D X = \text{vector } c'o' = 0.235 \text{ kg-m}^2 \quad \text{......(i)}$$

– By measurement, the angular position of m_D is θ_D **= 251°** in the anticlockwise direction from mass m_B (*i.e.* 18 kg).



(a) Couple Polygon *(b) Force Polygon*

Fig. 3.12

– Now draw the force polygon, to some suitable scale, as shown in Fig. 3.11 (b), from the data given in Table 3.5 (column 5), as discussed below :

i. Draw vector ob parallel to OB and equal to 1.08 kg-m.
ii. From point b, draw vector bc parallel to OC and equal to 0.75 kg-m.
iii. For the shaft to be in complete dynamic balance, the force polygon must be a closed. Therefore from point *c*, draw vector cd parallel to *OA* and from point *o* draw vector *od* parallel to *OD*. The vectors *cd* and *od* intersect at *d*. Since the vector *cd* is proportional to 0.08 m_A, therefore by measurement

$$0.08\ m_A = \text{vector } cd = 0.77 \text{ kg-m}$$

or $\mathbf{m_A = 9.625\ kg.}$

– and vector *do* is proportional to 0.08 m_D, therefore by measurement,

$$0.08\ m_D = \text{vector do} = 0.65 \text{ kg-m}$$

or $\mathbf{m_D = 8.125\ kg.}$

– Distance between planes A and D

From equation (i),

$$0.08\ m_D.X = 0.235 \text{ kg-m}^2$$

$$0.08 \times 8.125 \times X = 0.235 \text{ kg-m}^2$$

$$\boldsymbol{X = 0.3615\ m}$$

$$\mathbf{= 361.5\ mm}$$

Example 3.7: A rotating shaft carries four masses A, B, C and D which are radially attached to it. The mass centers are 30 mm, 40 mm, 35 mm and 38 mm respectively from the axis of rotation. The masses A, C and D are 7.5 kg, 5 kg and 4 kg respectively. The axial distances between the planes of rotation of A and B is 400 mm and between B and C is 500 mm. The masses A and C are at right angles to each other. Find for a complete balance,

(i) the angles between the masses B and D from mass A,

(ii) the axial distance between the planes of rotation of C and D, and

(iii) the magnitude of mass B.

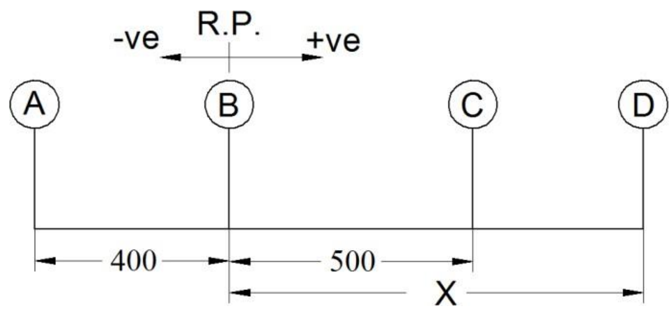
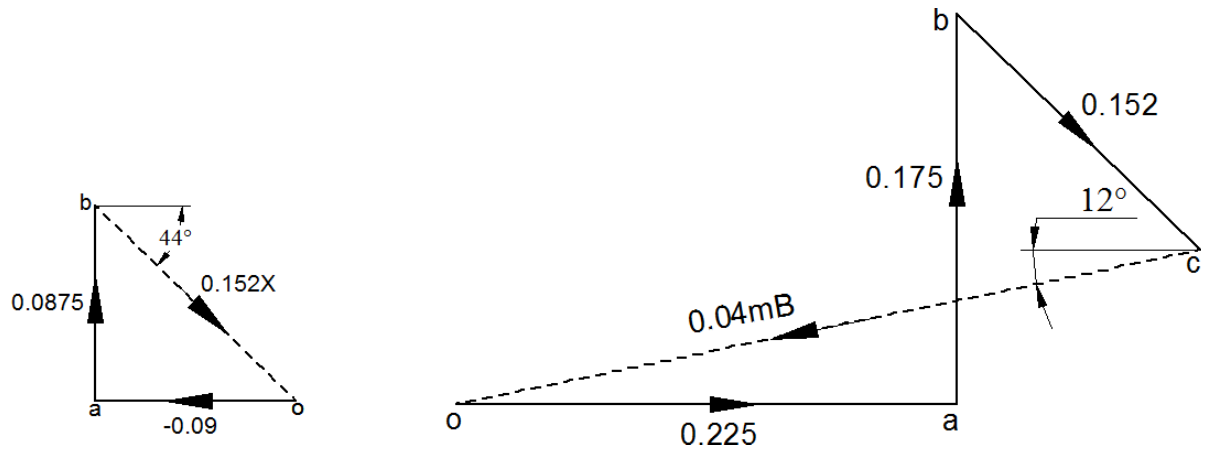


Fig. 3.13 Position of planes

Table 3.6

Plane	Angle	Mass (m) kg	Radius (r) m	Cent.force ÷ ω^2 *(mr)* kg-m	Distance from Ref. Plane *(l)* m	Couple ÷ ω^2 *(mrl)* kg-m^2
A	0°	7.5	0.03	0.225	– 0.4	– 0.09
B (R.P.)	θ_B	m_B	0.04	0.04 m_B	0	0
C	90°	5	0.035	0.175	0.5	0.0875
D	θ_D	4	0.038	0.152	X	0.152X



(a) Couple Polygon *(b) Force Polygon*

Fig. 3.14

– First of all, draw the couple polygon from the data given in Table 3.6 (column 7) as shown in Fig. 3.14 (a) to some suitable scale. The vector *bo* represents the balanced couple. Since the balanced couple is proportional to 0.152X, therefore by measurement,

$$0.152X = \text{vector } bo$$
$$= 0.13 \text{ kg-m}^2$$
or $$X = 0.855 \text{ m.}$$

The axial distance between the planes of rotation of **C and D = 855 – 500 = 355 mm**

- By measurement, the angular position of m_D is θ_D = **360° – 44° = 316°** in the anticlockwise direction from mass m_A (*i.e*. 7.5 kg).

- Now draw the force polygon from the data given in Table 3.6 (column 5) as shown in Fig. 3.14 (b). The vector *co* represents the balanced force. Since the balanced force is proportional to 0.04 m_B, therefore by measurement,

$$0.04\, m_B = \text{vector } co$$
$$= 0.34 \text{ kg-m}$$
or $$\boldsymbol{m_B = 8.5 \text{ kg.}}$$

- By measurement, the angular position of m_B is θ_B = **180° + 12° = 192°** in the anticlockwise direction from mass m_A (*i.e*. 7.5 kg).

Example 3.8: The four masses A, B, C and D revolve at equal radii are equally spaces along the shaft. The mass B is 7 kg and radii of C and D makes an angle of 90° and 240° respectively (counterclockwise) with radius of B, which is horizontal. Find the magnitude of A, C and D and angular position of A so that the system may be completely balance. Solve problem by analytically.

Table 3.7

Plane	Angle	Mass (m) kg	Radius (r) m	Cent.force ÷ ω^2 *(mr)* kg-m	Distance from Ref. Plane *(l)* m	Couple ÷ ω^2 *(mrl)* kg-m²
A (R.P.)	θ_A	m_A	X	m_A	0	0
B	0°	7	X	7	Y	$7Y$
C	90°	m_C	X	m_C	$2Y$	$2m_CY$
D	240°	m_D	X	m_D	$3Y$	$3m_DY$

mrlcosθ (H_C)	*mrl sin*θ (V_C)	*mrcos*θ (H_F)	*mr sin*θ (V_F)
0	0	$m_A\cos\theta_A$	$m_A\sin\theta_A$
$7Y$	0	7	0
0	$2m_CY$	0	m_C
$-1.5m_DY$	$-2.59m_DY$	$-0.5m_D$	$-0.866m_D$

$\Sigma H_C = 0$

$0 + 7Y + 0 - 1.5m_DY = 0$

$m_D = 7/1.5$

$\boldsymbol{m_D = 4.67 \text{ kg}}$

$\Sigma V_C = 0$

$0 + 0 + 2m_C Y - 2.59 m_D Y = 0$

$\mathbf{m_C = 6.047\ kg}$

$\Sigma H_F = 0$

$m_A \cos\theta_A + 7 + 0 - 0.5 m_D = 0$

$m_A \cos\theta_A = -4.665$

$\Sigma V_F = 0$

$m_A \sin\theta_A + 0 + m_C - 0.866 m_D = 0$

$m_A \sin\theta_A = -2.00278$

$$m_A = \sqrt{(-4.665)^2 + (-2.00278)^2}$$

$\mathbf{m_A = 5.076\ kg}$

$$\tan\theta_A = \frac{m_A \sin\theta_A}{m_A \cos\theta_A} = \frac{-2.00278}{-4.665} = 0.43$$

$\theta_A = 23.23°$

$\theta_A = 180° + 23.23°$

$\theta_A = 203.23°$

3.7 Balancing Machines

- A balancing machine is able to indicate whether a part is in balance or not and if it is not, then it measures the unbalance by indicating its magnitude and location.

3.7.1. Static Balancing Machines

- Static balancing machines are helpful for parts of small axial dimensions such as fans, gears and impellers, etc., in which the mass lies practically in a single plane.
- There are two machine which are used as static balancing machine: Pendulum type balancing machine and Cradle type balancing machine.

(i) Pendulum type balancing machine

- Pendulum type balancing machine as shown in Figure 3.15 is a simple kind of static balancing machine. The machine is of the form of a weighing machine.
- One arm of the machine has a mandrel to support the part to be balanced and the other arm supports a suspended deadweight to make the beam approximately horizontal.
- The mandrel is then rotated slowly either by hand or by a motor. As the mandrel is rotated, the beam will oscillate depending upon the unbalance of the part.
- If the unbalance is represented by a mass m at radius r, the apparent weight is greatest when m is at the position I and least when it is at B as the lengths of the arms in the two cases will be maximum and minimum.

- A calibrated scale along with the pointer can also be used to measure the amount of unbalance. Obviously, the pointer remains stationary in case the body is statically balanced.

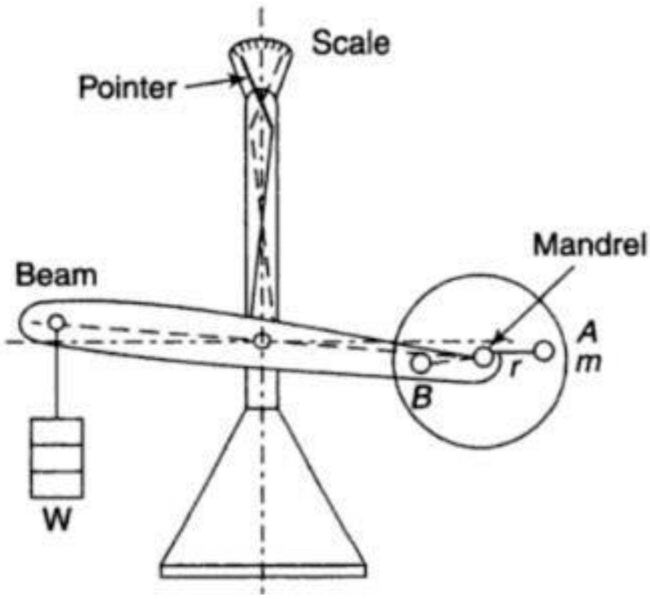


Fig. 3.15

(ii) Cradle type balancing machine

- Cradle type balancing machine as shown in fig. 3.16 is more sensitive machine than the pendulum type balancing machine.
- It consists of a cradle supported on two pivots P-P parallel to the axis of rotation of the part and held in position by two springs S-S.
- The part to be tested is mounted on the cradle and is flexibly coupled to an electric motor. The motor is started and the speed of rotation is adjusted so that it coincides with the natural frequency of the system.
- Thus, the condition of resonance is obtained under which even a small amount of unbalance generates large amplitude of the cradle.
- The moment due to unbalance = $(mr\omega^2 \cos\theta).l$ where ω is the angular velocity of rotation. Its maximum value is $mr\omega^2 l$. If the part is in static balance but dynamic unbalance, no oscillation of the cradle will be there as the pivots are parallel to the axis of rotation.

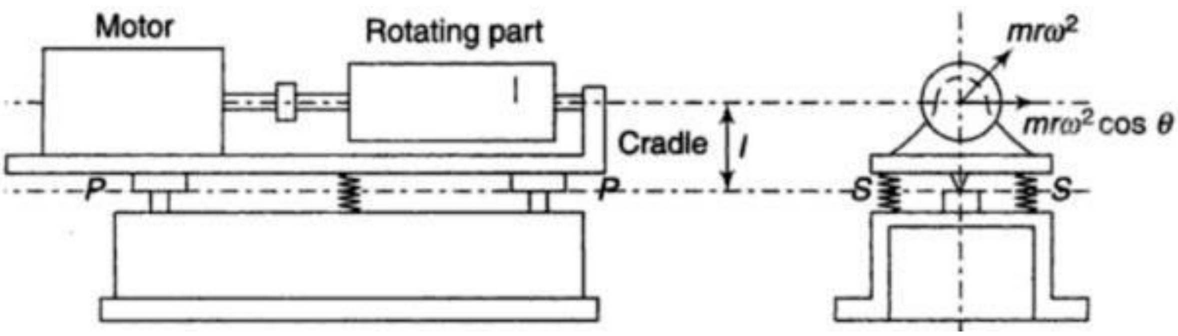


Fig. 3.16

3.7.2. Dynamic Balancing Machines

- For dynamic balancing of a rotor, two balancing or countermasses are required to be used in any two convenient planes. This implies that the complete unbalance of any rotor system can be represented by two unbalances in those two planes.
- Balancing is achieved by addition or removal of masses in these two planes, whichever is convenient. The following is a common type of dynamic balancing machine.

Pivoted-cradle Balancing Machine

- Fig 3.17 shows a pivot cradle type dynamic balancing machine. Here, part which is required to be balanced is to be mounted on cradle supported by supported rollers and it is connected to drive motor through universal coupling.
- Two planes are selected for dynamic balancing as shown in fig. 3.17 where pivots are provided about which the cradle is allowed to oscillate.
- As shown in fig 3.17, right pivot is released condition and left pivot is in locked position so as to allow the cradle and part to oscillate about the pivot.
- At the both ends of the cradle, the spring and dampers are attached such that the natural frequency can be adjusted and made equal to the motor speed. Two amplitude indicators are attached at each end of the cradle.
- The permanent magnet is mounted on the cradle which moves relative to stationary coil and generates a voltage which is directly proportional to the unbalanced couple. This voltage is amplified and read from the calibrated voltmeter and gives output in terms of kg-m.
- When left pivot is locked, the unbalanced in the right correction plane will cause vibration whose amplitude is measured by the right amplitude indicator.
- After that right pivot is locked and another set of measurement is made for left hand correction plane using the amplitude indicator of the left hand side.

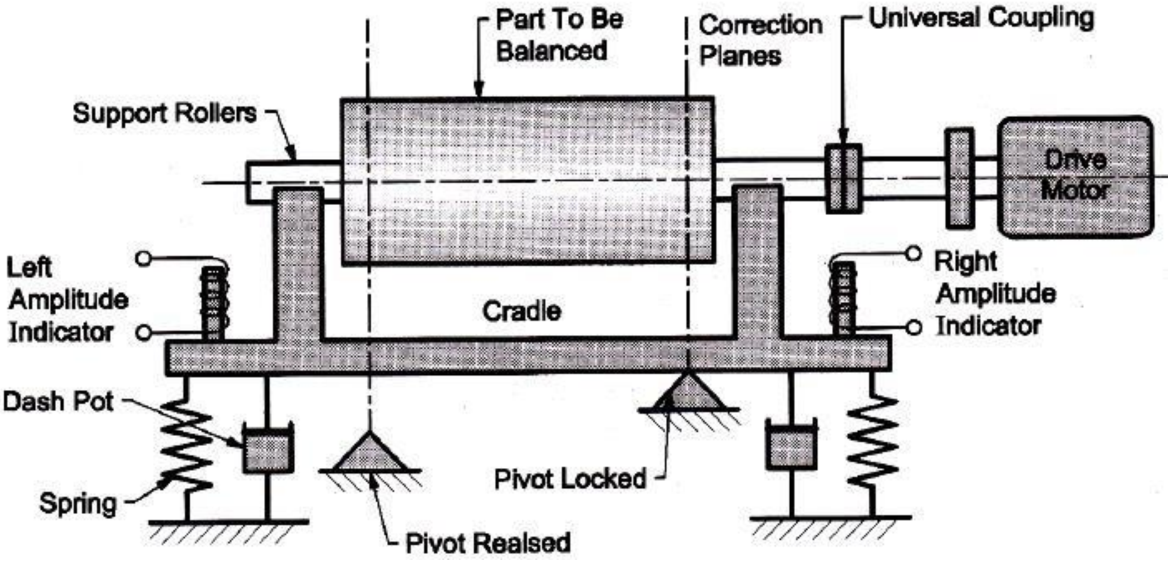


Fig. 3.17

3.8 Balancing of unbalanced forces in reciprocating masses

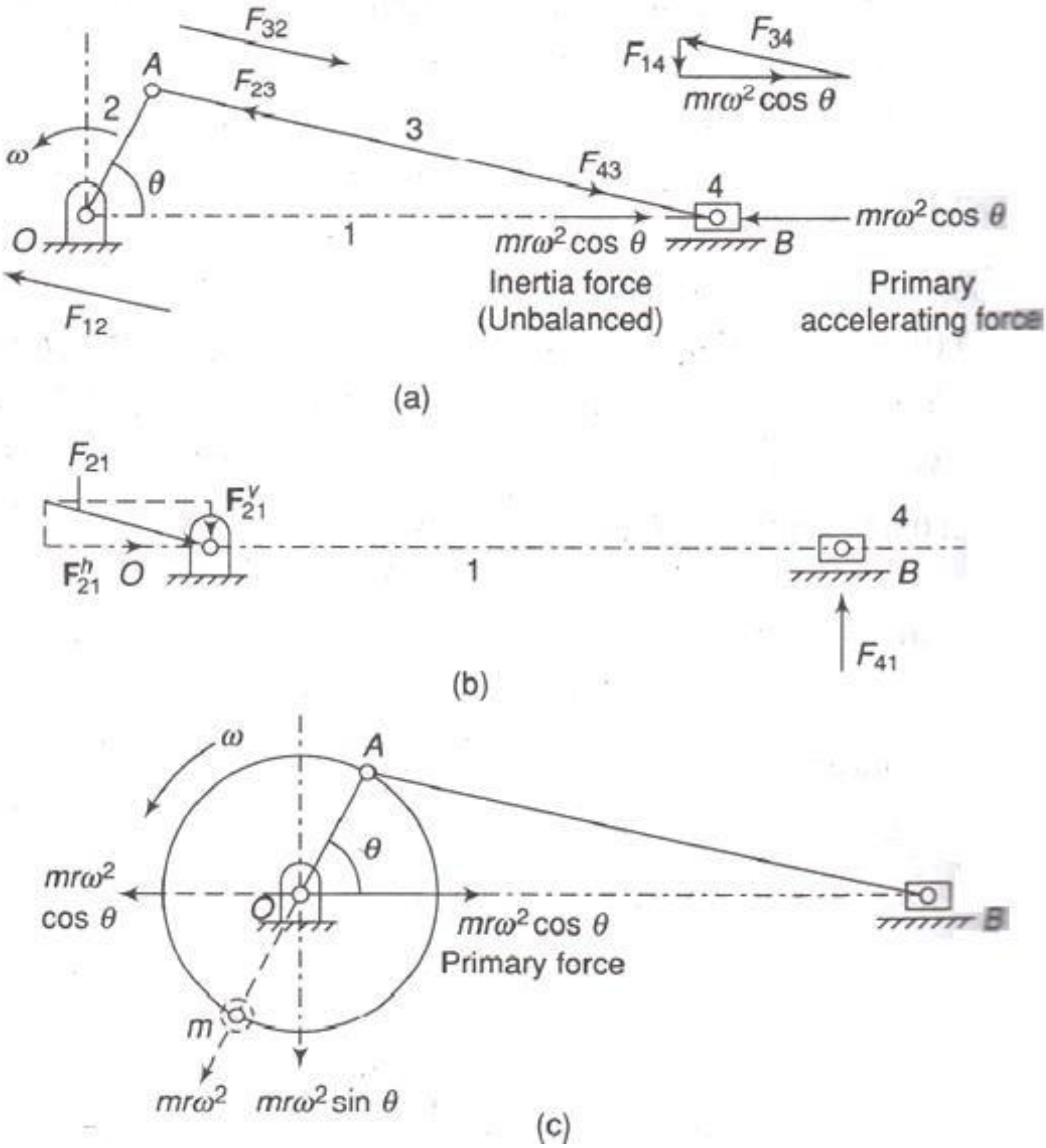


Fig. 3.18

– Acceleration of reciprocating mass of a slider-crank mechanism is given by

$$a = r\omega^2 \left(\cos\theta + \frac{\cos 2\theta}{n}\right)$$

– Therefore, the force required to accelerate mass *m* is

$$F = mr\omega^2 \left(\cos\theta + \frac{\cos 2\theta}{n}\right)$$

$$F = mr\omega^2 \cos\theta + mr\omega^2 \frac{\cos 2\theta}{n}$$

- $mr\omega^2\cos\theta$ is called the primary accelerating force and $mr\omega^2 \cos2\theta/n$ is called the secondary accelerating force.

  Maximum value of the primary force = $mr\omega^2$

  Maximum value of the secondary force =$mr\omega^2/n$

- As n is, usually, much greater than unity, the secondary force is small, compared with the primary force and can be safely neglected for slow-speed engines.
- The inertia force due to primary accelerating force is shown in Fig. 3.18(a). In Fig. 3.18(b), the forces acting on the engine frame due to this inertia force are shown. The force exerted by the crankshaft on the main bearings has two components, $F21^h$ and $F21^v$.
- The horizontal force $F21^h$ is an unbalanced shaking force. The vertical forces $F21^v$ and $F41^v$ balance each other, but form an unbalanced shaking couple. The magnitude and direction of this force and couple go on changing with the rotation of the crank angle θ.
- The shaking force produces linear vibration of the frame in the horizontal direction whereas the shaking couple produces an oscillating vibration.
- Thus, it is seen that the shaking force $F21^h$ is the only unbalanced force. It may hamper the smooth running of the engine and Thus, effort is made to balance the same. However, it is not at all possible to balance it completely and only some modification can be made.
- The usual approach of balancing the shaking force is by addition of a rotating countermass at radius r directly opposite the crank which however, provides only a partial balance. This countermass is in addition to the mass used to balance the rotating unbalance due to the mass at the crank pin.
- Fig. 3.18(c) shows the reciprocating mechanism with a countermass m at the radial distance r. The horizontal component of the centrifugal force due to the balancing mass is $mr\omega^2\cos\theta$ in the line of stroke.
- This neutralizes the unbalanced reciprocating force. But the rotating mass also has a component $mr\omega^2\sin\theta$ perpendicular to the line of stroke which remains unbalanced. The unbalanced force is zero at the ends of the stroke when 0 = 0° or 180° and maximum at the middle when θ = 90°.
- The magnitude or the maximum value of the unbalanced force remains the same, i.e., equal to $mr\omega^2$. Thus, instead of sliding to and fro on its mounting, the mechanism tends to jump up and down.
- To minimize the effect of the unbalanced force, a compromise is, usually, made, i.e., 2/3 of the reciprocating mass is balanced (or a value between one-half and three-quarters). If c is the fraction of the reciprocating mass Thus, balanced then

primary force balanced by the mass = $cmr\omega^2\cos\theta$
primary force unbalanced by the mass = $(1 - c)\, mr\omega^2\cos\theta$

- Vertical component of centrifugal force which remains unbalanced

$$= cmr\omega^2\sin\theta$$

- In fact, in reciprocating engines, unbalanced forces in the direction of the line of stroke are more dangerous than forces perpendicular to the line of stroke.
- Resultant unbalanced force at any instant

$$= \sqrt{[(1-c)mr\omega^2\cos\theta]^2 + [cmr\omega^2\sin\theta]^2}$$

- The resultant unbalanced force is minimum when $c = 1/2$.
- The method just discussed above to balance the disturbing effect of a reciprocating mass is just equivalent to as if a revolving mass at the crankpin is completely balanced by providing a countermass at the same radius diametrically opposite the crank.
- Thus, if mp is the mass at the crankpin and c is the fraction of the reciprocating mass m to be balanced, the mass at the crankpin may be considered as (cm + mp) which is to be completely balanced.

Example 3.9: The following data relate to a single - cylinder reciprocating engine:
Mass of reciprocating parts = 40 kg
Mass of revolving parts = 30 kg at crank radius
Speed = 150 rpm
Stroke = 350 mm
If 60% of the reciprocating parts and all the revolving parts are to be balanced, determine (i) balance mass required at a radius of 320 mm
(ii) unbalanced force when the crank has turned 45° from top dead centre.

$m = 40$ kg
$m_P = 30$ kg
$N = 150$ rpm
$r = l/2 = 175$ mm

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 150}{60}$$
$$= 15.7\ rad/s$$

(i) Mass to be balanced at crank pin = $cm + m_P$
$= 0.6 \times 40 + 30$
$= 54$ kg

$$m_C r_C = mr$$
$$m_C \times 320 = 54 \times 175$$
$$m_C = 29.53 \text{ kg.}$$

(ii) Unbalanced force (at θ = 45°)

$$= \sqrt{[(1-c)mr\omega^2\cos\theta]^2 + [cmr\omega^2\sin\theta]^2}$$

$$= \sqrt{[(1-0.6)\times 40\times 0.175\times (15.7)^2\cos 45°]^2 + [0.6\times 40\times 0.175\times (15.7)^2\sin 45°]^2}$$

= 880.1 N.

Example 3.10: A single cylinder reciprocating engine has speed 240 rpm, stroke 300 mm, mass of reciprocating parts 50 kg, mass of revolving parts at 150 mm radius 30 kg. If all the mass of revolving parts and two-third of the mass of reciprocating parts are to be balanced, find the balance mass required at radius of 400 mm and the residual unbalanced force when the crank has rotated 60° from IDC.

N = 240 rpm

l = 300 mm

m = 50 kg

m_P = 30 kg

r = l/2 =150 mm

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 240}{60}$$

$$= 25.13\, rad/s$$

(i) Mass to be balanced at crank pin = cm + m_P

= 2/3 x 50 + 30

= 63.33 kg

$m_C r_C$ = mr

m_C x 400 = 63.33 x 150

m_C = 23.75 kg.

(ii) Unbalanced force (at θ = 45°)

$$= \sqrt{[(1-c)mr\omega^2\cos\theta]^2 + [cmr\omega^2\sin\theta]^2}$$

$$= \sqrt{\left[\left(1-\frac{2}{3}\right)\times 50\times 0.15\times (25.13)^2\cos 60°\right]^2 + \left[\frac{2}{3}\times 50\times 0.15\times (25.13)^2\sin 60°\right]^2}$$

$$= \sqrt{(789.36)^2 + (2734.55)^2}$$

= 2846.2 N

3.9 Balancing of Locomotives

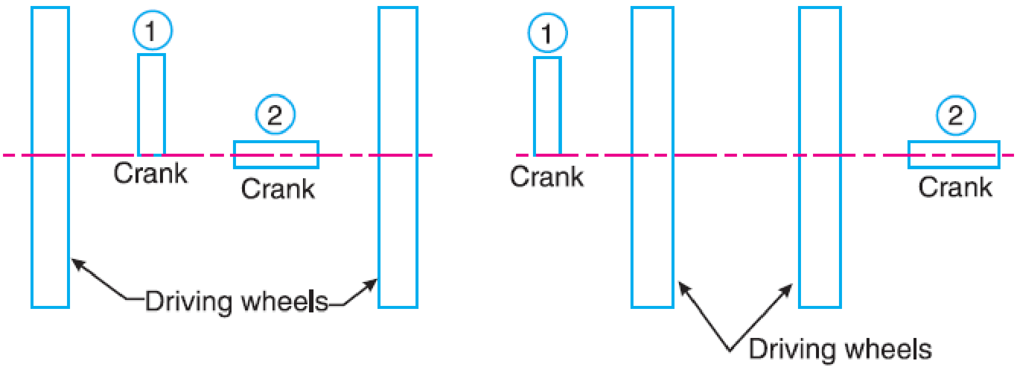
– Locomotives are of two types, coupled and uncoupled. If two or more pairs of wheels are coupled together to increase the adhesive force between the wheels and the track, it is called a coupled locomotive. Otherwise, it is an uncoupled locomotive.

– Locomotives usually have two cylinders. If the cylinders are mounted between the wheels, it is called an inside cylinder locomotive and if the cylinders are outside the wheels, it is an outside cylinder locomotive. The cranks of the two cylinders are set at 90° to each other so that the engine can be started easily after stopping in any position. Balance masses are placed on the wheels in both types.

– In coupled locomotives, wheels are coupled by connecting their crankpins with coupling rods. As the coupling rod revolves with the crankpin, its proportionate mass can be considered as a revolving mass which can be completely balanced.

– Thus, whereas in uncoupled locomotives, there are four planes for consideration, two of the cylinders and two of the driving wheels, in coupled locomotives there are six planes, two of cylinders, two of coupling rods and two of the wheels. The planes which contain the coupling rod masses lie outside the planes that contain the balance (counter) masses. Also, in case of coupled locomotives, the mass required to balance the reciprocating parts is distributed among all the wheels which are coupled. This results in a reduced hammer blow.

– Locomotives have become obsolete nowadays.



(a) Inside cylinder locomotives *(b) Outside cylinder locomotives*

Fig. 3.19

3.9.1 Effects of Partial Balancing in Locomotives

– Reciprocating parts are only partially balanced. Due to this partial balancing of the reciprocating parts, there is an unbalanced primary force along the line of stroke and also an unbalanced primary force perpendicular to the line of stroke.

I. Hammer-blow

– Hammer-blow is the maximum vertical unbalanced force caused by the mass provided to balance the reciprocating masses. Its value is $mr\omega^2$. Thus, it varies as a square of the speed. At high speeds, the force of the hammer-blow could exceed the static load on the wheels and the wheels can be lifted off the rail when the direction of the hammer-blow will be vertically upwards.

Hammer blow = $mr\omega^2$

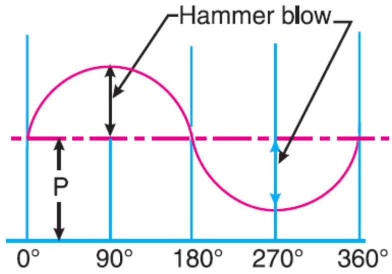


Fig. 3.20

II. Variation of Tractive Force

– A variation in the tractive force (effort) of an engine is caused by the unbalanced portion of primary force which acts along the line of stroke of a locomotive engine. If c is the fraction of the reciprocating mass that is balanced then

unbalanced primary force for cylinder 1 = $(1 - c)\, mr\omega^2\cos\theta$

unbalanced primary force for cylinder 2 = $(1 - c)\, mr\omega^2\cos(90° + \theta)$

$= -(1 - c)\, mr\omega^2\sin\theta$

Total unbalanced primary force or the variation in the tractive force

$= (1 - c)\, mr\omega^2(\cos\theta - \sin\theta)$

This is maximum when $(\cos\theta - \sin\theta)$ is maximum,

or when

$$\frac{d}{d\theta}(\cos\theta - \sin\theta) = 0$$

$$-\sin\theta - \cos\theta = 0$$

$$\sin\theta = -\cos\theta$$

$$\tan\theta = -1$$

$$\theta = 135° \text{ or } 315°$$

When $\theta = 135°$

Maximum variation in tractive force = $(1 - c)\, mr\omega^2(\cos 135° - \sin 135°)$

$$= (1 - c)mr\omega^2\left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right)$$

$$= -\sqrt{2}(1 - c)mr\omega^2$$

When $\theta = 315°$

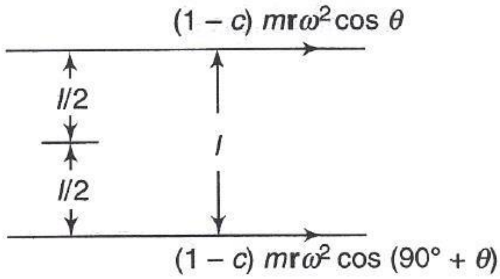
Maximum variation in tractive force = $(1 - c)\, mr\omega^2(\cos 315° - \sin 315°)$

$$= (1 - c)mr\omega^2\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)$$

$$= \sqrt{2}(1 - c)mr\omega^2$$

Thus, maximum variation $= \pm\sqrt{2}(1 - c)mr\omega^2$

III. Swaying Couple



Unbalanced primary forces along the lines of stroke are separated by a distance *l* apart and Thus, constitute a couple. This tends to make the leading wheels sway from side to side.

- Swaying couple = moments of forces about the engine center line

$$= \left[(1-c)mr\omega^2\cos\theta\right]\frac{l}{2} - \left[(1-c)mr\omega^2\cos(90^\circ+\theta)\right]\frac{l}{2}$$

$$= (1-c)mr\omega^2(\cos\theta + \sin\theta)\frac{l}{2}$$

- This is maximum when (cos θ + sin θ) is maximum.

i.e., when $\frac{d}{dt}(\cos\theta + \sin\theta) = 0$

$$-\sin\theta + \cos\theta = 0$$
$$\sin\theta = \cos\theta$$
$$\tan\theta = 1$$
$$\theta = 45^\circ \text{ or } 225^\circ$$

- When θ = 45°, maximum swaying couple $= \frac{1}{\sqrt{2}}(1-c)mr\omega^2 l$
- When θ = 225°, maximum swaying couple $= -\frac{1}{\sqrt{2}}(1-c)mr\omega^2 l$
- Thus, maximum swaying couple $= \pm\frac{1}{\sqrt{2}}(1-c)mr\omega^2 l$

Example 3.11: An inside cylinder locomotive has its cylinder center lines 0.7 m apart and has a stroke of 0.6 m. The rotating masses per cylinder are equivalent to 150 kg at the crank pin, and the reciprocating masses per cylinder to 180 kg. The wheel center lines are 1.5 m apart. The cranks are at right angles.

The whole of the rotating and 2/3 of the reciprocating masses are to be balanced by masses placed at a radius of 0.6 m. Find the magnitude and direction of the balancing masses.

Find the fluctuation in rail pressure under one wheel, variation of tractive effort and the magnitude of swaying couple at a crank speed of 300 r.p.m.

$l_B = l_C = 0.6$ m or $r_B = r_C = 0.3$ m;
$m_1 = 150$ kg $r_A = r_D = 0.6$ m;
$m_2 = 180$ kg; $c = 2/3$
$N = 300$ r.p.m.

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 300}{60}$$
$$= 31.42\ rad/s$$

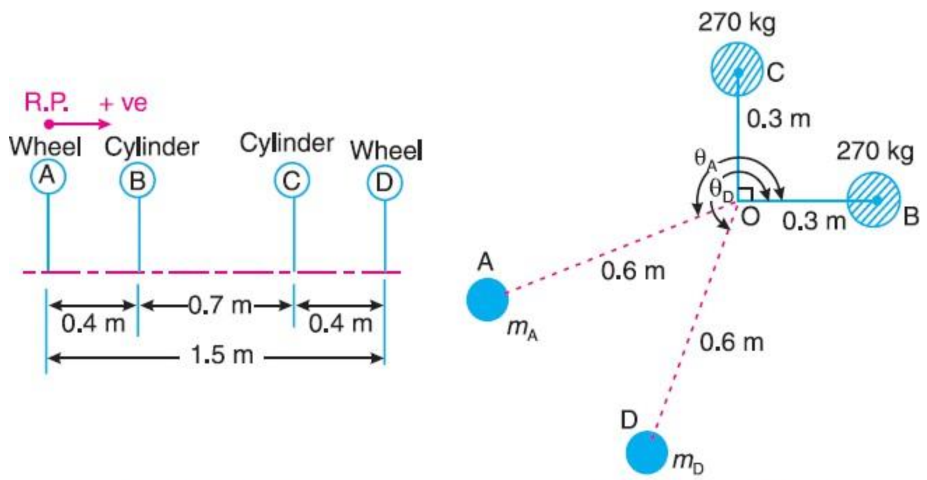
Equivalent mass of the rotating parts to be balanced per cylinder at the crank pin,

$$m = m_B = m_C = m_1 + c.m_2$$

$$= 150 + \frac{2}{3} \times 180 = 270 \text{ kg}$$

The magnitude and direction of balancing masses may be determined graphically as below:

- First of all, draw the space diagram to show the positions of the planes of the wheels and the cylinders, as shown in Fig. 3.21(*a*). Since the cranks of the cylinders are at right angles, therefore assuming the position of crank of the cylinder *B* in the horizontal direction, draw *OC* and *OB* at right angles to each other as shown in Fig. 3.21 (b).



(*a*) Position of planes. (*b*) Angular position of masses.

Fig. 3.21

- Tabulate the data as given in the following table. Assume the plane of wheel *A* as the reference plane.

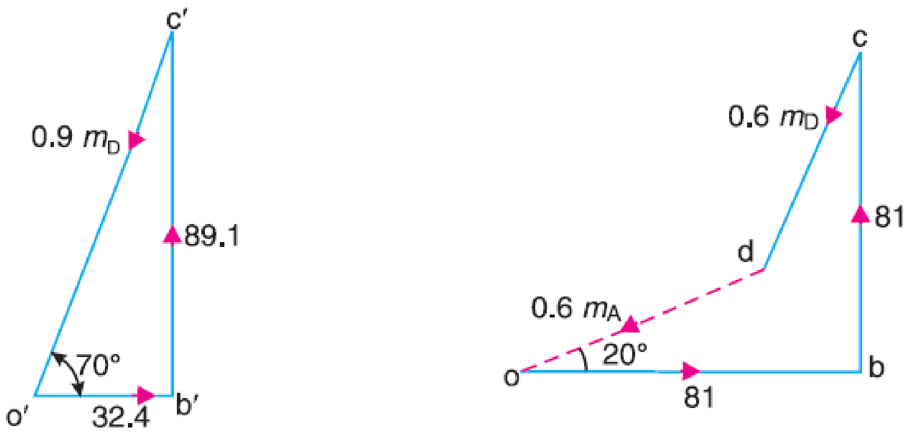
Table 3.8

Plane	Angle	Mass (m) kg	Radius (r) m	Cent.force ÷ ω^2 *(mr)* kg-m	Distance from Ref. Plane *(l)* m	Couple ÷ ω^2 *(mrl)* kg-m^2
A (R.P.)	θ_A	m_A	0.6	0.6 m_A	0	0
B	0°	270	0.3	81	0.4	32.4
C	90°	270	0.3	81	1.1	89.1
D	θ_D	m_D	0.6	0.6 m_D	1.5	0.9 m_D

- Now, draw the couple polygon from the data given in Table 3.8 (column 7), to some suitable scale, as shown in Fig 3.22 (a). The closing side *c′o′* represents the balancing couple and it is proportional to 0.9 m_D. Therefore, by measurement,

$$0.9\, m_D = \text{vector } c'o' = 94.5 \text{ kg-m}^2$$

$$\boldsymbol{m_D = 105 \text{ kg}}$$



(*a*) Couple polygon. (*b*) Force polygon.

Fig. 3.22

- By measurement, the angular position of m_D is $\boldsymbol{\theta_D}$ **= 250°** in the anticlockwise direction from mass m_B .
- In order to find the balancing mass *A*, draw the force polygon from the data given in Table 3.8 (column 5), to some suitable scale, as shown in Fig. 3.22 (b), The vector *do* represents the balancing force and it is proportional to 0.6 m_A. Therefore by measurement,

$$0.6\, m_A = \text{vector } do = 63 \text{ kg-m}$$

$$\boldsymbol{m_A = 105 \text{ kg}}$$

- By measurement, the angular position of m_A is $\boldsymbol{\theta_A}$ **= 200°** in the anticlockwise direction from mass m_B.

Each balancing mass = 105 kg

Balancing mass for rotating masses, $M = \dfrac{m_1}{m} \times 105 = \dfrac{150}{270} \times 105$

= 58.3 kg

Balancing mass for reciprocating masses, $M' = \dfrac{cm_2}{m} \times 105 = \dfrac{2}{3} \times \dfrac{180}{270} \times 105$

= 46.6 kg

Balancing mass of 46.6 kg for reciprocating masses gives rise to centrifugal force.

∴ Fluctuation in rail pressure or hammer blow = $M' r\omega^2$

$= 46.6 \times 0.6 \times (31.42)^2$

= 27602 N

- **Variation of tractive effort**

Maximum variation of tractive effort $= \pm \sqrt{2}(1-c)mr\omega^2$

$$= \sqrt{2}\left(1-\frac{2}{3}\right) \times 180 \times 0.3 \times (31.42)^2$$

= 25.123 KN

– **Swaying couple**

$$\text{Maximum swaying couple} = \pm\frac{1}{\sqrt{2}}(1-c)mr\omega^2 l$$

$$= \frac{1}{\sqrt{2}}(1-\frac{2}{3})\times 180\times 0.3\times (31.42)^2\times 0.7$$

= 8797 N.m

Example 3.12: The following data refers to two-cylinder uncoupled locomotive:

Rotating mass per cylinder	= 280 kg
Reciprocating mass per cylinder	= 300 kg
Distance between wheels	= 1400 mm
Distance between cylinder centers	= 600 mm
Diameter of treads of driving wheels	= 1800 mm
Crank radius	= 300 mm
Radius of centre of balance mass	= 620 mm
Locomotive speed	= 50 km/hr
Angle between cylinder cranks	= 90°
Dead load on each wheel	= 3.5 tonne

Determine

i. Balancing mass required in planes of driving wheels if whole of the revolving and 2/3 of reciprocating mass are to be balanced
ii. Swaying couple
iii. Variation in tractive force
iv. Maximum and minimum pressure on the rails
v. Maximum speed of locomotive without lifting the wheels from rails.

m_1 = 280 kg $\qquad r_B = r_C = 0.3$ m;

m_2 = 300 kg $\qquad r_A = r_D = 0.62$ m;

v = 50 km/hr $\qquad c = 2/3$

Dead load, W = 3.5 tonne

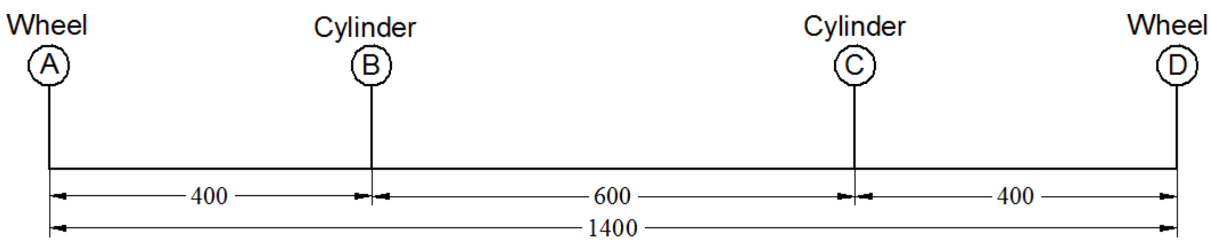


Fig. 3.23

Total mass to be balanced per cylinder, $m_B = m_C = m_1 + c.m_2$

$$= 280 + \frac{2}{3}\times 300$$

$$= 480\ kg$$

Table 3.9

Plane	Angle	Mass (m) kg	Radius (r) m	Cent.force ÷ ω^2 *(mr)* kg-m	Distance from Ref. Plane *(l)* m	Couple ÷ ω^2 *(mrl)* kg-m^2
A (R.P.)	θ_A	m_A	0.62	$0.62m_A$	0	0
B	0°	480	0.3	144	0.4	57.6
C	90°	480	0.3	144	1	144
D	θ_D	m_D	0.62	$0.62m_D$	1.4	$0.868m_D$

- Draw the couple polygon from the data given in Table 3.9 (column 7), to some suitable scale, as shown in Fig 3.24 (a). The closing side *bo* represents the balancing couple and it is proportional to $0.868m_D$. Therefore, by measurement,

$$0.868m_D = \text{vector bo} = 155.1 \text{ kg-m}^2$$

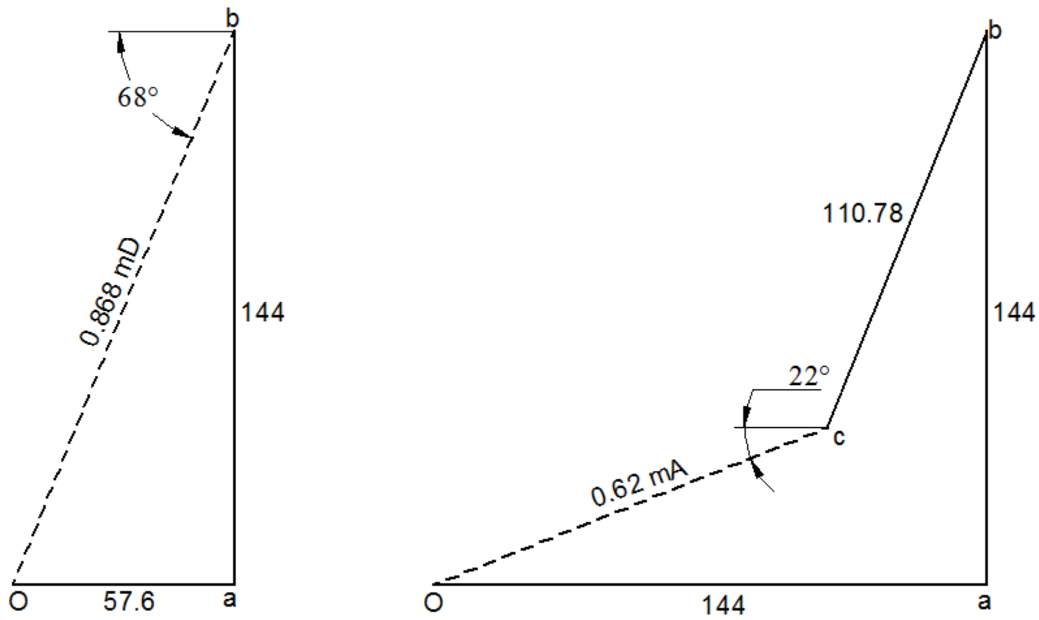
$$\boldsymbol{m_D = 178.68 \text{ kg}}$$

- By measurement, the angular position of m_D is $\boldsymbol{\theta_D = 248°}$ in the anticlockwise direction from mass m_B.
- Draw the force polygon from the data given in Table 3.9 (column 5), to some suitable scale, as shown in Fig 3.24 (b). The closing side *co* represents the balancing couple and it is proportional to $0.62m_A$. Therefore, by measurement,

$$0.62m_A = \text{vector co} = 110.78 \text{ kg-m}^2$$

$$\boldsymbol{m_A = 178.68 \text{ kg}}$$

- By measurement, the angular position of m_A is $\boldsymbol{\theta_A = 202°}$ in the anticlockwise direction from mass m_B.



(a) Couple Polygon *(b) Force Polygon*

Fig. 3.24

v = rω

$$\omega = \frac{v}{r} = \frac{50 \times 10^6}{60 \times 60} \times \frac{1}{1800/2} = 15.43\, rad/s$$

$$\text{Swaying couple} = \pm\frac{1}{\sqrt{2}}(1-c)mr\omega^2 l$$

$$= \frac{1}{\sqrt{2}}(1-\frac{2}{3})\times 300\times 0.3\times(15.43)^2\times 0.6$$

= 3030.3 N.m

$$\text{Variation of tractive effort} = \pm\sqrt{2}(1-c)mr\omega^2$$

$$= \sqrt{2}(1-\frac{2}{3})\times 300\times 0.3\times(15.43)^2$$

= 10100 N

Balance mass for reciprocating parts only

$$= 178.7\times\frac{\frac{2}{3}\times 300}{480} = 74.46\ kg$$

Hammer blow = $mr\omega^2$

$= 74.46 \times 0.62 \times (15.43)^2 = 10991$ N

Dead load = 3.5 x 1000 x 9.81

= 34335 N

Maximum pressure on rails = 34335 + 10991

= 45326 N

Minimum pressure on rails = 34335 – 10991

= 23344 N

Maximum speed of the locomotive without lifting the wheels from the rails will be when the dead load becomes equal to the hammer blow

$74.46 \times 0.62 \times \omega^2 = 34335$

$\omega = 27.27$ rad/s

Velocity of wheels = $r\omega$

$$= \left(\frac{1.8}{2}\times 27.27\right) m/s$$

$$= \left(27.27\times\frac{1.8}{2}\times\frac{60\times 60}{1000}\right) km/hr$$

V = 88.36 km / hr

Example 3.13: The three cranks of a three-cylinder locomotive are all on the same axle and are set at 120°. The pitch of the cylinders is 1 meter and the stroke of each piston is 0.6 m. The reciprocating masses are 300 kg for inside cylinder and 260 kg for each outside cylinder and the planes of rotation of the balance masses are 0.8 m from the inside crank.

If 40% of the reciprocating parts are to be balanced, find:

(i) The magnitude and the position of the balancing masses required at a radius of 0.6 m

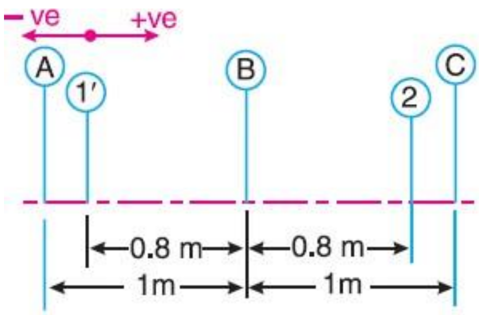
(ii) The hammer blow per wheel when the axle makes 6 r.p.s.

$l_A = l_B = l_C = 0.6$ m or $r_A = r_B = r_C = 0.3$ m ;

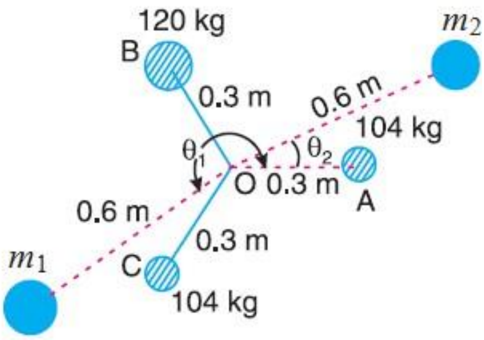
$m_I = 300$ kg $r_1 = r_2 = 0.6$ m ;

$m_O = 260$ kg $c = 40\% = 0.4$;

$N = 6$ r.p.s. $= 6 \times 2\pi = 37.7$ rad/s



(a) Position of planes.



(b) Position of cranks.

Fig. 3.25

Since 40% of the reciprocating masses are to be balanced, therefore mass of the reciprocating parts to be balanced for each outside cylinder,

$$m_A = m_C = c \times m_O = 0.4 \times 260$$
$$= 104 \text{ kg}$$

and mass of the reciprocating parts to be balanced for inside cylinder,

$$m_B = c \times m_I = 0.4 \times 300$$
$$= 120 \text{ kg}$$

Table 3.10

Plane	**Angle**	**Mass (m) Kg**	**Radius (r) m**	**Cent.force ÷ ω^2 *(mr)* kg-m**	**Distance from Ref. Plane *(l)* m**	**Couple ÷ ω^2 *(mrl)* kg-m^2**
A	0°	104	0.3	31.2	–0.2	–6.24
1 (R.P.)	θ_1	m_1	0.6	0.6 m_1	0	0
B	120°	120	0.3	36	0.8	28.8
2	θ_2	m_2	0.6	0.6 m_2	1.6	0.96 m_2
C	240°	104	0.3	31.2	1.8	56.16

– Draw the couple polygon with the data given in Table 3.10 (column 7), to some suitable scale, as shown in Fig. 3.26 (a). The closing side *c′o′* represents the balancing couple and it is proportional to 0.96 m_2. Therefore, by measurement,

$$0.96\, m_2 = \text{vector } c'o' = 55.2 \text{ kg-m}^2$$

$$\mathbf{m_2 = 57.5 \text{ kg.}}$$

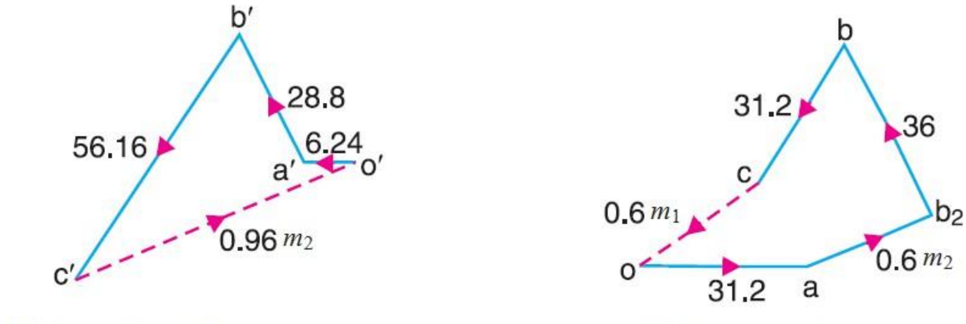
– By measurement, the angular position of m_2 is $\boldsymbol{\theta_2 = 24°}$ in the anticlockwise direction from mass m_A.

– Draw the force polygon with the data given in Table 3.10 (column 5), to some suitable scale, as shown in Fig. 3.26 (b). The closing side *co* represents the balancing force and it is proportional to 0.6 m_1. Therefore, by measurement,

$0.6\ m_1$ = vector co= 34.5 kg-m

m_1 = 57.5 kg.

- By measurement, the angular position of m_1 is **θ_1= 215°** in the anticlockwise direction from mass m_A.



(a) Couple polygon. (b) Force polygon.

Fig. 3.26

Hammer blow per wheel = $mr\omega^2$

$= 57.5 \times 0.6 \times (37.7)^2$

= 49 035 N.

Example 3.14: A two-cylinder locomotive has the following specifications;

Reciprocating mass per cylinder = 300 Kg

Crank radius = 300 mm

Angle between cranks = 90°

Driving wheels diameter = 1800 mm

Distance between cylinder centers = 650 mm

Distance between driving wheel planes = 1550 mm

Determine (a) The fraction of reciprocating masses to be balanced, if the hammer blow is not to exceed 46 KN at 96.5 Km/hr.

(b) The variation in tractive force.

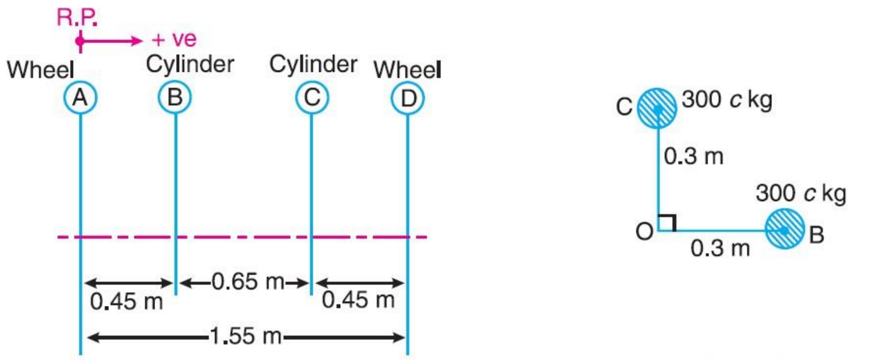
(c) The maximum swaying couple.

m = 300 kg | D = 1.8 m or R = 0.9 m

r = 0.3 m | Hammer blow = 46 kN

v = 96.5 km/h = 26.8 m/s

The mass of the reciprocating parts to be balanced = $c.m$ = 300c kg



(a) Position of planes. (b) Position of cranks.

Fig. 3.27

Table 3.11

Plane	Angle	Mass (m) kg	Radius (r) m	Cent.force ÷ ω^2 *(mr)* kg-m	Distance from Ref. Plane *(l)* m	Couple ÷ ω^2 *(mrl)* kg-m^2
A (R.P.)	θ_A	m_A	r_A	$m_A r_A$	0	0
B	0°	300*c*	0.3	90*c*	0.45	40.5*c*
C	90°	300*c*	0.3	90*c*	1.1	99*c*
D	θ_D	m_D	r_D	$m_D r_D$	1.55	$1.55 m_D r_D$

Now the couple polygon, to some suitable scale, may be drawn with the data given in Table 3.11 (column 7), as shown in Fig. 3.28. The closing side of the polygon (vector *c′o′*) represents the balancing couple and is proportional to 1.55 *B.b*.

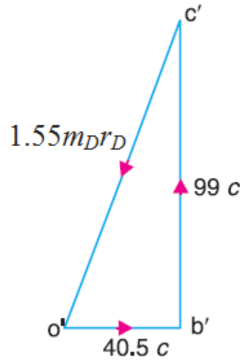


Fig. 3.28

From the couple polygon,

$$1.55 m_D r_D = \sqrt{(40.5c)^2 + (99c)^2} = 107c$$

$$m_D r_D = 69c$$

Angular speed, $\omega = v/R$

$= 26.8/0.9$

$= 29.8$ rad/s

Hammer blow $= m_D r_D \omega^2$

$46000 = 69c(29.8)^2$

$\boldsymbol{c = 0.751}$

$$\text{Variation of tractive effort} = \pm\sqrt{2}(1-c)mr\omega^2$$

$$= \sqrt{2}(1-0.751)\times 300\times 0.3\times(29.8)^2$$

= 28140 N

$$\text{Swaying couple} = \pm\frac{1}{\sqrt{2}}(1-c)mr\omega^2 l$$

$$= \frac{1}{\sqrt{2}}(1-0.751)\times 300\times 0.3\times(29.8)^2\times 0.65 = 9148\ N.m$$

Example 3.15: The following data apply to an outside cylinder uncoupled locomotive:

Mass of rotating parts per cylinder = 360 kg
Mass of reciprocating parts per cylinder = 300 kg
Angle between cranks = 90°
Crank radius = 0.3 m
Cylinder centres = 1.75 m
Radius of balance masses = 0.75 m
Wheel centres = 1.45 m.

If whole of the rotating and two-thirds of reciprocating parts are to be balanced in planes of the driving wheels, find:

(a) Magnitude and angular positions of balance masses,

(b) Speed in km/hr at which the wheel will lift off the rails when the load on each driving wheel is 30 kN and the diameter of tread of driving wheels is 1.8 m, and

(c) Swaying couple at speed arrived at in (b) above.

$m_1 = 360$ kg $\quad\quad r_A = r_D = 0.3$ m

$m_2 = 300$ kg $\quad\quad r_B = r_C = 0.75$ m

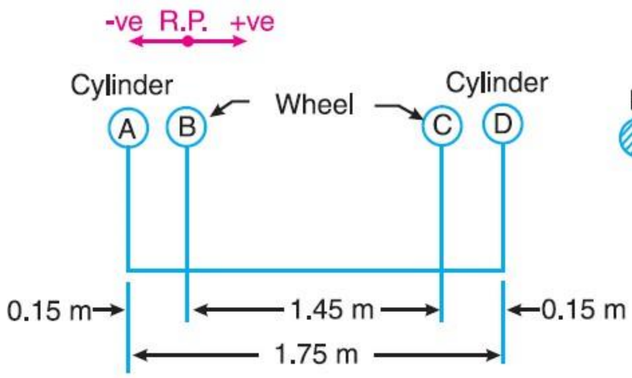
$c = 2 / 3.$

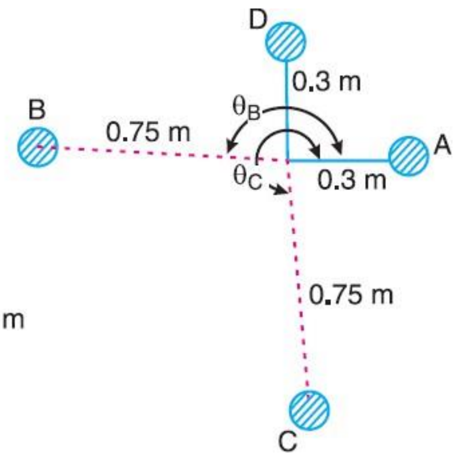
The equivalent mass of the rotating parts to be balanced per cylinder,

$$m = m_A = m_D = m_1 + c.m_2$$

$$= 360 + \frac{2}{3} \times 300$$

$$= 560 \text{ kg}$$





(a) Position of Planes *(b) Position of Masses*

Fig. 3.29

Table 3.12

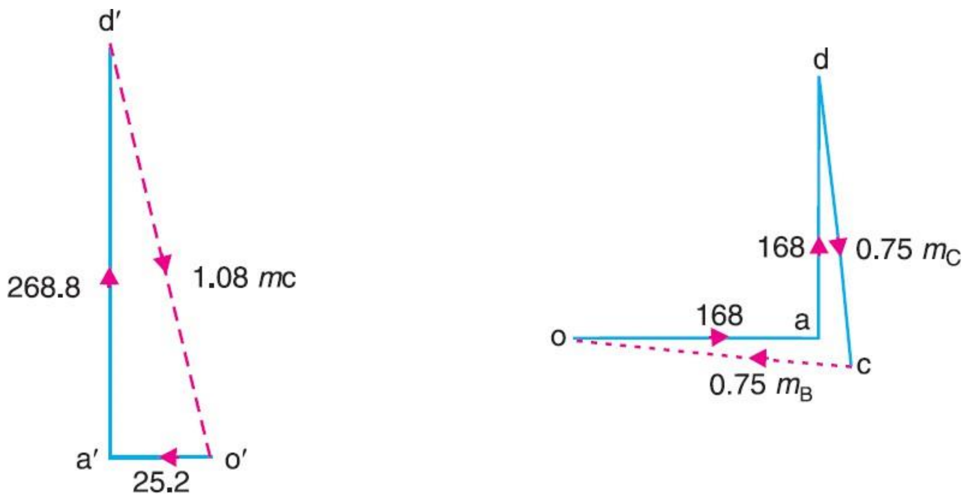
Plane	Angle	Mass (m) Kg	Radius (r) m	Cent.force ÷ ω^2 *(mr)* kg-m	Distance from Ref. Plane *(l)* m	Couple ÷ ω^2 *(mrl)* kg-m^2
A	0°	560	0.3	168	−0.15	−25.2
B (R.P.)	θ_B	m_B	0.75	0.75 m_B	0	0
C	θ_C	m_C	0.75	0.75 m_C	1.45	1.08 m_C
D	90°	560	0.3	168	1.6	268.8

– Draw the couple polygon with the data given in Table 3.12 column (7), to some suitable scale as shown in Fig. 3.30(a). The closing side d'o' represents the balancing couple and it is proportional to 1.08 m_C. Therefore, by measurement,

$$1.08\, m_C = 269.6 \text{ kg-m}^2$$

$$\mathbf{m_C = 249 \text{ kg}}$$

– By measurement, the angular position of m_C is $\boldsymbol{\theta_C = 275°}$ in the anticlockwise direction from mass m_A.



(a) Couple Polygon *(b) Force Polygon*

Fig. 3.30

– Draw the force polygon with the data given in Table 3.12 column (5), to some suitable scale as shown in Fig. 3.30(b). The closing side *co* represents the balancing force and it is proportional to 0.75 m_B. Therefore, by measurement,

$$0.75\ m_B = 186.75 \text{ kg-m}$$

$$\mathbf{m_B = 249\ kg}$$

– By measurement, the angular position of m_B is $\boldsymbol{\theta_B = 174.5°}$ in the anticlockwise direction from mass m_A.

Speed at which the wheel will lift off the rails

Given : P = 30 kN = 30000 N D = 1.8 m

ω = Angular speed at which the wheels will lift off the rails in rad/s, and

v = Corresponding linear speed in km/h.

Each balancing mass = m_B = m_C = 249 kg

Balancing mass for reciprocating parts, $M = \frac{cm_2}{m} \times 249 = \frac{2}{3} \times \frac{300}{560} \times 249$ = 89 kg.

$$\omega = \sqrt{\frac{P}{Mr}} = \sqrt{\frac{30 \times 10^3}{89 \times 0.75}} = 21.2 \text{ rad/s} \qquad (r = r_B = r_C)$$

v = ω×D/2 = 21.2×1.8/2

= 19.08 m/s

= 19.08 × 3600/ 1000

= 68.7 km/h

Swaying couple at speed ω = 21.1 rad/s

$$\text{Swaying couple} = \pm \frac{1}{\sqrt{2}}(1-c)m_2 r\omega^2 l$$

$$= \frac{1}{\sqrt{2}}(1-\frac{2}{3}) \times 300 \times 0.3 \times (21.2)^2 \times 1.75$$

= 16687 N.m

3.10 Balancing of Multi Cylinder Engine

➢ Balancing of Primary force and couple

- The multi-cylinder engines with the cylinder center lines in the same plane and on the same side of the center line of the crankshaft are known as In-line engines.
- The following two conditions must be satisfied in order to give the primary balance of the reciprocating parts of a multi-cylinder engine:
  (a) The algebraic sum of the primary forces must be equal to zero. In other words, the primary force polygon must close and

$$\text{Primary force, } F_{PF} = \Sigma m r \omega^2 \cos\theta$$

  (b) The algebraic sum of the couples about any point in the plane of the primary forces must be equal to zero. In other words, primary couple polygon must close.

$$\text{Primary couple, } F_{PC} = \Sigma m r l \omega^2 \cos\theta$$

- The primary unbalanced force due to the reciprocating masses is equal to the component, parallel to the line of stroke, of the centrifugal force produced by the equal mass placed at the crankpin and revolving with it. Therefore, in order to give the primary balance of the reciprocating parts of a multi-cylinder engine, it is convenient to imagine the reciprocating masses to be transferred to their respective crankpins and to treat the problem as one of revolving masses.

➢ Balancing of Secondary force and couple

- When the connecting rod is not too long (i.e. when the obliquity of the connecting rod is considered), then the secondary disturbing force due to the reciprocating mass arises.
- The following two conditions must be satisfied in order to give a complete secondary balance of an engine:
  (a) The algebraic sum of the secondary forces must be equal to zero. In other words, the secondary force polygon must close, and

$$\text{Secondary force, } F_{SF} = \Sigma m r \omega^2 \frac{\cos 2\theta}{n}$$

  (b) The algebraic sum of the couples about any point in the plane of the secondary forces must be equal to zero. In other words, the secondary couple polygon must close.

$$\text{Secondary couple, } F_{SC} = \Sigma m r \omega^2 l \frac{\cos 2\theta}{n}$$

Example 3.16: A four crank engine has the two outer cranks set at 120° to each other, and their reciprocating masses are each 400 kg. The distance between the planes of rotation of adjacent cranks are 450 mm, 750 mm and 600 mm. If the engine is to be in complete primary balance, find the reciprocating mass and the relative angular position for each of the inner cranks. If the length of each crank is 300 mm, the length of each connecting rod is 1.2 m and the speed of rotation is 240 r.p.m., what is the maximum secondary unbalanced force?

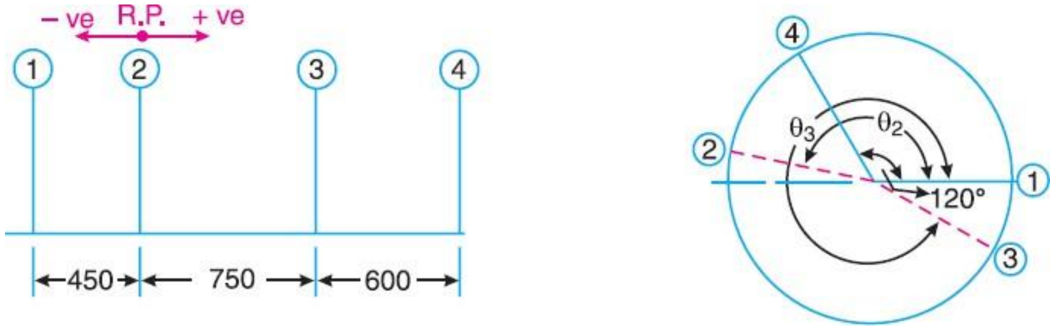
$m_1 = m_4 = 400$ kg

$r = 300$ mm $= 0.3$ m

$l = 1.2$ m

N = 240 r.p.m

$$\omega = \frac{2\pi N}{60} = \frac{2\times\pi\times 240}{60} = 25.14 \text{ rad/s}$$



(a) Position of planes *(b) Primary crank position*

Fig. 3.31

Table 3.13

Plane	Angle		Mass (m) Kg	Radius (r)m	Cent.force÷ω^2 *(mr)* kg-m	Distance from Ref. Plane*(l)* m	Couple÷ω^2 *(mrl)* kg-m^2
	θ	2θ					
1	0°	0°	400	0.3	120	-0.45	-54
2 (R.P.)	θ_2	336°	m_2	0.3	0.3 m_2	0	0
3	θ_3	292°	m_3	0.3	0.3 m_3	0.75	0.225 m_3
4	120°	240°	400	0.3	120	1.35	162

(a) Primary couple polygon *(b) Primary force polygon*

Fig. 3.32

Since the engine is to be in complete primary balance, therefore the primary couple polygon and the primary force polygon must close. First of all, the primary couple polygon, as shown in Fig. 3.32 (a), is drawn to some suitable scale from the data given in Table 3.13 (column 8), in order to find the reciprocating mass for crank 3. Now by measurement, we find that

$$0.225\, m_3 = 196 \text{ kg-m}^2$$

$$\mathbf{m_3 = 871\ kg.}$$

and its angular position with respect to crank 1 in the anticlockwise direction,

$$\boldsymbol{\theta_3 = 326°.}$$

Now in order to find the reciprocating mass for crank 2, draw the primary force polygon, as shown in Fig. 3.32 (b), to some suitable scale from the data given in Table 3.13 (column 6). Now by measurement, we find that

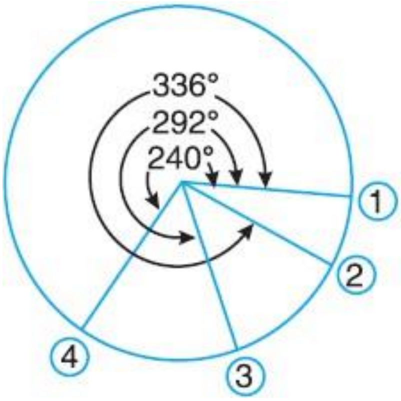
$$0.3\ m_2 = 284 \text{ kg-m}$$

$$\mathbf{m_2 = 947\ kg.}$$

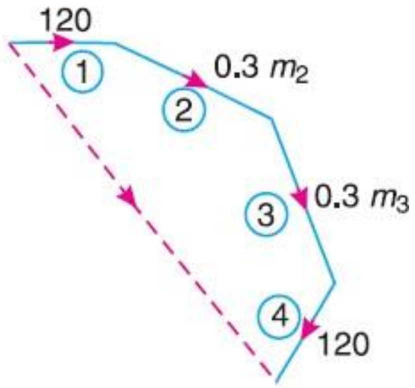
and its angular position with respect to crank 1 in the anticlockwise direction,

$$\boldsymbol{\theta_2 = 168°.}$$

Maximum secondary unbalanced force



(a) Secondary crank positions



(b) Secondary force polygon

Fig. 3.33

The secondary crank positions obtained by rotating the primary cranks at twice the angle, is shown in Fig. 3.33 (a). Now draw the secondary force polygon, as shown in Fig. 3.33 (b), to some suitable scale, from the data given in Table 3.13 (column 6). The closing side of the polygon shown dotted in Fig. 3.33 (b) represents the maximum secondary unbalanced force. By measurement, we find that the maximum secondary unbalanced force is proportional to 582 kg-m.

∴ Maximum Unbalanced Secondary Force,

$$\text{U.S.F.} = 582 \times \frac{\omega^2}{n}$$

$$\text{U.S.F.} = 582 \times \frac{(25.14)^2}{1.2/0.3}$$

U.S.F. = 91960 N

Example 3.17: The intermediate cranks of a four cylinder symmetrical engine, which is in complete primary balance, are 90° to each other and each has a reciprocating mass of 300 kg. The center distance between intermediate cranks is 600 mm and between extreme cranks it is 1800 mm. Lengths of the connecting rod and cranks are 900 mm and 300 mm respectively. Calculate the masses fixed to the extreme cranks with their relative angular positions. Also find the magnitudes of secondary forces and couples about the center line of the system if the engine speed is 1500 rpm.

$m_2 = m_3 = 300$ kg N = 1500 r.p.m

r = 300 mm = 0.3 m $\qquad \omega = \frac{2\pi N}{60} = \frac{2\times\pi\times1500}{60} = 157.08$ rad/s

l = 0.9 m

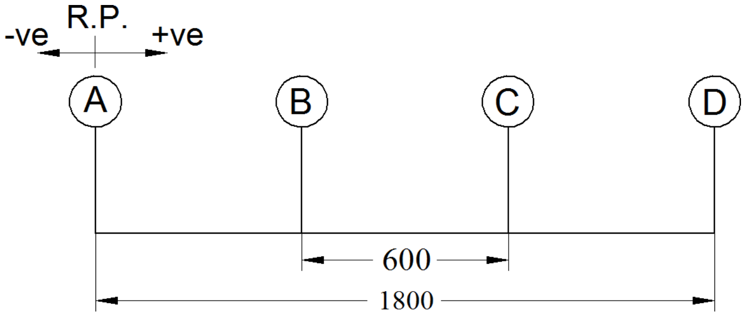
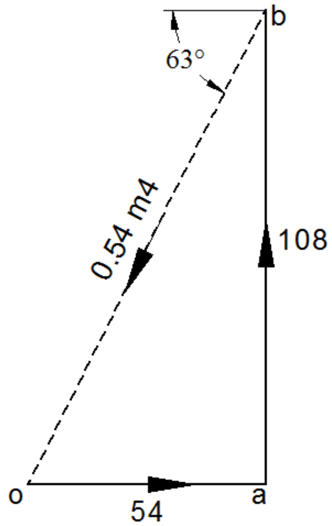


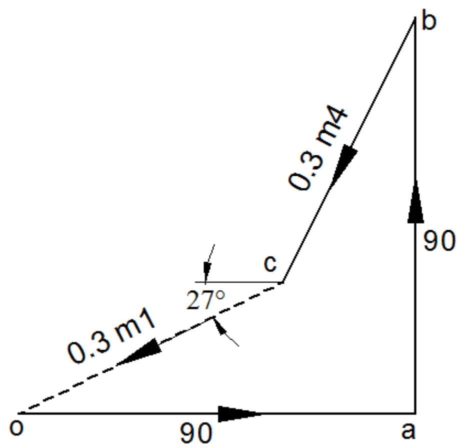
Fig. 3.34 Position of planes

Table 3.14

Plane	Angle		Mass (m) kg	Radius (r)m	Cent.force÷ω^2 *(mr)* kg-m	Distance from Ref. Plane*(l)* m	Couple÷ω^2 *(mrl)* kg-m^2
	θ	2θ					
1 (R.P.)	θ_1	54°	m_1	0.3	0.3 m_1	0	0
2	0°	0°	300	0.3	90	0.6	54
3	90°	180°	300	0.3	90	1.2	108
4	θ_4	126°	m_4	0.3	0.3 m_4	1.8	0.54 m_4



(a) Primary couple polygon



(b) Primary force polygon

Fig. 3.35

Since the engine is to be in complete primary balance, therefore the primary couple polygon and the primary force polygon must close. First of all, the primary couple polygon, as shown in Fig. 3.35 (a), is drawn to some suitable scale from the data given in Table 3.14 (column 8), in order to find the reciprocating mass for crank 4. Now by measurement, we find that

$$0.54\ m_4 = 120.75 \text{ kg-m}^2$$

$$\mathbf{m_4 = 223.61\ kg.}$$

and its angular position with respect to crank 2 in the anticlockwise direction,

$$\boldsymbol{\theta_4 = 180° + 63° = 243°.}$$

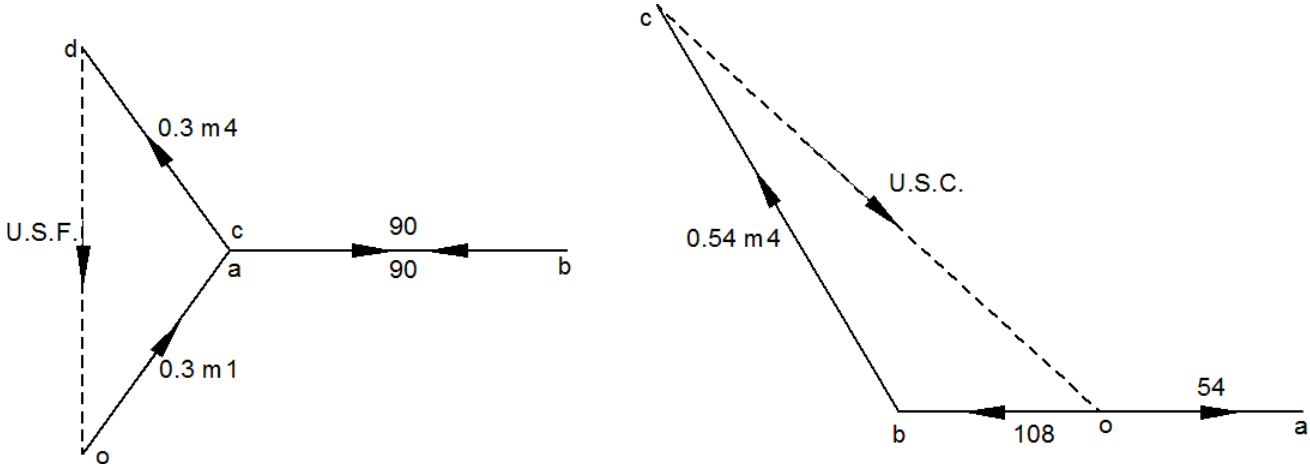
Now in order to find the reciprocating mass for crank 2, draw the primary force polygon, as shown in Fig. 3.35 (b), to some suitable scale from the data given in Table 3.14 (column 6). Now by measurement, we find that

$$0.3\ m_1 = 67.08 \text{ kg-m}$$

$$\mathbf{m_1 = 223.6 \text{ kg.}}$$

and its angular position with respect to crank 1 in the anticlockwise direction,

$$\boldsymbol{\theta_1 = 180^\circ + 27^\circ = 207^\circ.}$$



(a) Secondary force polygon *(b) Secondary couple polygon*

Fig. 3.36

The secondary crank positions obtained by rotating the primary cranks at twice the angle. Now draw the secondary force polygon, as shown in Fig. 3.36 (a), to some suitable scale, from the data given in Table 3.14 (column 6). The closing side of the polygon shown dotted in Fig. 3.36 (a) represents the maximum secondary unbalanced force. By measurement, we find that the maximum secondary unbalanced force is proportional to 108.54 kg-m.

∴ Maximum Unbalanced Secondary Force,

$$\text{U.S.F.} = 108.54 \times \frac{\omega^2}{n}$$

$$\text{U.S.F.} = 108.54 \times \frac{(157.08)^2}{0.9/0.3}$$

U.S.F. = 892.71 KN

Now draw the secondary couple polygon, as shown in Fig. 3.36 (b), to some suitable scale, from the data given in Table 3.14 (column 8). The closing side of the polygon shown dotted in Fig. 3.36 (b) represents the maximum secondary unbalanced couple. By measurement, we find that the maximum secondary unbalanced couple is proportional to 160.47 kg-m^2.

∴ Maximum Unbalanced Secondary Couple,

$$U.S.C. = 160.47 \times \frac{\omega^2}{n}$$

$$U.S.C. = 160.47 \times \frac{(157.08)^2}{0.9/0.3}$$

U.S.C. = 1319.82 KN.m

Example 3.18: The cranks and connecting rods of a 4-cylinder in-line engine running at 1800 r.p.m. are 60 mm and 240 mm each respectively and the cylinders are spaced 150 mm apart. If the cylinders are numbered 1 to 4 in sequence from one end, the cranks appear at intervals of 90° in an end view in the order 1-4-2-3. The reciprocating mass corresponding to each cylinder is 1.5 kg.

Determine: (i) Unbalanced primary and secondary forces, if any, and (ii) Unbalanced primary and secondary couples with reference to central plane of the engine.

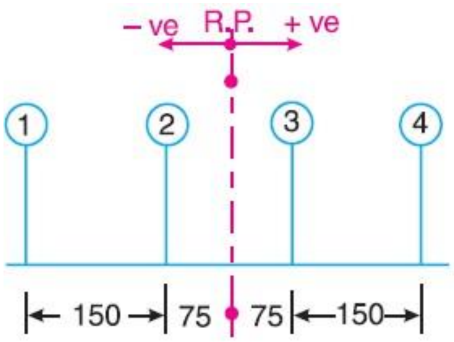
r = 60 mm N = 1800 r.p.m

l = 240 mm $\omega = \dfrac{2\pi N}{60} = \dfrac{2\times\pi\times1800}{60}$

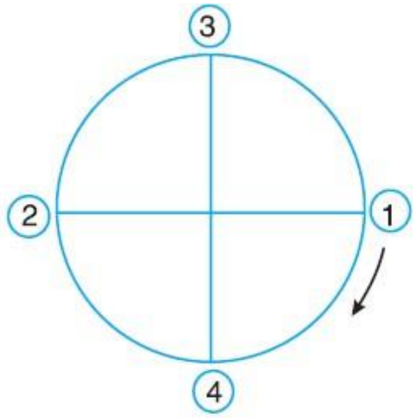
m = 1.5 kg =188.5 rad/s

Table 3.15

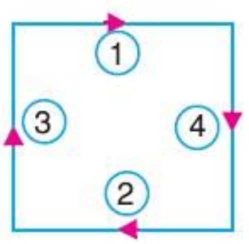
Angle 2θ	Angle θ	Plane	Mass (m) kg	Radius (r) m	Cent.force ÷ ω^2 *(mr)* kg-m	Distance from Ref. Plane *(l)* m	Couple ÷ ω^2 *(mrl)* kg-m^2
0°	0°	1	1.5	0.06	0.09	-0.225	-0.02025
360°	180°	2	1.5	0.06	0.09	-0.075	-0.00675
540°	270°	3	1.5	0.06	0.09	0.075	0.00675
180°	90°	4	1.5	0.06	0.09	0.225	0.02025



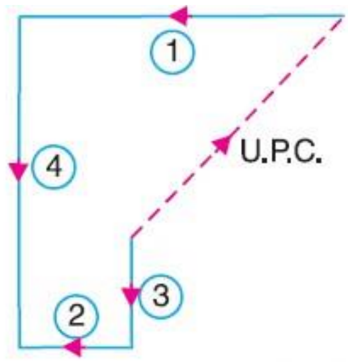
(*a*) Cylinder plane positions.



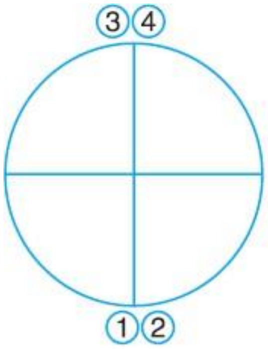
(*b*) Primary crank positions.



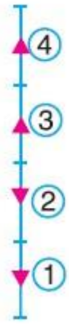
(*c*) Primary force polygon.



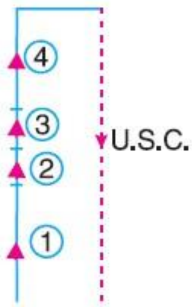
(*d*) Primary couple polygon.



(e) Secondary crank positions.



(f) Secondary force polygon.



(g) Secondary couple polygon.

Fig. 3.37

Unbalanced primary forces and couples

The position of the cylinder planes and cranks is shown in Fig. 3.37 (a) and (b) respectively. With reference to central plane of the engine, the data may be tabulated as above:

The primary force polygon from the data given in Table 3.15 (column 6) is drawn as shown in Fig. 3.37 (c). Since the primary force polygon is a closed figure, therefore there are no unbalanced primary forces,

∴ Unbalanced Primary Force, **U.P.F. = 0.**

The primary couple polygon from the data given in Table 3.15 (column 8) is drawn as shown in Fig. 3.37 (d). The closing side of the polygon, shown dotted in the figure, represents unbalanced primary couple. By measurement, the unbalanced primary couple is proportional to 0.0191 kg-m^2.

∴Unbalanced Primary Couple,

$$U.P.C = 0.0191 \times \omega^2 = 0.0191\ (188.52)^2$$

U.P.C = 678.81 N-m.

Unbalanced secondary forces and couples

The secondary crank positions, taking crank 3 as the reference crank, as shown in Fig. 2.24 (e). From the secondary force polygon as shown in Fig. 3.37 (f), it is a closed figure. Therefore, there are no unbalanced secondary forces.

∴ Unbalanced Secondary Force, **U.S.F. = 0.**

The secondary couple polygon is shown in Fig. 3.37 (g). The unbalanced secondary couple is shown by dotted line. By measurement, we find that unbalanced secondary couple is proportional to 0.054 kg-m^2.

∴Unbalanced Secondary Couple,

$$U.S.C. = 0.054 \times \frac{\omega^2}{n} = 0.054 \times \frac{(188.52)^2}{0.24/0.06}$$

U.S.C. = 479.78 N.m $(n = l / r)$

Example 3.19: The successive cranks of a five cylinder in-line engine are at 144° apart. The spacing between cylinder center lines is 400 mm. The lengths of the crank and the connecting rod are 100 mm and 450 mm respectively and the reciprocating mass for each cylinder is 20 kg. The engine speed is 630 r.p.m. Determine the maximum values of the primary and secondary forces and couples and the position of the central crank at which these occur.

l = 450 mm = 0.45 m　　　r = 0.1 m

m = 20 kg　　　N = 630 r.p.m.

n = l/r = 4.5　　　$\omega = \frac{2\pi N}{60} = \frac{2\times\pi\times630}{60}$ = 65.97 rad/s

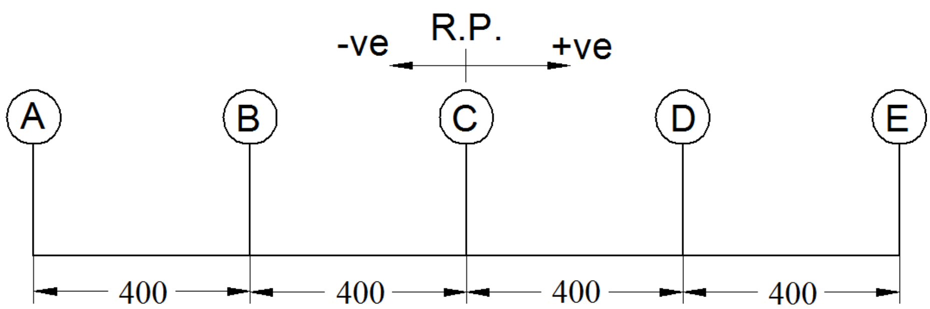


Fig. 3.38 Cylinder plane position

Table 3.16

Angle 2θ	Angle θ	Plane	Mass (m) Kg	Radius (r) m	Cent.force ÷ ω^2 *(mr)* kg-m	Distance from Ref. Plane *(l)* m	Couple ÷ ω^2 *(mrl)* kg-m^2
0°	0°	1	20	0.1	2	-0.8	-1.6
288°	144°	2	20	0.1	2	-0.4	-0.8
216°	288°	3	20	0.1	2	0	0
144°	72°	4	20	0.1	2	0.4	0.8
72°	216°	5	20	0.1	2	0.8	1.6

Unbalanced primary forces and couples

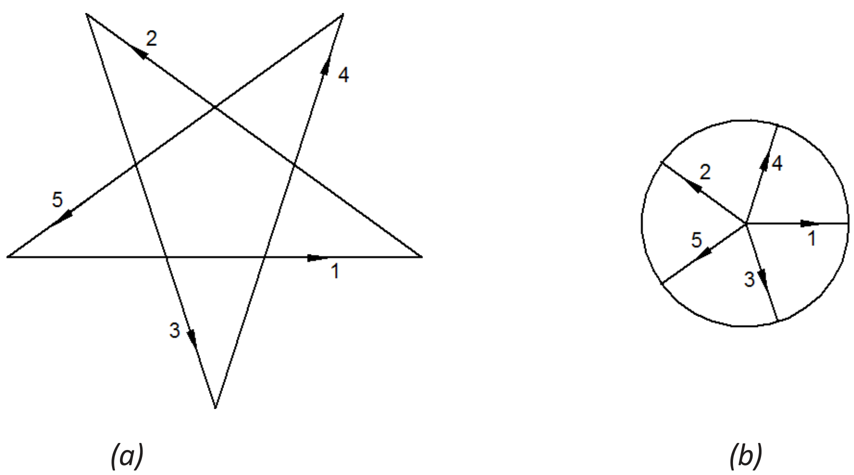


Fig. 3.39 Primary force polygon

The position of the cylinder planes and cranks is shown in Fig. 3.38 and Fig. 3.39 (b) respectively. With reference to central plane of the engine, the data may be tabulated as above:

The primary force polygon from the data given in Table 3.16 (column 6) is drawn as shown in Fig. 3.39 (a). Since the primary force polygon is a closed figure, therefore there are no unbalanced primary forces,

∴ Unbalanced Primary Force, **U.P.F. = 0.**

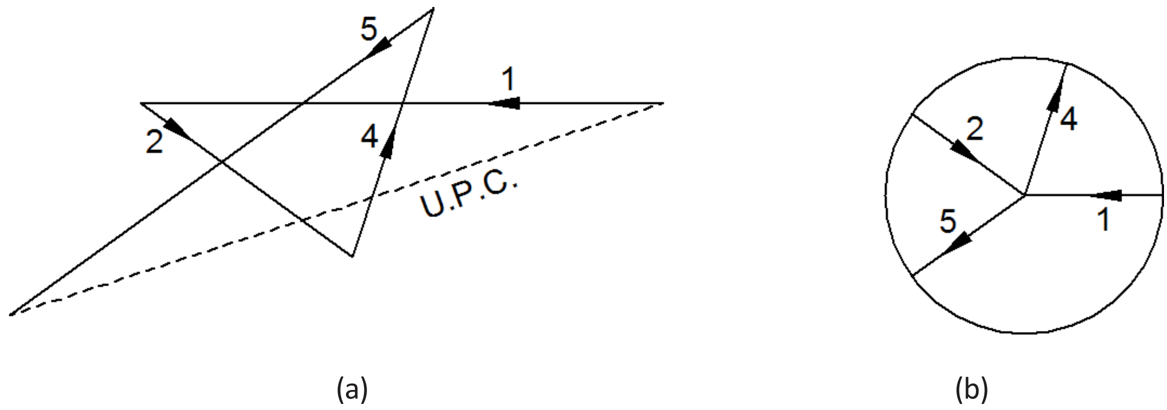


Fig. 3.40 Primary couple polygon

The primary couple polygon from the data given in Table 3.16 (column 8) is drawn as shown in Fig. 3.40 (a). The closing side of the polygon, shown dotted in the figure, represents unbalanced primary couple. By measurement, the unbalanced primary couple is proportional to 2.1 kg-m^2.

∴Unbalanced Primary Couple,

$$U.P.C = 2.1 \times \omega^2 = 2.1\ (65.97)^2$$

$$\mathbf{U.P.C = 9139.3\ N\text{-}m}.$$

Unbalanced secondary forces and couples

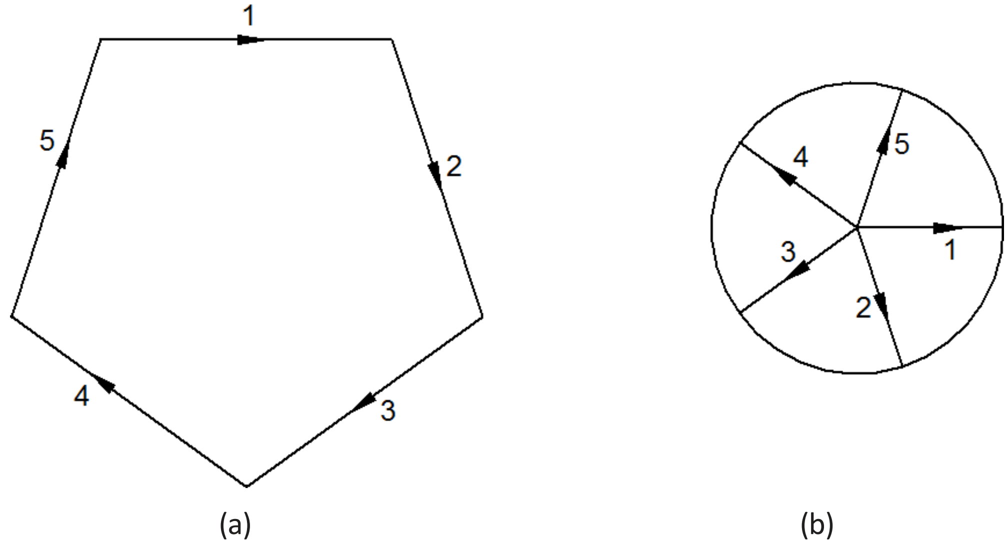
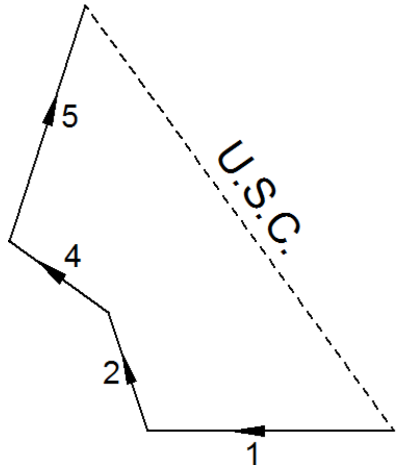


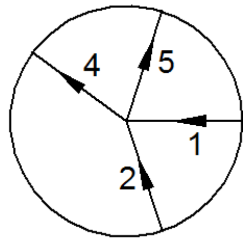
Fig. 3.41 Secondary force polygon

The secondary crank positions are shown in Fig. 3.41 (b). From the secondary force polygon as shown in Fig. 3.41 (a), it is a closed figure. Therefore, there are no unbalanced secondary forces.

$\therefore$ Unbalanced Secondary Force, **U.S.F. = 0.**



(a)



(b)

Fig. 3.42 Secondary couple polygon

The secondary couple polygon is shown in Fig. 3.42 (a). The unbalanced secondary couple is shown by dotted line. By measurement, we find that unbalanced secondary couple is proportional to 3.4 kg-m^2.

$\therefore$Unbalanced Secondary Couple,

$$U.S.C. = 3.4 \times \frac{\omega^2}{n} = 3.4 \times \frac{(65.97)^2}{0.45/0.1}$$

U.S.C. = 3288.2 N.m $\quad\quad$ ($n = l / r$)

Example 3.20: A four stroke five cylinder in-line engine has a firing order of 1-4-5-3-2-1. The centers lines of cylinders are spaced at equal intervals of 15 cm, the reciprocating parts per cylinder have a mass of 15 kg, the piston stroke is 10 cm and the connecting rods are 17.5 cm long. The engine rotates at 600 rpm. Determine the values of maximum primary and secondary unbalanced forces and couples about the central plane.

l = 10cm = 0.1 m $\quad$ or $\quad$ r = 0.05 m

m = 15 kg $\quad$ N = 600 r.p.m.

n = l / r = 17.5/5 = 3.5 $\quad$ $\omega = \frac{2\pi N}{60} = \frac{2\times\pi\times 600}{60}$ = 62.83 rad/s

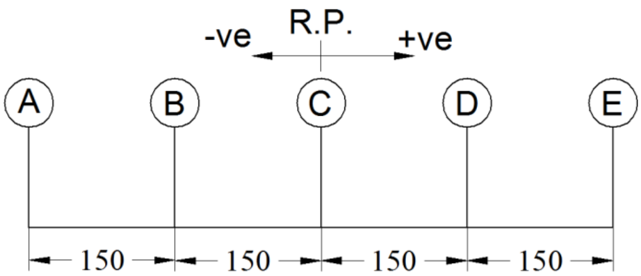
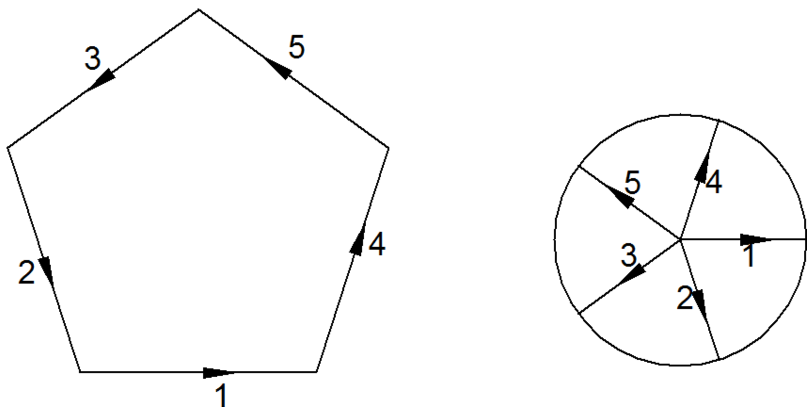


Fig. 3.43 Cylinder plane position

Table 3.17

Angle 2θ	Angle θ	Plane	Mass (m) kg	Radius (r) m	Cent.force ÷ ω^2 *(mr)* kg-m	Distance from Ref. Plane *(l)* m	Couple ÷ ω^2 *(mrl)* kg-m^2
0°	0°	1	15	0.05	0.75	-0.3	-0.225
216°	288°	2	15	0.05	0.75	-0.15	-0.1125
72°	216°	3	15	0.05	0.75	0	0
144°	72°	4	15	0.05	0.75	0.15	0.1125
288°	144°	5	15	0.05	0.75	0.3	0.225

Unbalanced primary forces and couples



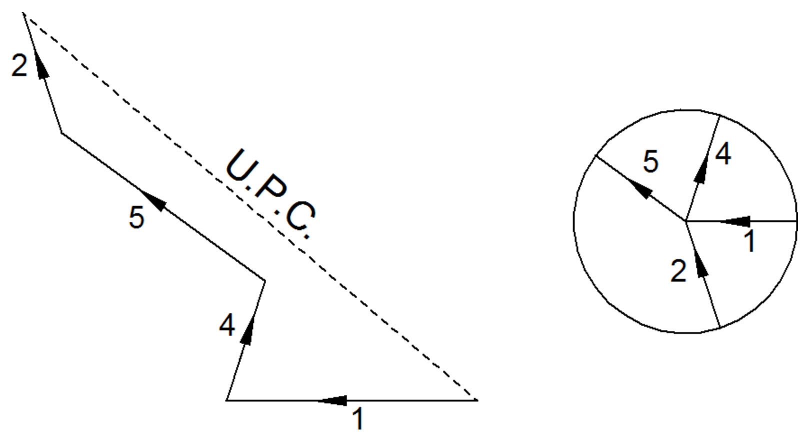
(a) *(b)*

Fig. 3.44 Primary force polygon

The position of the cylinder planes and cranks is shown in Fig. 3.43 and Fig. 3.44(b) respectively. With reference to central plane of the engine, the data may be tabulated as above:

The primary force polygon from the data given in Table 3.17 (column 6) is drawn as shown in Fig. 3.44 (a). Since the primary force polygon is a closed figure, therefore there are no unbalanced primary forces,

∴ Unbalanced Primary Force, **U.P.F. = 0**.



(a) (b)

Fig. 3.45 Primary couple polygon

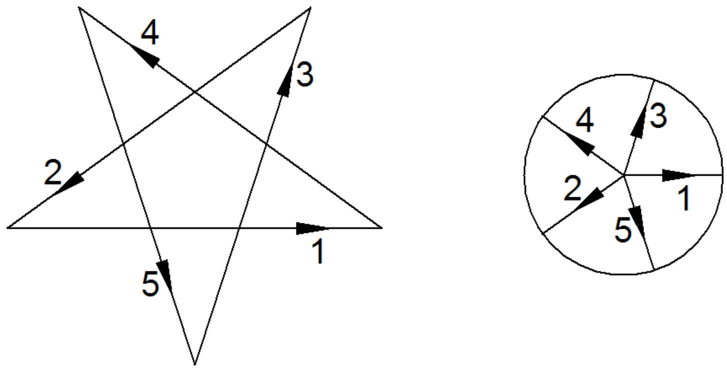
The primary couple polygon from the data given in Table 3.17 (column 8) is drawn as shown in Fig. 3.45 (a). The closing side of the polygon, shown dotted in the figure, represents unbalanced primary couple. By measurement, the unbalanced primary couple is proportional to 0.53 kg-m^2.

∴Unbalanced Primary Couple,

U.P.C. = 0.53 $\times \omega^2$ = 0.53 $(62.83)^2$

U.P.C. = 2092.23 N-m.

Unbalanced secondary forces and couples

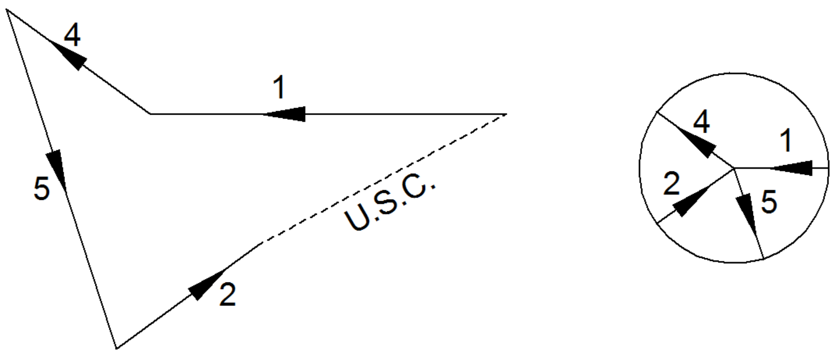


(a) (b)

Fig. 3.46 Secondary force polygon

The secondary crank positions are shown in Fig. 3.46 (b). From the secondary force polygon as shown in Fig. 3.46 (a), it is a closed figure. Therefore, there are no unbalanced secondary forces.

∴ Unbalanced Secondary Force, **U.S.F. = 0.**



(a) (b)

Fig. 3.47 Secondary couple polygon

The secondary couple polygon is shown in Fig. 3.47 (a). The unbalanced secondary couple is shown by dotted line. By measurement, we find that unbalanced secondary couple is proportional to 0.18 kg-m^2.

∴Unbalanced Secondary Couple,

$$U.S.C. = 0.18 \times \frac{\omega^2}{n}$$

$$U.S.C. = 0.18 \times \frac{(62.83)^2}{3.5}$$

U.S.C. = 203 N.m ($n = l / r$)

Example 3.21: In an in-line six-cylinder engine working on two stroke cycle, the cylinder center lines are spaced at 600 mm. In the end view, the cranks are 60° apart and in the order 1-4-5-2-3-6. The stroke of each piston is 400 mm and the connecting rod length is 1 m. The mass of the reciprocating parts is 200 kg per cylinder and that of rotating parts 100 kg per crank. The engine rotates at 300 r.p.m. Examine the engine for the balance of primary and secondary forces and couples. Find maximum unbalanced forces and couples.

L = 400 mm or r = L / 2 = 200 mm = 0.2 m

l = 1 m N = 300 r.p.m.

m_1 = 200 kg $\omega = \dfrac{2\pi N}{60} = \dfrac{2\times\pi\times300}{60}$ = 31.42 rad/s

m_2 = 100 kg

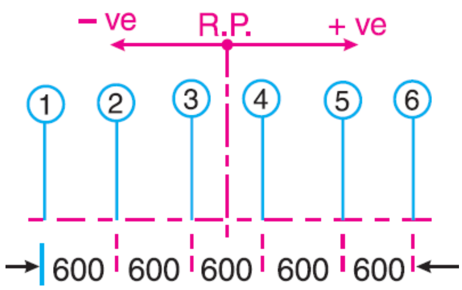
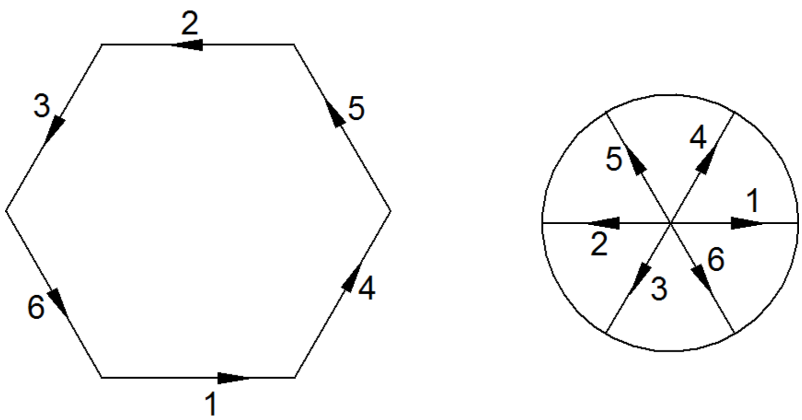


Fig. 3.48 Positions of planes of cylinders

Table 3.18

Angle 2θ	Angle θ	Plane	Mass (m) kg	Radius (r) m	Cent. force ÷ ω^2 *(mr)* kg-m	Distance from Ref. Plane *(l)* m	Couple ÷ ω^2 *(mrl)* kg-m^2
0°	0°	1	300	0.2	60	-1.5	-90
360°	180°	2	300	0.2	60	-0.9	-54
120°	240°	3	300	0.2	60	-0.3	-18
120°	60°	4	300	0.2	60	0.3	18
240°	120°	5	300	0.2	60	0.9	54
240°	300°	6	300	0.2	60	1.5	90

Unbalanced primary forces and couples



(a) *(b)*

Fig. 3.49 Primary force polygon

The position of the cylinder planes and cranks is shown in Fig. 3.48 and Fig. 3.49 (b) respectively. With reference to central plane of the engine, the data may be tabulated as above:

The primary force polygon from the data given in Table 3.18 (column 6) is drawn as shown in Fig. 3.49 (a). Since the primary force polygon is a closed figure, therefore there are no unbalanced primary forces,

∴ Unbalanced Primary Force, **U.P.F. = 0**.

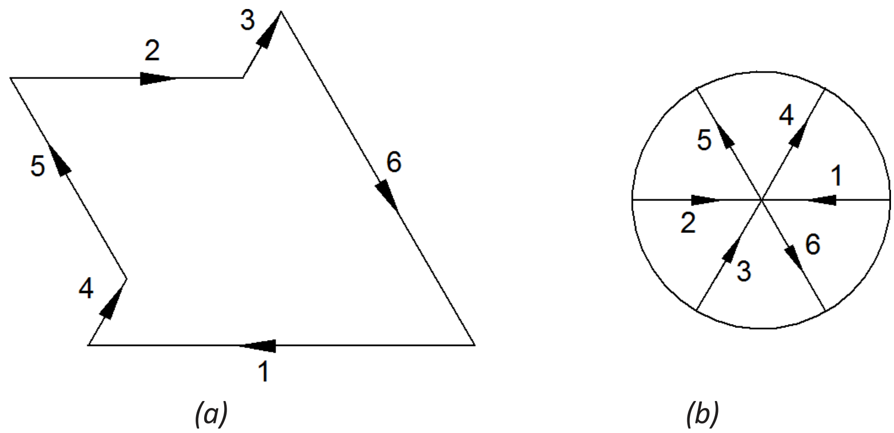


Fig. 3.50 Primary couple polygon

The primary couple polygon from the data given in Table 3.18 (column 8) is drawn as shown in Fig. 3.50 (a). Since the primary couple polygon is a closed figure, therefore there are no unbalanced primary couples,

∴Unbalanced Primary Couple, **U.P.C. = 0**

Unbalanced secondary forces and couples

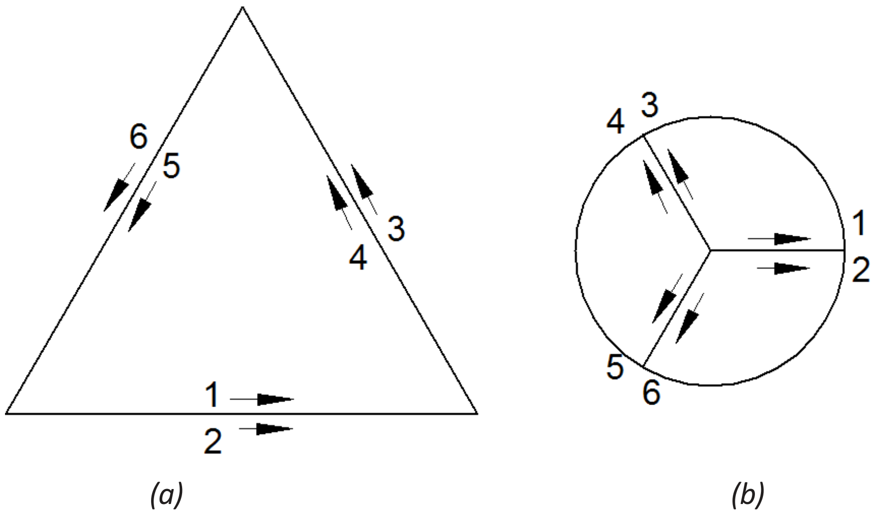
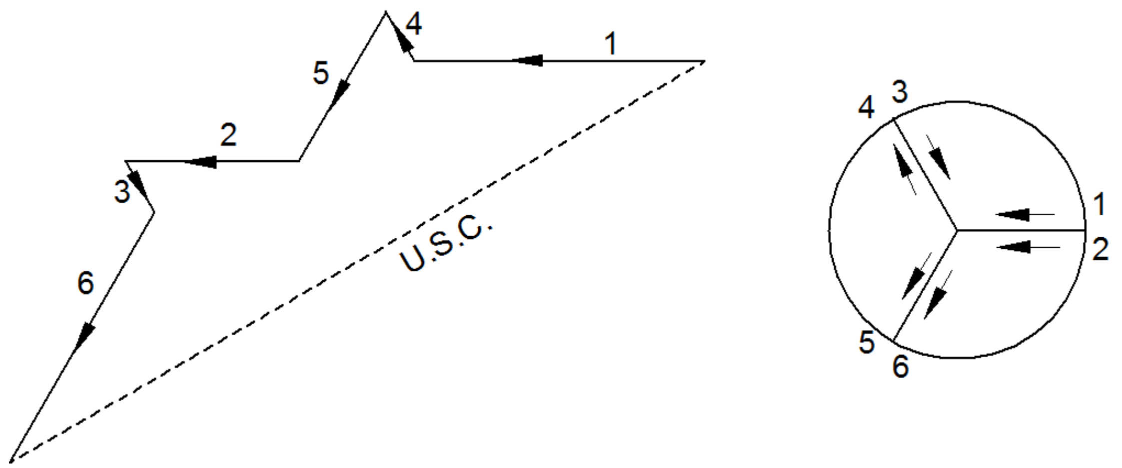


Fig. 3.51 Secondary force polygon

The secondary crank positions are shown in Fig. 3.51 (b). From the secondary force polygon as shown in Fig. 3.51 (a), it is a closed figure. Therefore, there are no unbalanced secondary forces.

∴ Unbalanced Secondary Force, **U.S.F. = 0.**



(a) *(b)*

Fig. 3.52 Secondary couple polygon

The secondary couple polygon is shown in Fig. 3.52 (a). The unbalanced secondary couple is shown by dotted line. By measurement, we find that unbalanced secondary couple is proportional to 249 kg-m^2.

∴Unbalanced Secondary Couple,

$$U.S.C. = 249 \times \frac{\omega^2}{n}$$

$$U.S.C. = 249 \times \frac{(31.42)^2}{1/0.2}$$

U.S.C. = 49163.37 N.m ($n = l / r$)

Example 3.22: The firing order in a 6 cylinder vertical four stroke in-line engine is 1-4-2-6-3-5. The piston stroke is 100 mm and the length of each connecting rod is 200 mm. The pitch distances between the cylinder center lines are 100 mm, 100 mm, 150 mm, 100 mm, and 100 mm respectively. The reciprocating mass per cylinder is 1 kg and the engine runs at 3000 r.p.m. Determine the out-of-balance primary and secondary forces and couples on this engine, taking a plane midway between the cylinder 3 and 4 as the reference plane.

L = 100 mm or r = L / 2 = 50 mm = 0.05 m

l = 200 mm N = 3000 r.p.m.

m = 1 kg $\omega = \frac{2\pi N}{60} = \frac{2 \times \pi \times 3000}{60}$ = 314.16 rad/s

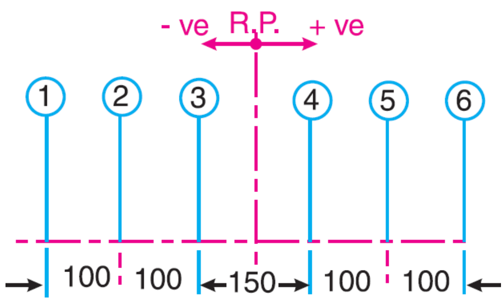
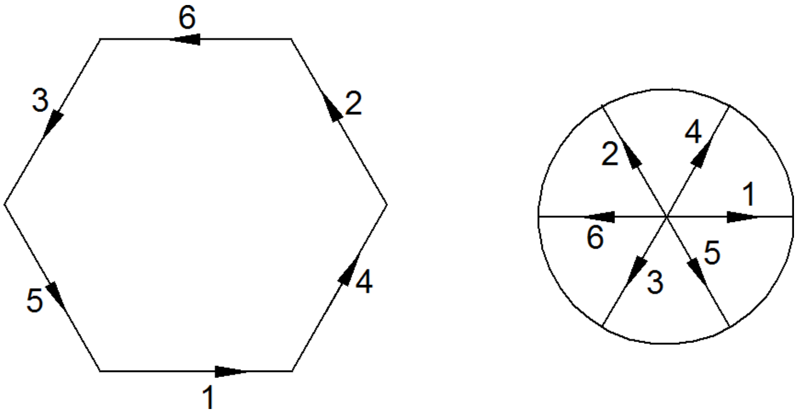


Fig. 3.53 Positions of planes

Table 3.19

Angle 2θ	Angle θ	Plane	Mass (m) kg	Radius (r) m	Cent.force ÷ ω^2 *(mr)* kg-m	Distance from Ref. Plane *(l)* m	Couple ÷ ω^2 *(mrl)* kg-m^2
0°	0°	1	1	0.05	0.05	-0.275	-0.01375
240°	120°	2	1	0.05	0.05	-0.175	-0.00875
120°	240°	3	1	0.05	0.05	-0.075	-0.00375
120°	60°	4	1	0.05	0.05	0.075	0.00375
240°	300°	5	1	0.05	0.05	0.175	0.00875
360°	180°	6	1	0.05	0.05	0.275	0.01375

Unbalanced primary forces and couples



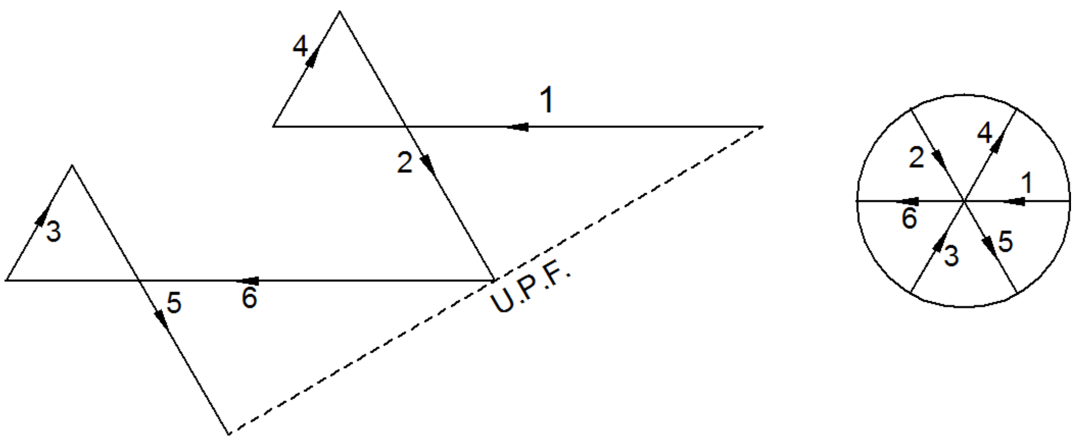
(a) *(b)*

Fig. 3.54 Primary force polygon

The position of the cylinder planes and cranks is shown in Fig. 3.53 and Fig. 3.54 (b) respectively. With reference to central plane of the engine, the data may be tabulated as above:

The primary force polygon from the data given in Table 3.19 (column 6) is drawn as shown in Fig. 3.54 (a). Since the primary force polygon is a closed figure, therefore there are no unbalanced primary forces,

∴ Unbalanced Primary Force, **U.P.F. = 0.**



(a) *(b)*

Fig. 3.55 Primary couple polygon

The primary couple polygon from the data given in Table 3.19 (column 8) is drawn as shown in Fig. 3.55 (a). The closing side of the polygon, shown dotted in the figure, represents unbalanced primary couple. By measurement, the unbalanced primary couple is proportional to 0.01732 kg-m^2.

∴Unbalanced Primary Couple,

$$\text{U.P.C.} = 0.01732 \times \omega^2 = 0.01732\,(314.16)^2$$

$$\textbf{U.P.C. = 1709.42 N-m.}$$

Unbalanced secondary forces and couples

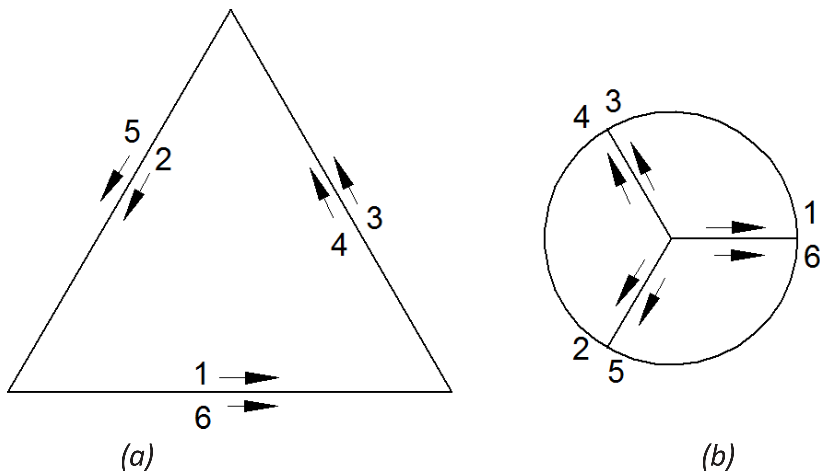


Fig. 3.56 Secondary force polygon

The secondary crank positions are shown in Fig. 3.56 (b). From the secondary force polygon as shown in Fig. 3.56 (a), it is a closed figure. Therefore, there are no unbalanced secondary forces.

∴ Unbalanced Secondary Force, **U.S.F. = 0.**

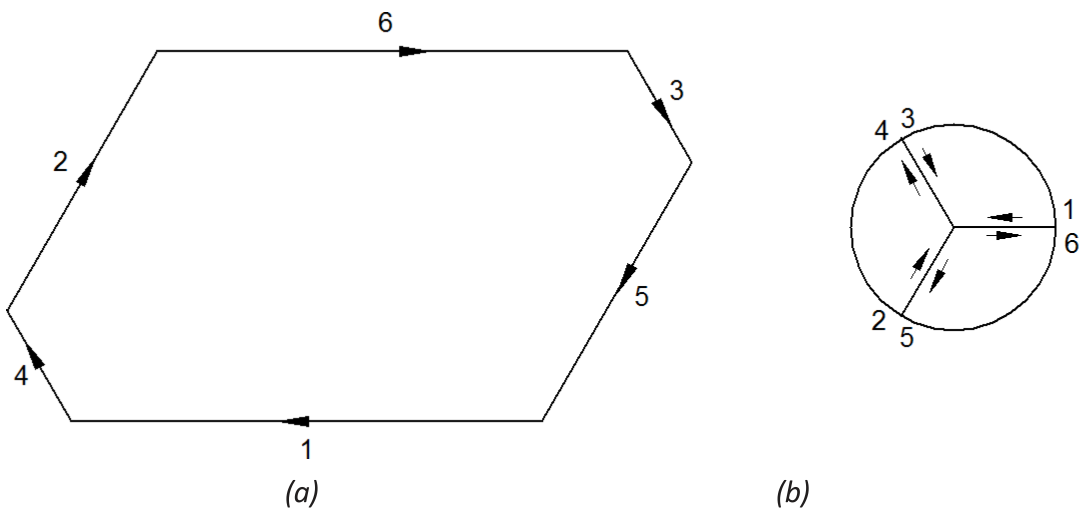


Fig. 3.57 Secondary couple polygon

The secondary couple polygon is drawn as shown in Fig. 3.57 (a). Since the secondary couple polygon is a closed figure, therefore there are no unbalanced secondary couples,

∴Unbalanced Secondary Couple, **U.S.C. = 0**

3.11 Balancing of V – Engine

In V–engines, a common crank OA is operated by two connecting rods AB_1, and AB_2. Fig. 3.58 shows a symmetrical two-cylinder V-cylinder, the center lines of which are inclined at an angle α to the x-axis.

Let θ be the angle moved by the crank from the x-axis.

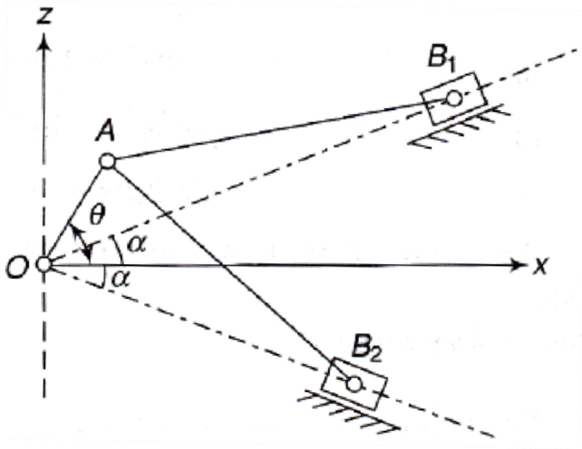


Fig. 3.58

I. Primary force

Primary force of 1 along line of stroke

$$OB_1 = mr\omega^2 \cos(\theta - \alpha)$$

Primary force of 1 along x–axis

$$= mr\omega^2 \cos(\theta - \alpha) \cos\alpha$$

Primary force of 2 along line of stroke $OB_2 = mr\omega^2 \cos(\theta + \alpha)$

Primary force of 2 along x–axis $= mr\omega^2 \cos(\theta + \alpha) \cos\alpha$

Total primary force along x–axis

$$= mr\omega^2 \cos\alpha\,[\cos(\theta - \alpha) + \cos(\theta + \alpha)]$$

$$= mr\omega^2 \cos\alpha\,[(\cos\theta \cos\alpha + \sin\theta \sin\alpha) + (\cos\theta \cos\alpha - \sin\theta \sin\alpha)]$$

$$= 2mr\omega^2 \cos\alpha \cos\theta \cos\alpha$$

$$= 2mr\omega^2 \cos^2\alpha \cos\theta$$

Similarly, total primary force along z–axis

$$= mr\omega^2 \sin\alpha\,[\cos(\theta - \alpha) - \cos(\theta + \alpha)]$$

$$= mr\omega^2 \sin\alpha\,[(\cos\theta \cos\alpha + \sin\theta \sin\alpha) - (\cos\theta \cos\alpha - \sin\theta \sin\alpha)]$$

$$= 2mr\omega^2 \sin\alpha \sin\theta \sin\alpha$$

$$= 2mr\omega^2 \sin^2\alpha \sin\theta$$

Resultant primary force $= \sqrt{(2mr\omega^2 \cos^2\alpha\ \cos\theta)^2 + (2mr\omega^2 \sin^2\alpha\ \sin\theta)^2}$

$$= 2mr\omega^2 \sqrt{(\cos^2\alpha\ \cos\theta)^2 + (\sin^2\alpha\ \sin\theta)^2}$$

It will be at an angle β with the x–axis given by

$$\tan\beta = \frac{\sin^2\alpha\ \sin\theta}{\cos^2\alpha\ \cos\theta}$$

If 2α = 90°, resultant force $= 2mr\omega^2 \sqrt{(\cos^2 45^\circ\ \cos\theta)^2 + (\sin^2 45^\circ\ \sin\theta)^2}$

$$= mr\omega^2$$

$$\tan\beta = \frac{\sin^2 45^\circ\ \sin\theta}{\cos^2 45^\circ\ \cos\theta} = \tan\theta$$

i.e., $\beta = \theta$ or it acts along the crank and, therefore, can be completely balanced by a mass at a suitable radius diametrically opposite to the crank such that $m_r r_r = mr$.
For a given value of α, the resultant primary force is maximum when

$$(\cos^2\alpha\cos\theta)^2 + (\sin^2\alpha\sin\theta)^2 \text{ is maximum}$$

$$(\cos^4\alpha\cos^2\theta + \sin^4\alpha\sin^2\theta) \text{ is maximum}$$

$$\frac{d}{d\theta}(\cos^4\alpha\cos^2\theta + \sin^4\alpha\sin^2\theta) = 0$$

$$-\cos^4\alpha.2\cos\theta\sin\theta + \sin^4\alpha.2\sin\theta\cos\theta = 0$$

$$-\cos^4\alpha.\sin2\theta + \sin^4\alpha.\sin2\theta = 0$$

$$\sin2\theta(\sin^4\alpha - \cos^4\alpha) = 0$$

As α is not zero, therefore, for a given value of α, the resultant primary force is maximum when θ is zero degree.

II. Secondary force

Secondary force of 1 along OB_1 $= \dfrac{mr\omega^2}{n}\cos2(\theta-\alpha)$

Secondary force of 1 along x-axis $= \dfrac{mr\omega^2}{n}\cos2(\theta-\alpha)\cos\alpha$

Secondary force of 2 along OB_2 $= \dfrac{mr\omega^2}{n}\cos2(\theta+\alpha)$

Secondary force of 2 along x-axis $= \dfrac{mr\omega^2}{n}\cos2(\theta+\alpha)\cos\alpha$

Total secondary force along x-axis

$$= \frac{mr\omega^2}{n}\cos\alpha[\cos2(\theta-\alpha) + \cos2(\theta+\alpha)]$$

$$= \frac{mr\omega^2}{n}\cos\alpha[(\cos2\theta\cos2\alpha + \sin2\theta\sin2\alpha) + (\cos2\theta\cos2\alpha - \sin2\theta\sin2\alpha)]$$

$$= \frac{2mr\omega^2}{n}\cos\alpha\cos2\theta\cos2\alpha$$

Similarly, secondary force along z-axis $= \dfrac{2mr\omega^2}{n}\sin\alpha\sin2\theta\sin2\alpha$

Resultant secondary force $= \dfrac{2mr\omega^2}{n}\sqrt{(\cos\alpha\cos2\theta\cos2\alpha)^2 + (\sin\alpha\sin2\theta\sin2\alpha)^2}$

$$\tan\beta' = \frac{\sin\alpha\sin2\theta\sin2\alpha}{\cos\alpha\cos2\theta\cos2\alpha}$$

If $2\alpha = 90°$ or $\alpha = 45°$

$$\text{Secondary force } = \frac{2mr\omega^2}{n}\sqrt{\left(\frac{\sin 2\theta}{\sqrt{2}}\right)^2}$$

$$= \frac{\sqrt{2}mr\omega^2}{n}\sin 2\theta$$

$$\tan\beta' = \infty, \beta' = 90°$$

This means that the force acts along z-axis and is a harmonic force and special methods are needed to balance it.

Example 3.23: Reciprocating mass per cylinder in 60° V–twin engine is 1.5 kg. The stroke and connecting rod length are 100 mm and 250 mm respectively. If the engine runs at 2500 rpm, determine the maximum and minimum values of primary and secondary forces.

2α = 60° L = 250 mm

l = 100 mm or r = 50 mm

m = 1.5 kg N = 2500 rpm

$$\omega = \frac{2\pi N}{60} = \frac{2\times\pi\times 2500}{60} = 261.8\, rad/s$$

$$\text{Resultant primary force, } F_P = 2mr\omega^2\sqrt{(\cos^2\alpha\ \cos\theta)^2 + (\sin^2\alpha\ \sin\theta)^2}$$

$$= 2mr\omega^2\sqrt{(\cos^2 30°\ \cos\theta)^2 + (\sin^2 30°\ \sin\theta)^2}$$

$$= 2mr\omega^2\sqrt{\left(\frac{3}{4}\cos\theta\right)^2 + \left(\frac{1}{4}\sin\theta\right)^2}$$

$$= \frac{mr\omega^2}{2}\sqrt{\left(9\cos^2\theta\right) + \left(\sin^2\theta\right)} \quad \text{... .... (i)}$$

The primary force is maximum, when θ = 0°. Therefore, substituting θ = 0° in equation (i), maximum primary force,

$$F_{P(\max)} = \frac{mr\omega^2}{2}\times 3 = \frac{1.5\times 0.05\times(261.8)^2}{2}\times 3$$

= 7710.7 N

The primary force is minimum, when θ = 90°. Therefore, substituting θ = 90° in equation (i), minimum primary force,

$$F_{P(\min)} = \frac{mr\omega^2}{2} = \frac{1.5\times 0.05\times(261.8)^2}{2}$$

= 2570.2 N

$$\text{Resultant secondary force, } F_S = \frac{2mr\omega^2}{n}\sqrt{(\cos\alpha\ \cos 2\theta\ \cos 2\alpha)^2 + (\sin\alpha\ \sin 2\theta\ \sin 2\alpha)^2}$$

$$= \frac{2mr\omega^2}{n}\sqrt{(\cos 30°\ \cos 2\theta\ \cos 60°)^2 + (\sin 30°\ \sin 2\theta\ \sin 60°)^2}$$

$$= \frac{2mr\omega^2}{n}\sqrt{\left(\frac{\sqrt{3}}{2}\times\frac{1}{2}\cos 2\theta\right)^2 + \left(\frac{1}{2}\times\frac{\sqrt{3}}{2}\sin 2\theta\right)^2}$$

$$= \frac{\sqrt{3}}{2}\times\frac{mr\omega^2}{n}$$

$$= \frac{\sqrt{3}}{2}\times\frac{1.5\times 0.05\times(261.8)^2}{0.25/0.05} \qquad (n = l/r)$$

Resultant secondary force, F_S = 890.3 N

Example 3.24: A V-twin engine has the cylinder axes at right angles and the connecting rods operate a common crank. The reciprocating mass per cylinder is 11.5 kg and the crank radius is 75 mm. The length of the connecting rod is 0.3 m. Show that the engine may be balanced for primary forces by means of a revolving balance mass. If the engine speed is 500 rpm, what is the value of maximum resultant secondary force?

$2\alpha = 90°$ m = 11.5 kg

r = 75 mm = 0.075 m l = 0.3 m

N = 500 r.p.m. $\omega = \frac{2\pi N}{60} = \frac{2\times\pi\times 500}{60} = 52.37\ rad/s$

Primary force, $F_P = 2mr\omega^2\sqrt{(\cos^2\alpha\ \cos\theta)^2 + (\sin^2\alpha\ \sin\theta)^2}$

$$= 2mr\omega^2\sqrt{(\cos^2 45°\ \cos\theta)^2 + (\sin^2 45°\ \sin\theta)^2}$$

$$= 2mr\omega^2\sqrt{\left(\frac{\cos\theta}{2}\right)^2 + \left(\frac{\sin\theta}{2}\right)^2}$$

$$= mr\omega^2$$

Since the resultant primary force $mr\omega^2$ is the centrifugal force of a mass m at the crank radius r when rotating at ω rad/s, therefore, the engine may be balanced by a rotating balance mass.

Secondary force, $F_S = \frac{\sqrt{2}mr\omega^2}{n}\sin 2\theta$

This is maximum, when sin 2θ is maximum i.e. when sin 2θ = ± 1 or θ = 45° or 135°. ∴ Maximum resultant secondary force,

$$F_{S(\max)} = \frac{\sqrt{2}mr\omega^2}{n} \qquad \text{(Substituting } \theta = 45°\text{)}$$

$$F_{S(\max)} = \frac{\sqrt{2}\times 11.5\times 0.075\times(52.37)^2}{0.3/0.075} \qquad (n = l/r)$$

= 836 N

3.12 Balancing of Radial Engine

A radial engine is a multi-cylinder engine in which all the connecting rods are connected to a common crank. The analysis of forces in such type of engines is much simplified by using the method of direct and reverse cranks. As all the forces are in the same plane, no unbalance couples exist.

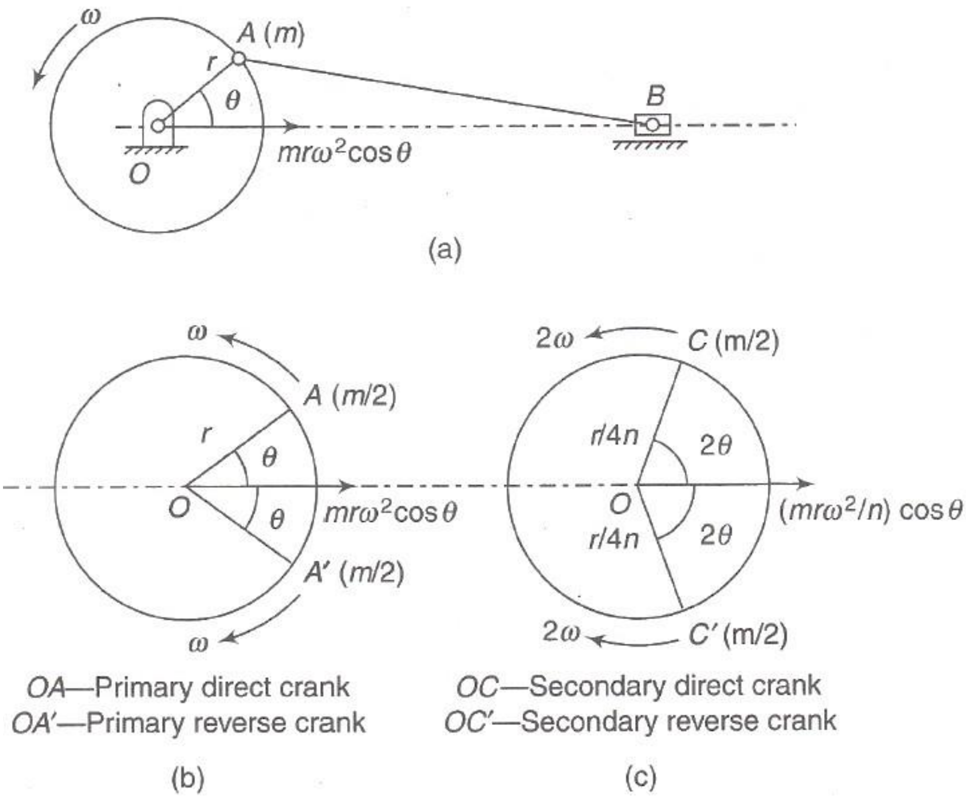


Fig. 3.59

In a reciprocating engine [Fig. 3.59(a)],

Primary force: $mr\omega^2\cos\theta$ (along line of stroke)

In the method of direct and reverse cranks, a force identical to this force is generated by two masses in the following way:

- A mass m/2, placed at the crank pin A and rotating at an angular velocity ω in the given direction [Fig. 3.59(b)].
- A mass m/2, placed at the crank pin of an imaginary crank OA' at the same angular position as the real crank but in the opposite direction of the line of stroke. This imaginary crank is assumed to rotate at the same angular velocity ω in the opposite direction to that of the real crank. Thus, while rotating; the two masses coincide only on the cylinder center line. Now, the components of centrifugal force due to rotating masses along line of stroke are

Due to mass at A $= \frac{m}{2} r\omega^2 \cos\theta$

Due to mass at A' $= \frac{m}{2} r\omega^2 \cos\theta$

Thus, total force along line of stroke = $mr\omega^2\cos\theta$ which is equal to the primary force. At any instant, the components of the centrifugal forces of these two masses normal to the line of stroke will be equal and opposite.

The crank rotating in the direction of engine rotation is known as the direct crank and the imaginary crank rotating in the opposite direction is known as the reverse crank.

Secondary accelerating force $= mr\omega^2 \frac{\cos 2\theta}{n} = mr(2\omega)^2 \frac{\cos 2\theta}{4n}$

$$= m\frac{r}{4n}(2\omega)^2 \cos 2\theta \qquad \text{(along line of stroke)}$$

This force can also be generated by two masses in a similar way as follows:

- A mass m/2, placed at the end of direct secondary crank of length r/(4n) at angle 2θ and rotating at an angular velocity 2ω in the given direction [Fig. 3.59(c)].
- A mass m/2, placed at the end of reverse secondary crank of length r/(4n) at angle -2θ rotating at an angular velocity 2ω in the opposite direction. Now, the components of centrifugal force due to rotating masses along line of stroke are

Due to mass at $C = \frac{m}{2}\frac{r}{4n}(2\omega)^2 \cos 2\theta = \frac{mr\omega^2}{2n}\cos 2\theta$

Due to mass at $C' = \frac{m}{2}\frac{r}{4n}(2\omega)^2 \cos 2\theta = \frac{mr\omega^2}{2n}\cos 2\theta$

Thus total force along line of stroke $= 2 \times \frac{m}{2}\frac{r}{4n}(2\omega)^2 \cos 2\theta = \frac{mr\omega^2}{n}\cos 2\theta$

Which is equal to the secondary force.

5.1 Introduction

Power is transmitted from one shaft to another by means of Belt, rope, chain and gears.

Salient Features:

- Belt, rope and chain are used where the distance between the shaft is large. For small distance, gears are preferred.
- Belt, rope and chain are the flexible types of connectors, i.e., they are bent easily.
- The flexibility of belt and rope is due to the property of their materials whereas chains have a number of small rigid elements having relative motion between the two elements.
- Belt and rope transmit power due to friction between them and the pulley. If the power transmitted exceeds the force of friction, the belt or rope slips over the pulley.
- Belts and ropes are strained during motion as tensions are developed in them.
- Owing to slipping and straining action belts and ropes are not positive drive, i.e., velocity ratio are not constant. Chain and gears have a constant velocity ratio.

5.2 Selection of Belt drive

Following are the factors which affect the selection of belt drive:

- Speed of driving and driven shafts.
- Power to be transmitted.
- Space available.
- Service conditions.
- Centre distance between the shafts.
- Speed reduction ratio.

5.3 Types of Belt drives

a) Light drives

- Small power.
- $V \leq 10$ m/s Agricultural machine, Small machine.

b) Medium drives

- Medium power
- $22 < V > 10$ m/s, Machine tool.

c) Heavy drives

- Large power
- $V > 22$ m/s Compressor, generator.

5.3.1 Belt drives & its materials

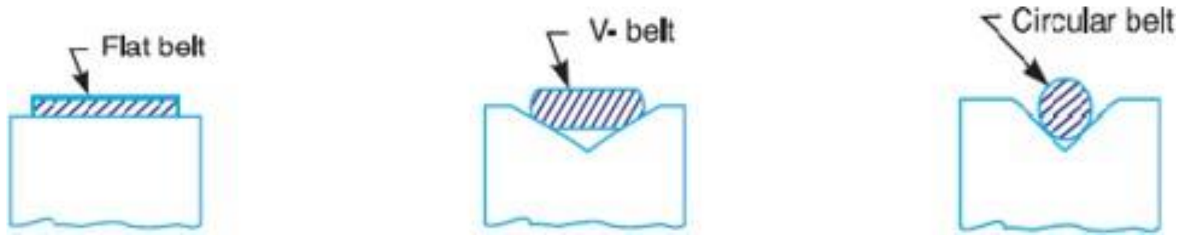


Fig.5.1 - Types of Belts

5.3.1.1 Flat Belt

- Used in the industry where a moderate amount of power is transmitted.
- Dist. x ≤ 8m or 10m apart with 22 m/sec.
- Materials are leather, rubber, canvas, cotton & rubber Balata (higher strength than rubber belt).

5.3.1.2 V- Belt

- Used in the industry where a moderate amount of power to be transmitted.
- Connect the shaft up to 4m.
- Speed ratio can be up to 7 to 1 and belt speed 24 m/sec.
- Made of rubber impregnated fabric with the angle of V between 30° to 40°.

  Note: In multiple V – belt drive all the belt should be stretch at the same rate so that load is equally divided. When one of the selves of belt break, the entire set should be replaced at the same drive. If one belt is replaced the new unworn and unstressed will be more tightly stretched and will more with different velocity.

5.3.1.3 Ropes

- Used where a higher amount of power to be transmitted distance up to 30m apart.
- Operating speed is less than 3 m/sec.
- Materials for rope are cotton, hemp, manila or wire.

5.3.2 Types of Flat Belt Drives

5.3.2.1 Open Belt Drive

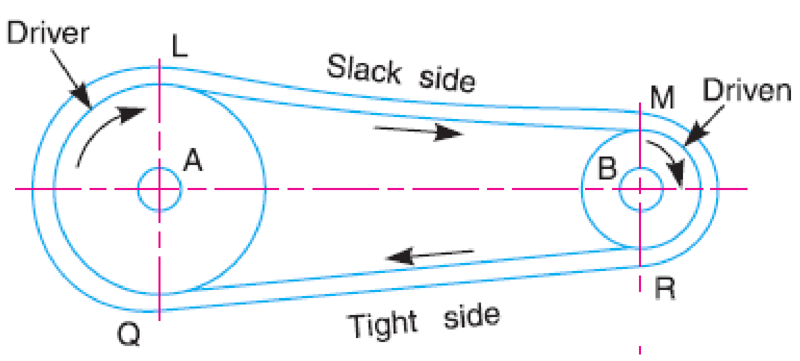


Fig.5.2 - Open belt drive

- An open belt drive is used when the driven pulley is desired to rotate in the same direction.
- Generally, the centre distance for open belt drive is 14 – 16 m. if the distance is too large, the belt whips i.e. vibrate in a direction perpendicular to the direction of motion.
- For very shorter distance, the belt slips increase.
- While transmitting power, one side of the belt is more tightened (known as a tight side) as compared to other (known as a slack side).
- In the case of horizontal drives, it is always desired that the tight side is at the lower side of two pulleys. This is because the sag of the belt will be more on the upper side than the lower side. This slightly increases the angle of wrap of the belts on the two pulleys than if the belt had been perfectly straight between the pulleys.
- In case the tight side of the belt is on the upper side, the sag will be greater at the lower side, reducing the angle of wrap and slip could occur earlier. This ultimately affects the power to be transmitted.

5.3.2.2 Crossed Belt Drive

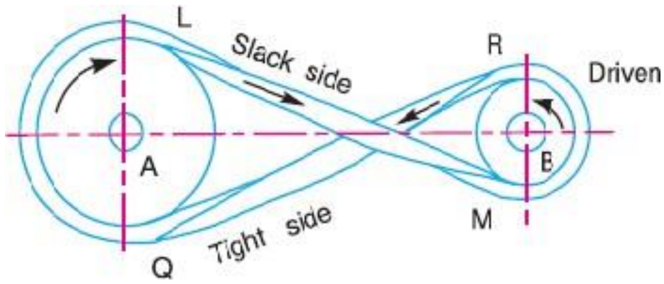


Fig.5.3 - Crossed belt drive

- A crossed belt drive is used when the driven pulley is to be rotated in the opposite direction to that of the driving pulley.
- A crossed belt drive can transmit more power than an open belt drive as the angle of wrap is more.
- However, the belt has to bend in two different planes and it wears out more. To avoid this the shaft should be placed at a max dist. 20 b where b = width of belt and speed should be less than 15 m /sec.

5.3.2.3 Quarter Turn Belt Drive / Right Angle Belt Drive

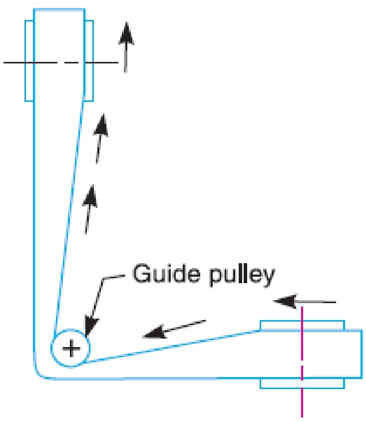


Fig.5.4 - Quarter turn belt drive with guide pulley

- A guide pulley is used to connect two non-parallel shafts in such a way that they may run in either direction and still making the pulley to deliver the belt properly in accordance with the law of belting.
- A guide pulley can also be used to connect even intersecting shaft also.

5.3.2.4 Belt Drive with Idler Pulley

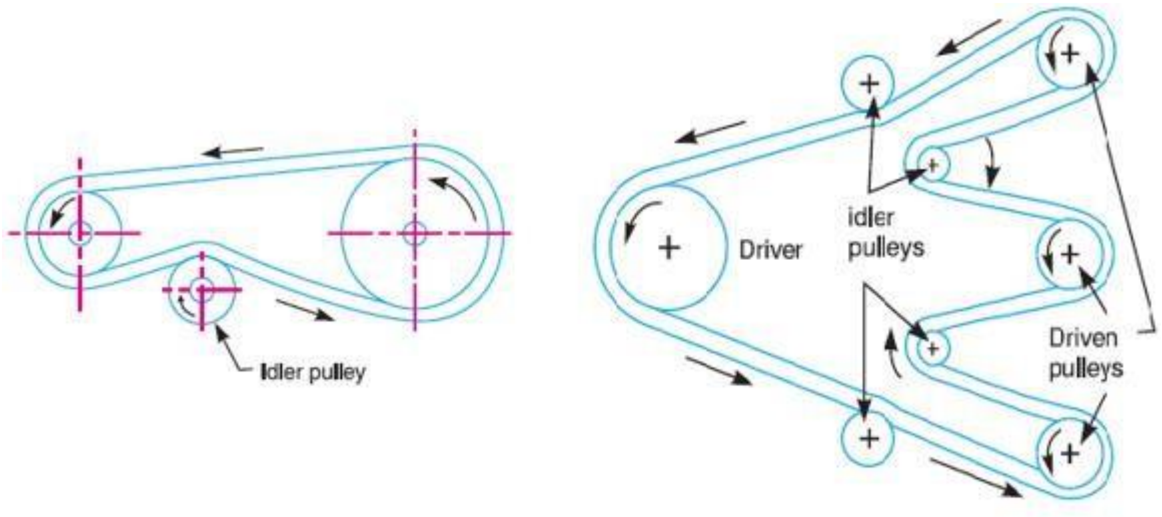


Fig.5.5 - Belt drive with Idler pulley

- With constant use, the belt is permanently stretched in a little longer. This reduces the initial tension in the belt leading to lower power transmission capacity. However, the tension in the belt can be restored to the original value.
- A bell – crank lever, hinged on the axis of the smaller pulley, supports adjustable weights on its one arm and the axis of a pulley on the other. The pulley is free to rotate on its axis and is known as an idler pulley. Owing to weights on one arm of the lever, the pulley exerts pressure on the belt increasing the tension and the angle of contact. Thus, the life of the belt is increased and the power capacity is restored to the original value.
- The motion of one shaft can be transmitted to two or more than two shafts by using a number of the idler pulley.

5.3.2.5 Compound Belt Drive / Intermediate Pulley

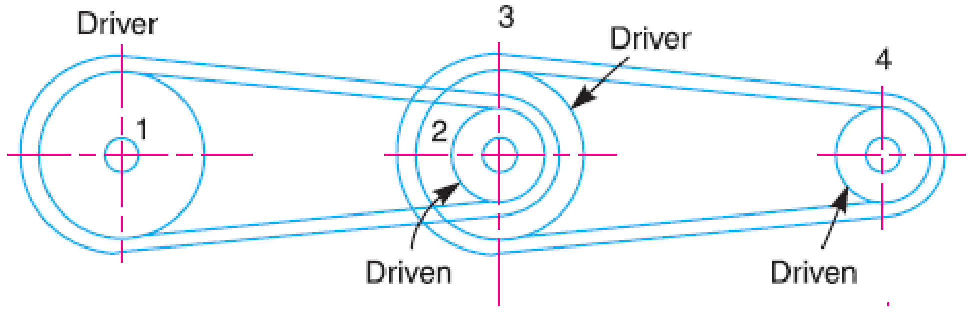


Fig.5.6 - Compound belt drive

- When it is required to have large velocity ratios, ordinarily the size of the larger pulley will be quite big. However, by using an intermediate (counter-shaft) pulley, the size can be reduced.

5.3.2.6 *Stepped / Cone Pulley Drive*

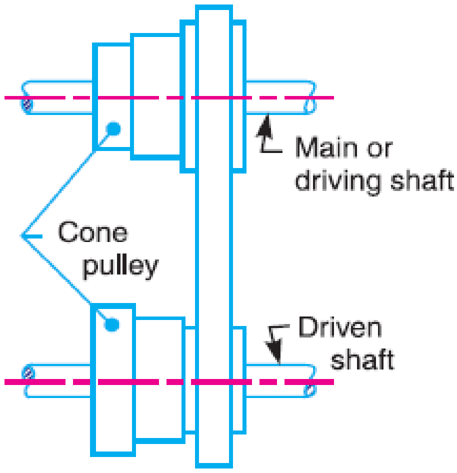


Fig.5.7 - Stepped / Cone pulley drive

- A stepped cone pulley drive is used for changing the speed of the driven shaft while the main or driving shaft runs at a constant speed.
- This is done by shifting the belt from one part of the step to the other.

5.3.2.7 *Fast and Loose Pulley drive*

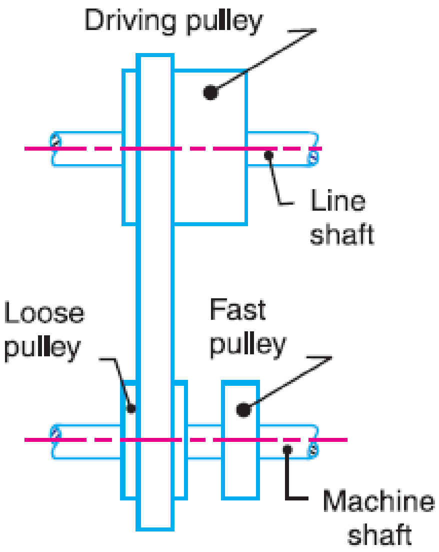


Fig.5.8 - Fast and loose pulley drive

- Many times, it is required to drive several machines from a single main shaft. In such a case, some arrangement to link or delink a machine to or from the main shaft has to be incorporated as all the machines may not be operating simultaneously. The arrangement usually provided is that of using a loose pulley along with a fast pulley.
- A fast pulley is keyed to the shaft and rotates with it at the same speed and thus transmits power.
- A loose pulley is not keyed to the shaft and thus is unable to transmit any power.
- Whenever a machine is to be driven, the belt is mounted on the fast pulley and when it is not required to transmit any power, the belt is pushed to the loose pulley placed adjacent to the fast pulley.

5.4 Law of Belting

- The law of belting states that centre line of the belt when it approaches a pulley must lie in the midplane of that pulley. However, a belt leaving a pulley may be drawn out of the plane of the pulley.
- By following this law, non – parallel shafts may be connected by a flat belt.
- It should be observed that it is not possible to operate the belt in the reverse direction without violating the law of belting. Thus, in the case of non – parallel shafts, motion is possible only in one direction. Otherwise, the belt is thrown off the pulley.
- However, it is possible to run a belt in either direction on the pulley of two non – parallel or intersecting shafts with the help of guide pulleys. The law of belting is satisfied.

5.4.1 Velocity Ratio of Belt Drive

- Velocity ratio is the ratio of the speed of driven pulley (N_2) to that of a driving pulley (N_1)

Let,

N_1 = Speed of driving pulley

N_2 = Speed of driven pulley

D_1 = Diameter of the driving pulley

D_2 = Diameter of the driven pulley

T = thickness of the belt

- Neglecting slip between belt & pulley and consider belt to be inelastic.

Let, the speed of belt on driving pulley = speed of belt on driven pulley

$$\pi \frac{D_1 N_1}{60} = \pi \frac{D_2 N_2}{60}$$

$$\left(D_1 + 2\frac{t}{2}\right) N_1 = \left(D_2 + 2\frac{t}{2}\right) N_2$$

Or Velocity Ratio (VR)

$$(VR) = \frac{N_2}{N_1} = \frac{D_1 + t}{D_2 + t}$$

5.4.2 Velocity Ratio of Compound Belt Drive

Let,

D_1, D_2, D_3, D_4 = Diameter of pulley

N_1, N_2, N_3, N_4 = Speed of pulley

For pulley 1 & 2

$$\frac{N_2}{N_1} = \frac{D_1}{D_2} \qquad \textit{Eq. (5.1)}$$

For pulley 3 & 4

$$\frac{N_4}{N_3} = \frac{D_3}{D_4} \qquad \textit{Eq. (5.2)}$$

Multiplying *Eq. (5.1)* & *Eq. (5.2)*

$$\frac{N_2}{N_1} \times \frac{N_4}{N_3} = \frac{D_1}{D_2} \times \frac{D_3}{D_4}$$

$$\mathbf{\frac{N_4}{N_1} = \frac{D_1}{D_2} \times \frac{D_3}{D_4}}$$

5.5 Slip of Belt

- Sometimes, due to insufficient frictional grip, there may be a chance of forwarding motion of belt without carrying the driven pulley with it. This is called slip of belt and is expressed as a percentage.
- Result of slipping is reduced the velocity ratio of the system.
- Let,

  S_1% = Slip between driven & belt

  S_2% = Slip between belt & follower

  Let the velocity of the belt passing over driver per second

  $$V = \frac{\pi d_1 N_1}{60} - \frac{\pi d_1 N_1}{60} \times \frac{S_1}{100} = \frac{\pi d_1 N_1}{60}\left[1 - \frac{S_1}{100}\right] \qquad \textit{Eq. (5.3)}$$

  The velocity of the belt passing over follower per second

  $$\frac{\pi d_2 N_2}{60} = V - V \cdot \frac{S_2}{100} = V\left[1 - \frac{S_2}{100}\right] \qquad \textit{Eq. (5.4)}$$

  From Eq. (5.3) & Eq. (5.4)

  $$\frac{\pi d_2 N_2}{60} = \frac{\pi d_1 N_1}{60}\left[1 - \frac{S_1}{100}\right] \times \left[1 - \frac{S_2}{100}\right]$$

  $$\frac{N_2}{N_1} = \frac{d_1}{d_2}\left[1 - \frac{S_1}{100} - \frac{S_2}{100}\right] \ldots\ldots\ldots\ldots\ldots (\text{Neglecting } \frac{S_1 S_2}{100 \times 100})$$

  $$= \frac{d_1}{d_2}\left[1 - \frac{(S_1 + S_2)}{100}\right]$$

  $$\frac{N_2}{N_1} = \frac{d_1}{d_2}\left[1 - \frac{S}{100}\right]$$

  Where $S = S_1 + S_2 - 0.01\,S_1S_2$ = Total Percentage of slip

  $$\mathbf{\frac{N_2}{N_1} = \frac{d_1 + t}{d_2 + t}\left[1 - \frac{S}{100}\right]}$$

5.6 Creep of Belt

When the belt passes from slack side to tight side, a certain portion of the belt extends and it contracts again when the belt passes from tight side to slack side. Due to these change of length, there is a relative motion between belt & pulley surface. This relative motion is called "Creep".

The total effect of creep is to reduce slightly the speed of the driven pulley or follower.

Considering the creep,

$$\text{Velocity ratio } \frac{N_2}{N_1} = \frac{d_1}{d_2}\left[\frac{E + \sqrt{\sigma_2}}{E + \sqrt{\sigma_1}}\right]$$

Where

σ_1 = Stress on the tight side of the belt

σ_2 = Stress on the slack side of the belt

E = Young modulus for the belt material

5.7 Length of an Open Belt Drive

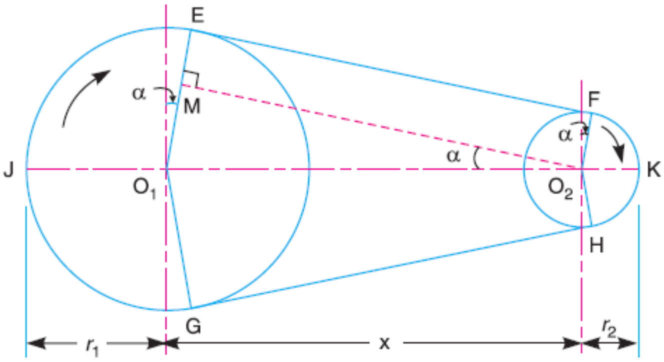


Fig.5.9 - 9 Length of an open belt drive

Let r_1 , r_2 = Radius of larger & smaller pulley.

x = Distance between centre of two pulley.

L = Total Length.

Let the length of the belt,

$$L = \text{Arc GJE} + \text{EF} + \text{Arc FKH} + \text{GH}$$

$$= 2\,(\text{Arc JE} + \text{EF} + \text{Arc FK}) \qquad \textit{Eq. (5.5)}$$

Let

$$\text{Arc EJG} = 2\,\text{Arc JE} = 2\left(\frac{\pi}{2} + \alpha\right)\cdot r_1$$

$$\text{Arc FKH} = 2\,\text{Arc FK} = 2\left(\frac{\pi}{2} - \alpha\right)\cdot r_2$$

$$\sin\alpha = \frac{r_1 - r_2}{x}$$

$$\text{EF} = \text{MO}_2 = \sqrt{(O_1O_2)^2 - (O_1M)^2}$$

$$= \sqrt{(x)^2 - (r_1 - r_2)^2}$$

$$= x\sqrt{1 - \left(\frac{r_1 - r_2}{x}\right)^2}$$

Expanding by Binominal Theorem

$$= x\left[1 - \frac{1}{2}\left(\frac{r_1 - r_2}{x}\right)^2 + \cdots\cdots\cdots\right] = x - \frac{1}{2}\left(\frac{r_1 - r_2}{x}\right)^2$$

Putting value in *Eq. (5.5)*

$$\therefore L = 2\ [(\frac{\pi}{2} + \alpha)\, r_1 + x - \frac{(r_1 - r_2)^2}{2x} + (\frac{\pi}{2} - \alpha)\, r_2]$$

$$= 2\ [r_1 \cdot \frac{\pi}{2} + r_1 \cdot \alpha + x - \frac{(r_1 - r_2)^2}{2x} + r_2 \cdot \frac{\pi}{2} - r_2 \cdot \alpha\,]$$

$$= 2\ [\frac{\pi}{2}(r_1 + r_2) + \alpha\,(r_1 - r_2) + x - \frac{(r_1 - r_2)^2}{2x}]$$

$$= \pi\,(r_1 + r_2) + 2\,\alpha\,(r_1 - r_2) + 2x - \frac{(r_1 - r_2)^2}{x}$$

$$\left[\begin{array}{c} \sin\alpha = \dfrac{r_1 - r_2}{x} \\ \alpha \text{ is small} \\ \therefore\ \sin\alpha = \alpha \end{array}\right]$$

$$= \pi\,(r_1 + r_2) + 2\,(\frac{r_1 - r_2}{x})\,(r_1 - r_2) + 2x - \frac{(r_1 - r_2)^2}{x}$$

$$= \pi\,(r_1 + r_2) + 2\,\frac{(r_1 - r_2)^2}{x} + 2x - \frac{(r_1 - r_2)^2}{x}$$

$$L = \pi\,(r_1 + r_2) + 2x + \frac{(r_1 - r_2)^2}{x} \quad \text{In terms of Radius}$$

$$\mathbf{L = \frac{\pi}{2}(d_1 + d_2) + 2x + \frac{(d_1 - d_2)^2}{4x}} \quad \textbf{In terms of diameter}$$

5.8 Length of Crossed Belt Drive

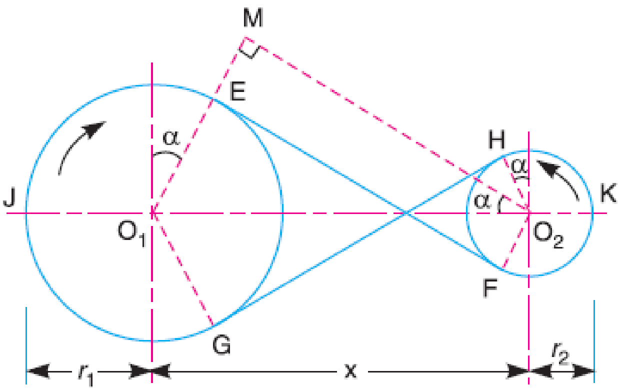


Fig.5.10 - Length of crossed belt drive

Let r_1, r_2 = Radius of larger & smaller pulley

x = Distance between centre of two pulley

L = Total Length

$$L = \text{Arc GJE} + \text{Arc FKH} + EF + GH \quad \textit{Eq. (5.6)}$$

$$\text{Arc GJE} = 2\,\text{Arc JE} = 2\left(\frac{\pi}{2} + \alpha\right) \cdot r_1 \quad \textit{Eq. (5.7)}$$

$$\text{Arc FKH} = 2\,\text{Arc HK} = 2\left(\frac{\pi}{2} + \alpha\right) \cdot r_2 \quad \textit{Eq. (5.8)}$$

$$EF = GH = MO_2 = \sqrt{(O_1O_2)^2 - (O_2M)^2}$$

$$= \sqrt{x^2 - (r_1 + r_2)^2}$$

$$= x\sqrt{1 - \left(\frac{r_1 + r_2}{x}\right)^2}$$

Expanding by Binominal Theorem

$$= x\left[1 - \frac{1}{2}\left(\frac{r_1 + r_2}{x}\right)^2 + \cdots\cdots\cdots\right]$$

$$= x - \frac{(r_1 + r_2)^2}{2x} \quad \textit{Eq. (5.9)}$$

Putting value of *Eq. (5.7)*, *Eq. (5.8)* & *Eq. (5.9)* in *Eq. (5.6)*

$$\therefore L = 2\left[r_1\left(\frac{\pi}{2} + \alpha\right) + x - \frac{(r_1 + r_2)^2}{2x} + r_2\left(\frac{\pi}{2} + \alpha\right)\right]$$

$$= 2\left[r_1 \cdot \frac{\pi}{2} + r_1 \cdot \alpha + x - \frac{(r_1 + r_2)^2}{2x} + r_2 \cdot \frac{\pi}{2} + r_2 \cdot \alpha\right]$$

$$= 2\left[\frac{\pi}{2}(r_1 + r_2) + \alpha(r_1 + r_2) + x - \frac{(r_1 + r_2)^2}{2x}\right]$$

$$= \pi(r_1 + r_2) + 2\alpha(r_1 + r_2) + 2x - \frac{(r_1 + r_2)^2}{x}$$

$$\left[\begin{array}{c} \sin\alpha = \alpha \\ \text{As } \alpha \text{ is very small} \\ \sin\alpha = \dfrac{r_1 + r_2}{x} \end{array}\right]$$

$$= \pi(r_1 + r_2) + 2\left(\frac{r_1 + r_2}{x}\right)(r_1 + r_2) + 2x - \frac{(r_1 + r_2)^2}{x}$$

$$= \pi(r_1 + r_2) + 2\,\frac{(r_1 + r_2)^2}{x} + 2x - \frac{(r_1 + r_2)^2}{x}$$

$$L = \pi(r_1 + r_2) + 2x + \frac{(r_1 + r_2)^2}{x} \quad \text{In terms of Radius}$$

$$\mathbf{L = \frac{\pi}{2}(d_1 + d_2) + 2x + \frac{(d_1 + d_2)^2}{4x}} \quad \textbf{In terms of diameter}$$

5.9 Ratio of Friction Tensions

5.9.1 Flat Belt

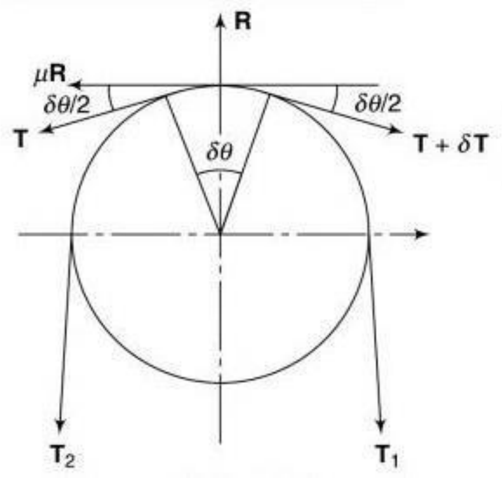


Fig.5.11 - Ratio of Friction Tensions for Flat Belt

T_1 = Tensions on tight side.

T_2 = Tensions on slack side.

θ = Angle of Lap of the belt over the pulley.

μ= Coefficient of friction between belt & pulley.

Consider a short length of belt PQ subtending an angle of $\delta\theta$ at the centre of the pulley.

R = Normal (Radial) reaction between element length of belt & pulley.

T = Tension on the slack side of the element.

δT = increase in tension on the tight side than that of the slack side.

$T + \delta T$ = Tension on the tight side of the element.

Tensions T and $(T + \delta T)$ act in directions perpendicular to the radii drawn at the end of elements. The friction force $F = \mu R$ will act tangentially to the pulley rim resisting the slipping of the elementary belt on the pulley.

- Resolving the forces in the tangential direction (Horizontally),

$$\mu R + T\ \cos\frac{\delta\theta}{2} - (T + \delta T)\cos\frac{\delta\theta}{2} = 0$$

$$\mu R + T\ - (T + \delta T)\ = 0$$

As $\delta\theta$ is small
$[\ \cos\frac{\delta\theta}{2}\ \approx 1\ ]$

$$R = \frac{\delta T}{\mu}$$ *Eq. (5.10)*

- Resolving the forces in Radial Direction (Vertically),

$$R\ - T\ \sin\frac{\delta\theta}{2} - (T + \delta T)\sin\frac{\delta\theta}{2} = 0$$

$$R - T\frac{\delta\theta}{2} - T\frac{\delta\theta}{2} - \frac{\delta T \cdot \delta\theta}{2} = 0$$

As $\delta\theta$ is small
$$\left[\sin\frac{\delta\theta}{2} \approx \frac{\delta\theta}{2}\right]$$
$$\frac{\delta T \cdot \delta\theta}{2} \rightarrow \text{Neglect}$$

$$R = 2\frac{T\delta\theta}{2} = T\,\delta\theta$$

Eq. (5.11)

Comparing *Eq. (5.10)* and *Eq. (5.11)*

$$\frac{\delta T}{\mu} = T\,\delta\theta$$

$$\therefore \frac{\delta T}{T} = \mu\,\delta\theta$$

Integrating between proper limits,

$$\int_{T_2}^{T_1} \frac{\delta T}{T} = \int_0^{\theta} \mu\,\delta\theta$$

$$\therefore \log_e \frac{T_1}{T_2} = \mu\theta$$

Or taking a log to the base 10

$$\frac{\mathbf{T_1}}{\mathbf{T_2}} = \mathbf{e^{\mu\theta}} \quad 2.3\log\frac{T_1}{T_2} = \mu\theta$$

Note: The above relation is valid only when the belt is on the point of slipping on the pulleys.

5.9.2 V – Belt or Rope

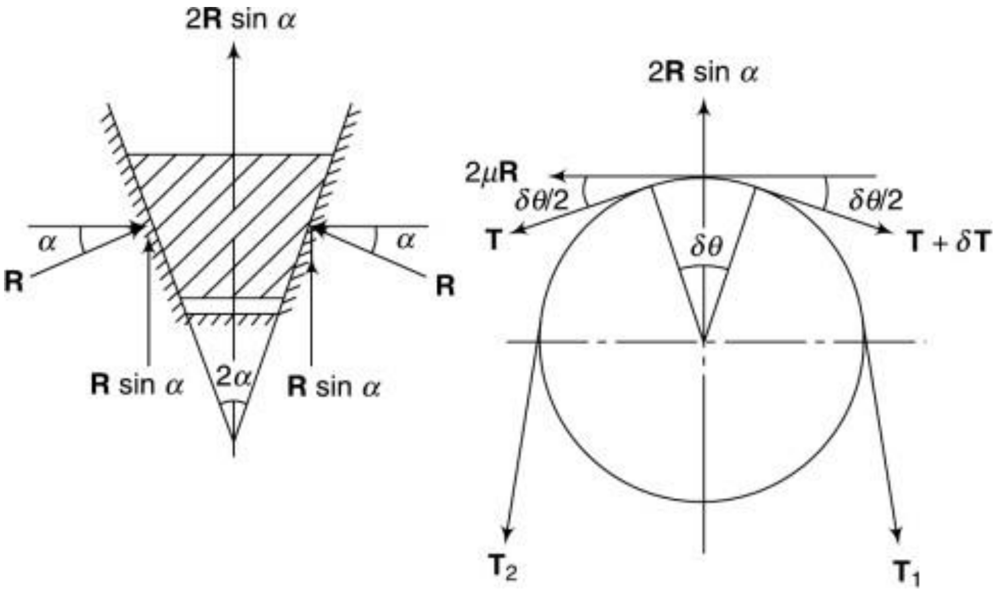


Fig.5.12 - Ratio of Friction Tensions for V-Belt or Rope

In the case of v belt or rope, there are two normal reactions, so the radial reaction is equal to

$$2R\ \sin\alpha$$

Thus total friction force = $2F = 2\mu R$

Resolving the forces tangentially,

$$2\mu R + T\ \cos\frac{\delta\theta}{2} - (T + \delta T)\cos\frac{\delta\theta}{2} = 0$$

$$2\mu R + T - T - \delta T = 0$$

$$\left[\begin{array}{c}\text{As } \delta\theta \text{ is small} \\ \cos\frac{\delta\theta}{2} \approx 1\end{array}\right]$$

$$\delta T = 2\mu R \qquad \textit{Eq. (5.12)}$$

Resolving the forces radially,

$$2R\sin\alpha - T\ \sin\frac{\delta\theta}{2} - (T + \delta T)\sin\frac{\delta\theta}{2} = 0$$

$$2R\sin\alpha - T\frac{\delta\theta}{2} - T\frac{\delta\theta}{2} - \frac{\delta T \cdot \delta\theta}{2} = 0$$

$$\left[\begin{array}{c}\text{As } \delta\theta \text{ is small} \\ \sin\frac{\delta\theta}{2} \approx \frac{\delta\theta}{2} \\ \frac{\delta T \cdot \delta\theta}{2} \rightarrow \text{Neglect}\end{array}\right]$$

$$2R\sin\alpha = 2\ \frac{T\delta\theta}{2}$$

$$\therefore R = \frac{T\delta\theta}{2\sin\alpha} \qquad \textit{Eq. (5.13)}$$

From *Eq. (5.12)* and *Eq. (5.13)*

$$\delta T = 2\ \mu\ \frac{T\delta\theta}{2\sin\alpha} \qquad \text{or} \qquad \frac{\delta T}{T} = \frac{\mu\ \delta\theta}{\sin\alpha}$$

Integrating between proper limits,

$$\int_{T_2}^{T_1}\frac{\delta T}{T} = \int_0^{\theta}\frac{\mu\ \delta\theta}{\sin\alpha}$$

$$\therefore\ \log_e\frac{T_1}{T_2} = \frac{\mu\theta}{\sin\alpha}$$

$$\mathbf{\frac{T_1}{T_2} = e^{\mu\theta/\sin\alpha} = e^{\mu\theta\ cosec\alpha}}$$

Notes:

- The expression is similar to that for a flat belt drive except that μ is replaced by $\mu/\sin\theta$, i.e., the coefficient of friction is increased by $1/\sin\theta$. Thus, the ratio T_1/T_2 is far greater in the case of V – belts & ropes for the same angle of lap θ and coefficient of friction μ.
- Above expression is derived on the assumption that the belt is on the point of slipping.

5.10 Power Transmitted by a belt

Let

T_1 = Tensions on tight side.

T_2 = Tensions on slack side.

V = Linear velocity of the belt

P = Power transmitted

$$P = \text{Net Force} \times \frac{\text{Distance moved}}{\text{second}}$$

$$= (T_1 - T_2)\,V$$

Note:-This relation gives the power transmitted irrespective of the fact whether the belt is on the point of slipping or not.

If it is the relationship between T_1 and T_2 for a flat belt is given by $T_1/T_2 = e^{\mu\theta}$. If it is not, no particular relation is available to calculate $T_1$and T_2.

5.11 Angle of Contact or Angle of Lap

5.11.1 Open Belt Drive

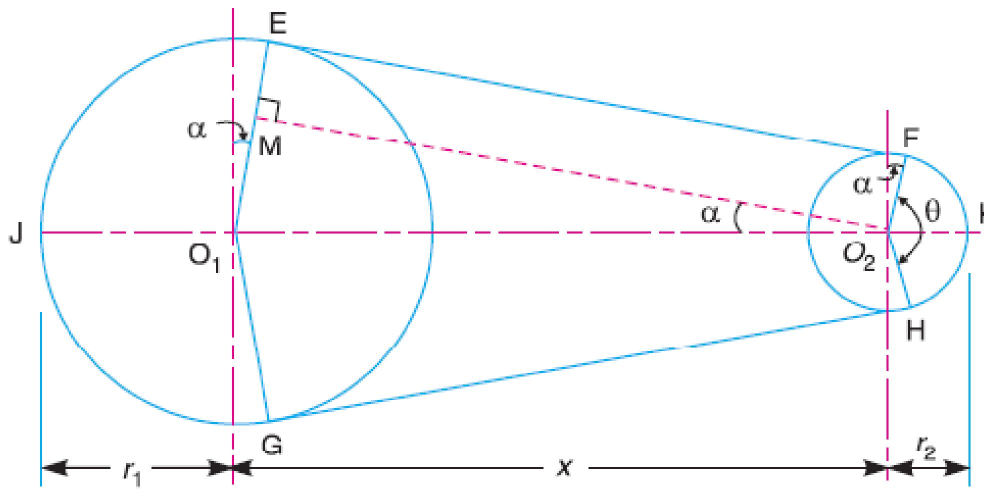


Fig.5.13 - Open Belt Drive

r_1 = Radius of the larger pulley
r_2 = Radius of the smaller pulley
x = Centre distance between two pulley

Angle of contact (θ)

$$\theta = (180 - 2\alpha)^\circ$$
$$= (180 - 2\alpha)\frac{\pi}{180} \text{ Radian}$$
$$\text{Where } \sin\alpha = \left(\frac{r_1 - r_2}{x}\right)$$

5.11.2 Cross Belt Drive

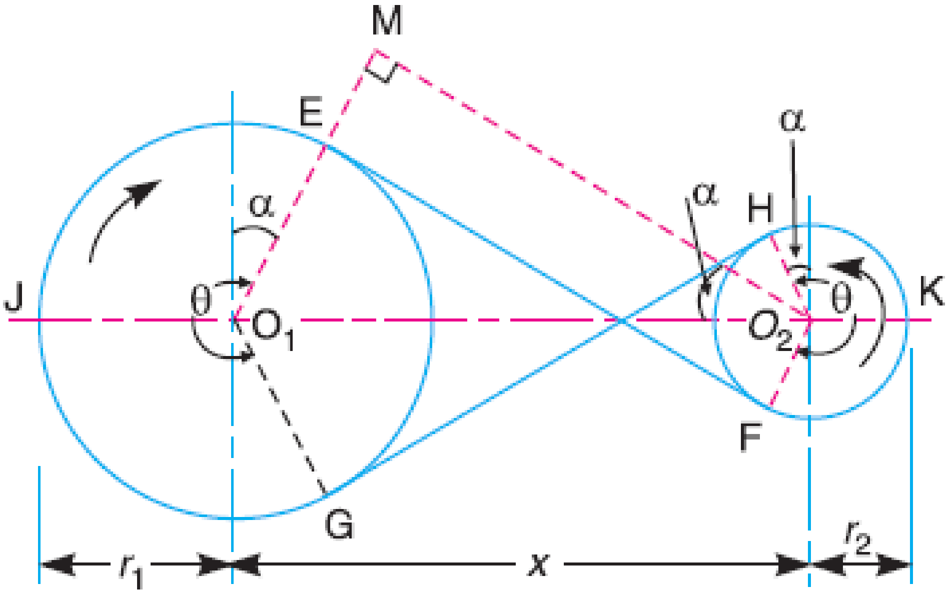


Fig.5.14 - Cross Belt Drive

Angle of contact (θ)

$$\theta = (180 + 2\alpha)^\circ$$
$$= (180 + 2\alpha)\frac{\pi}{180} \text{ Radian}$$
$$\text{Where } \sin\alpha = \left(\frac{r_1 + r_2}{x}\right)$$

5.12 Centrifugal effect on Belt

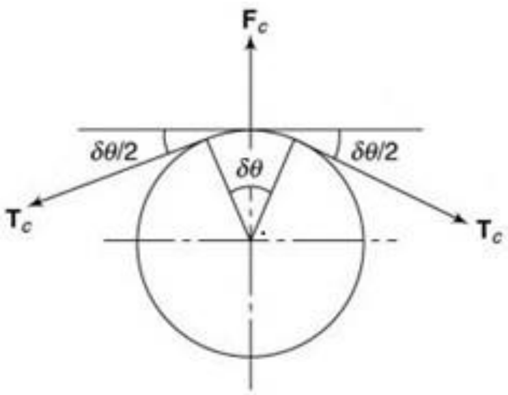


Fig.5.15 - Centrifugal effect on Belt

- While in motion, as a belt passes over a pulley, the centrifugal effect due to its own weight tends to lift the belt from the pulley.

- The centrifugal force produces equal tensions on the two side of the belt, i.e., on the tight and on the slack side.
- Let,

  m = mass of belt per meter length (Kg/m)

  v = Linear velocity of belt (m/sec)

  r = Radius of pulley

  T_C = Centrifugal tension on tight side & Slack side

  F_C = Centrifugal force

$$F_C = \text{mass of element} \times \text{Acceleration}$$

$$= (\text{Length of element} \times \text{mass per unit length}) \times \text{Acc.}$$

$$= (r\,\delta\theta \times m) \times \frac{v^2}{r} \qquad \textit{Eq. (5.14)}$$

From figure resolving forces radially,

$$F_C = 2\,T_C \sin\frac{\delta\theta}{2}$$

$$= 2\,T_C \frac{\delta\theta}{2}$$

$$\left[\begin{array}{c}\frac{\delta\theta}{2} \text{ is small} \\ \sin\frac{\delta\theta}{2} \approx \frac{\delta\theta}{2}\end{array}\right]$$

$$F_C = T_C\delta\theta \qquad \textit{Eq. (5.15)}$$

From *Eq. (5.14)* and *Eq. (5.15)*,

$$T_C\delta\theta = m\,v^2\delta\theta$$

$$\therefore\ \mathbf{T_C = m\,v^2}$$

∴ To depend on the only velocity of belt and mass of the belt.

$$\text{Also centrifugal stress in belt} = \frac{\text{centrifugal Tension}}{\text{c/s Area of belt}} = \frac{T_C}{a}$$

Or

$$T_C = \sigma \cdot A \text{ (Max tension in belt)}$$

Total Tension on tight side = Friction tension + Centrifugal tension

$$T = T_1 + T_C$$

Total Tension on slack side $= T_2 + T_C$

5.13 Maximum Power transmitted by a belt

If it is desired that belt transmit maximum power, the following two conditions must be satisfied.

1. Larger tension must reach the maximum permissible value for the belt.

2. The belt should be on the point of slipping i.e., the maximum frictional force is developed in the belt.

Let

$$P = (T_1 - T_2) \cdot v$$

$$= T_1 \left(1 - \frac{T_2}{T_1}\right) \cdot v$$

$$= T_1 \left(1 - \frac{1}{\left(\frac{T_1}{T_2}\right)}\right) \cdot v$$

$$\left[\begin{array}{l} \frac{T_1}{T_2} = e^{\mu\theta} \\ 1 - \frac{1}{e^{\mu\theta}} = k \text{ constant} \end{array} \right]$$

$$= T_1 k \cdot v$$

$$= (T - T_C) \cdot k \cdot v$$

$$\left[\begin{array}{l} T = T_1 + T_C \\ \therefore T_1 = T - T_C \end{array} \right]$$

$$= kTv - T_C \cdot kv$$

$$= kTv - (m v^2) \cdot kv$$

$$P = kTv - kmv^3$$

Here maximum tension T in the belt should not exceed permissible value. Hence treating T as constant and differentiating the power with respect to v and equating the same equal to zero.

$$\therefore \frac{d_P}{d_v} = kT - 3\,v^2(k\,m) = 0$$

$$\therefore kT = 3\,v^2\,k\,m$$

$$T = 3\,mv^2$$

$$T = 3\,T_C \quad \text{or} \quad \mathbf{T_C} = \frac{\mathbf{T}}{\mathbf{3}}$$

For maximum power transmission, the centrifugal tension in the belt is equal to 1/3 of the maximum allowable belt tension and belt should be on the point of slipping.

Also $T = T_1 + T_C$

$$\therefore T_1 = T - T_C$$

$$= T - \frac{T}{3} = \frac{2T}{3}$$

Also $T = 3T_C$

$$= 3 \cdot m\, v^2$$

$$\therefore V_{max} = \sqrt{\frac{T}{3m}}$$

5.14 Initial Tension in the Belt

When a belt is first fitted to a pair of the pulley, an initial tension T_0 is given to the belt when the system is stationary. When transmitting power, the tension on tight increases to T_1 and that on slack side decreases to T_2.

If it is assumed that the material of the belt is perfectly elastic i.e., the strain in the belt is proportional to stress in it and the total length of the belt remains unchanged, the tension on the tight side will increase by the same amount as the tension on the slack side decreases. If the change in the tension is δT,

Tension on the tight side $T_1 = T_0 + \delta T$

Tension on the slack side $T_2 = T_0 - \delta T$

$$\therefore T_0 = \frac{T_1 + T_2}{2} = \text{Mean of tight side \&}\ \textit{slack side tensions}$$

5.14.1 Initial tension with centrifugal tensions

Total tension on tight side = $T_1 + T_C$

Total tension on slack side = $T_2 + T_C$

Let,

$$T_0 = \frac{T_1 + T_2}{2} = \frac{(T_1 + T_C) + (T_2 + T_C)}{2} = \frac{T_1 + T_2 + 2T_C}{2}$$

$$T_0 = \frac{T_1 + T_2}{2} = T_C$$

$$\therefore T_1 + T_2 = 2(T_0 - T_C)$$

$$\left[\text{Let } \frac{T_1}{T_2} = e^{\mu\theta} = k\right]$$

$$k\,T_2 + T_2 = 2\,(T_0 - T_C)$$

$$T_2 = 2\,\frac{(T_0 - T_C)}{k + 1}$$

Let,

$$T_1 = T_2 \cdot k$$

$$T_1 = \frac{2\,k\,(T_0 - T_C)}{(k + 1)} \qquad \textit{Eq. (5.16)}$$

$$\therefore\ T_1 - T_2 = \frac{2\,k\,(T_0 - T_C)}{(k + 1)} - 2\,\frac{(T_0 - T_C)}{k + 1}$$

$$T_1 - T_2 = \frac{2\,(k - 1)(T_0 - T_C)}{(k + 1)}$$

$$\text{Power transmitted (P)} = (T_1 - T_2)v$$

$$= \frac{2\,(k - 1)(T_0 - T_C)}{(k + 1)} \cdot v$$

$$= \frac{2\,(k - 1)(T_0 - m\,v^2)}{k + 1} \cdot v$$

$$P = \frac{2\,(k - 1)(T_0 v - m\,v^3)}{k + 1}$$

To find the condition for maximum power transmission,

$$\frac{d_P}{d_v} = T_0 - 3mv^2 = 0$$

$$\therefore T_0 = 3mv^2$$

$$\therefore v = \sqrt{\frac{T_0}{3m}}$$

When the belt drive is started, v = 0 and thus $T_C = 0$ $(T_C = mv^2)$

$$T_1 = \frac{2\,k\,T_0}{k + 1} \qquad \textit{Eq. (5.17)}$$

5.15 Crowning of Pulley

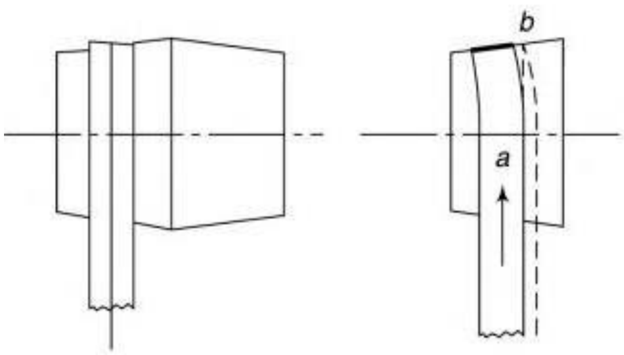


Fig.5.16 - Crowning of Pulley

Pulleys are provided with a slight dwell to prevent the belt from running off the pulley. This is known as the crowning of the pulley.

Crowning may be tapered or rounded. Normally crown height of 10 mm / m face width is provided.

5.16 Advantages & Disadvantages of V – Belt drive over Flat Belt drive

5.16.1 Advantages

- V - Belt drive gives compactness due to a small distance.
- The drive is positive because slip is less.
- V – Belts are made endless, no joint so smooth drive.
- Longer life 3 – 5 year.
- Easily installed & removed.
- The high-velocity ratio may be obtained.
- Power transmission is more due to wedging action of the belt in the groove.
- V – Belt may be operated in either direction.

5.16.2 Disadvantages

- V – Belt drive can't use for a large distance.
- V – Belts are not so durable as flat belts.
- Construction of pulley for V – Belt is more complicated than the pulleys for a flat belt.
- Since V – Belts are subjected to a certain amount of creep, so they are not suitable for constant speed application such as synchronous machines, timing devices etc.
- Belt life is effect with temperature changes, improper belt tension and mismatching of belt length.
- The centrifugal tension prevents the use of V – Belts at speed below 5 m/s and above 50 m/sec.

5.17 Rope drive

- Rope drive is widely used where a large amount of power is to be transmitted from one pulley to another over a considerable distance.
- Frictional grip in case of rope drives is more than V – drives.
- The number of separate drives may be taken from one driving pulley.

5.17.1 Types of Rope Drive

5.17.1.1 Fibre Rope Drive

- They are made from Fibrous material each as Hemp, Manila and cotton.
- Manila ropes are more durable and stronger than cotton rope. (Hempropes have less strength compared to Manila ropes).
- Cotton ropes are costlier than Manila ropes.

Fig.5.17 - Fibre Rope

5.17.1.2 Wire Rope Drive

Fig.5.18 - Wire Rope

- When a large amount of power is to be transmitted over a long distance ($\cong$ 150 m apart) then wire ropes are used.
- The wire ropes are used in elevators, mine hoists, crane, conveyors, and suspension bridges.
- The wire rope runs on grooved pulleys, but they rest on the bottom of the grooves and are not wedged between the sides of the grooves.
- Wire ropes are made from cold drawn wires, various materials are wrought iron, cast steel, alloy steel, copper, bronze, aluminium, alloys, S.S. etc.

5.17.1.2.1 Construction & Designation of Wire rope drive

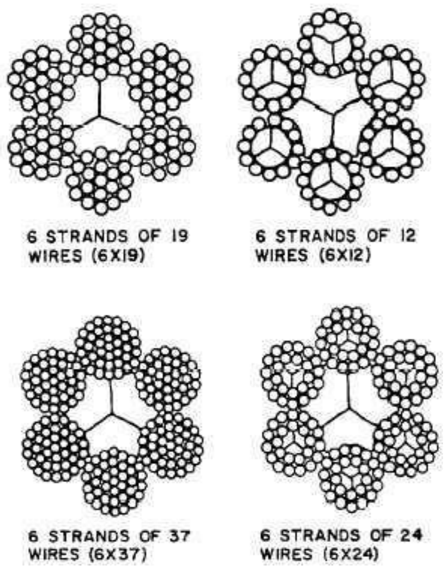


Fig.5.19 - Construction & Designation of Wire rope drive

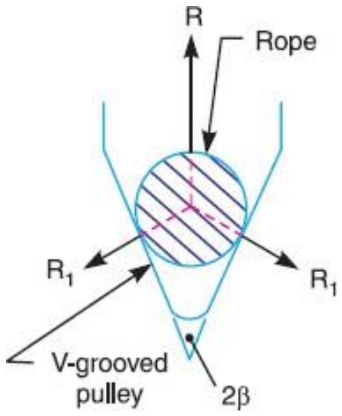


Fig.5.20 - Sheave for Fibre rope

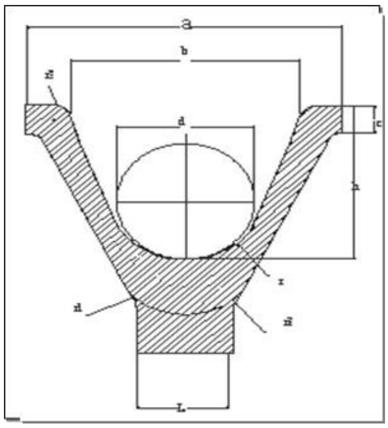


Fig.5.21 - Sheave for Wire rope

Ratio of Driving tensions $= 2.3\ \log\frac{T_1}{T_2} = \mu\,\theta\ \mathrm{cosec}\beta$

5.18 Chain Drive

- A chain is regarded in between the gear drive and the belt drive. Like gears, chains are made of metal, so it requires lesser space and gives a constant velocity ratio. As a belt, it is used for longer centre distances.

Advantages

1. Constant velocity due to no-slip, so it is a positive drive.
2. No effect on overload on the velocity ratio.
3. Oil or grease on the surface does not affect the velocity ratio.
4. Chains occupy less space as they made of metals.
5. Lesser loads are put on the shaft.
6. High transmission efficiency due to “No-slip”.
7. Through one chain only motion can be transmitted to several shafts.

Disadvantages

1. It is heavier as compared to the belt.
2. There are gradual stretching and an increase in the length of chains. From time to time some of its links have to be removed.
3. Lubrication of its parts is required.
4. Chains are costlier compared to belts.

5.18.1 Relation between Pitch and Pitch circle diameter for Chain

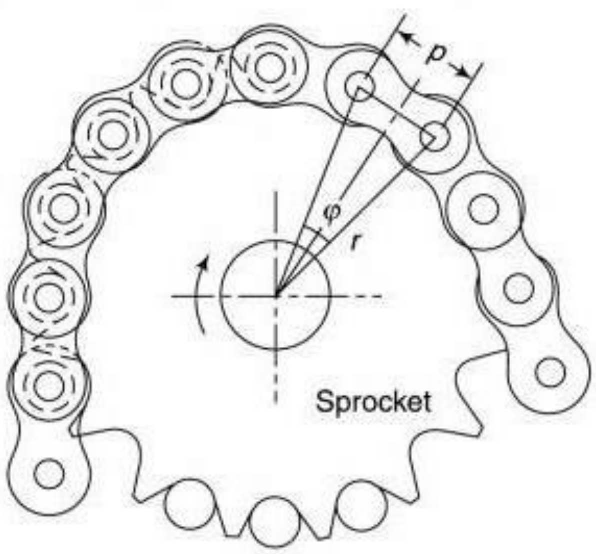


Fig.5.22 - Relation between Pitch and Pitch circle diameter for Chain

- The distance between roller centres of two adjacent links is known as pitch (*p*) of a chain.
- A circle through the roller centre of wrapped chain around a sprocket is called pitch circle and its diameter as pitch circle diameter (*d*).

Let,

T = No. of teeth on the sprocket

∅ = Angle subtended by the chord of a link at the centre of the sprocket

r= Radius of the pitch circle

Let,

$$\sin\frac{\emptyset}{2} = \frac{p/2}{r} \qquad \therefore\ \sin\frac{\emptyset}{2} = \frac{p}{2r}$$

$$\therefore\ p = 2r\sin\frac{\emptyset}{2} = 2r\sin\left(\frac{360°}{2T}\right) = 2r\sin\left(\frac{180°}{T}\right)$$

$$\therefore\ r = \frac{p}{2\sin\left(\frac{180°}{T}\right)} = \frac{p}{2}\ \mathrm{cosec}\ \left(\frac{180°}{T}\right)$$

or

$$\mathbf{d = p\ cosec}\left(\frac{\mathbf{180°}}{\mathbf{T}}\right)$$

5.18.2 Chain Length

- Length of a chain may be calculated the same way as for open belt drive.

R & r = Radius of the pitch circle of two sprockets having T & t teeth.

C = centre distance between sprocket = $k \cdot p$

p = Pitch of chain

Let,

$$L = \pi (R + r) + 2C + \frac{(R - r)^2}{C}$$

$$\left[\begin{array}{c} \text{Let} \\ R = \frac{p}{2} \operatorname{cosec} \left(\frac{180°}{T}\right) \\ r = \frac{p}{2} \operatorname{cosec} \left(\frac{180°}{t}\right) \end{array}\right]$$

$$L = \frac{pT + pt}{2} + 2(kp) + \frac{\left(\frac{p}{2} \operatorname{cosec} \left(\frac{180°}{T}\right) - \frac{p}{2} \operatorname{cosec} \left(\frac{180°}{t}\right)\right)^2}{kp}$$

$$\left[\begin{array}{c} \text{also circular pitch (p)} \\ p = \frac{\pi D}{T} = \frac{\pi 2R}{T} \\ \therefore \ \pi R = \frac{pT}{2} \\ \therefore \ \pi (R + r) = \frac{pT + pt}{2} \\ \text{Also } \ C = k \cdot p \end{array}\right]$$

$$= p \left[\frac{T + t}{2} + \frac{\left(\operatorname{cosec} \left(\frac{180°}{T}\right) - \operatorname{cosec} \left(\frac{180°}{t}\right)\right)^2}{4k} + 2k \right]$$

5.18.3 Classification of Chains

5.18.3.1 Hoisting Chain

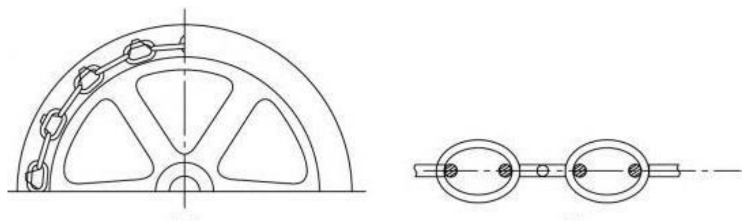


Fig.5.23 - Hoisting Chain

- Hoisting chain includes an oval link and stud link chains. An oval link is a common form of hoisting chain. It consists of an oval link and is also known as coil chain. Such chains are used for lower speed only.

5.18.3.2 Conveyor Chain

- Conveyor chain may be detachable or hook type or closed joint type. The sprocket teeth are so shaped that the chain should run onto and off the sprocket smoothly without interference.
- Such chains are used for low-speed agricultural machinery. The material of the link is usually malleable cast iron. The motion of the chain is not very smooth.

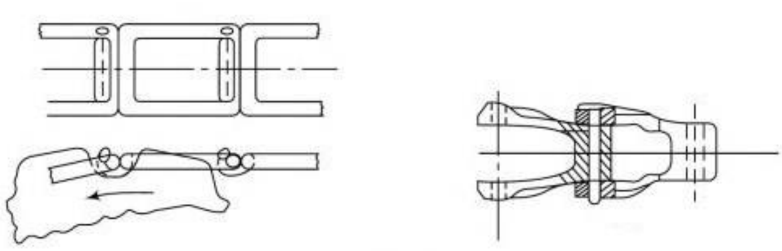


Fig.5.24 - Conveyor Chain

5.18.3.3 Power transmission chain

- These chains are made of steel in which the wearing parts are hardened. They are accurately machined and run on carefully designed sprockets.

a. Block Chain

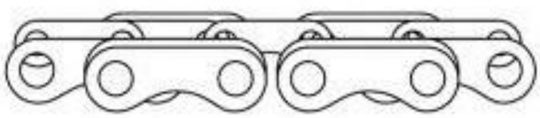


Fig.5.25 - Block Chain

- This type of chain is mainly used for transmission of power at low speeds. Sometimes they are also used as conveyor chains in place of malleable conveyor chains.

b. Roller Chain

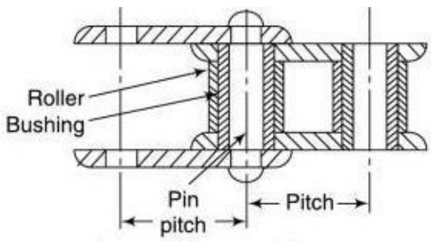


Fig.5.26 - Roller Chain

- A bushing is fixed to an inner link whereas the outer link has a pin fixed to it. There is only sliding motion between pin and bushing. The roller is made of hardened material and is free to turn on the bushing.

c. Silent Chain (Inverted tooth chain)

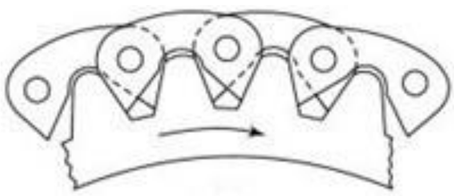


Fig.5.27 - Silent Chain

- Though roller chain can run quietly at fairly high speed. The silent chains are used where maximum quietness is desired. Silent chains do not have rollers. The links are so shaped as to engage directly with the sprocket teeth. The included angle is either 60° or 75°.

5.19 Problems

Ex. 5.1 **A shaft runs at 80 rpm and drives another shaft at 150 rpm through a belt drive. The diameter of the driving pulley is 600 mm. determine the diameter of the driven pulley in**

a) **Neglecting belt thickness.**
b) **Taking belt thickness as 5mm.**
c) **Assuming for case (ii) total slip of 4%.**
d) **Assuming for case (ii) a slip of 2% on each**

Solution: Given Data:

$$N_1 = 80\,\text{rpm}, D_1 = 600\,\text{mm}$$
$$N_2 = 150\,\text{rpm}, D_2 = ?$$

Case (i) Neglected belt thickness

$$\frac{N_2}{N_1} = \frac{D_1}{D_2} \qquad \therefore D_2 = \frac{N_1 D_1}{N_2} = \frac{80 \times 600}{150} = 320\,\text{mm}$$

Case (ii) Belt thickness = 5mm

$$\frac{N_2}{N_1} = \frac{D_1 + t}{D_2 + t} \qquad \therefore D_2 + t = \frac{N_1 (D_1 + t)}{N_2}$$

$$D_2 + 5 = \frac{80(605)}{150} \qquad \therefore D_2 = 317.67\,\text{mm}$$

Case (iii) Total Slip is 4%

$$\frac{N_2}{N_1} = \frac{D_1 + t}{D_2 + t}\left[1 - \frac{S}{100}\right]$$

$$\frac{150}{80} = \frac{(600+5)}{D_2 + t}\left[1 - \frac{4}{100}\right] \qquad \therefore D_2 = 304.76\,\text{mm}$$

Case (iv) Slip 2% on each pulley

$$\frac{N_2}{N_1} = \frac{D_1 + t}{D_2 + t}\left[1 - \frac{S}{100}\right]$$

$$\left\{\begin{aligned} S &= S_1 + S_2 - 0.01 S_1 S_2 \\ &= 2 + 2 - 0.01 \times 2 \times 2 \\ &= 3.96 \end{aligned}\right\}$$

$$\frac{150}{80} = \frac{(600+5)}{D_2 + t}\left[1 - \frac{3.96}{100}\right] \qquad \therefore D_2 = 304.88\,\text{mm}$$

Ex. 5.2 **Two parallel shafts connected by a crossed belt, are provided with pulleys 480 mm and 640 mm in diameters. The distance between the centre line of the shaft is 3 m. Find by how much the length of the belt should be changed if it is desired to alter the direction of**

Solution: Given Data:

R = 320 mm

r = 240 mm

X = C = 3 mtr

For cross belt drive,

$$\theta = (180 + 2\alpha)\frac{\pi}{180}$$

$$L_{cross} = (\pi + 2\alpha)(R + r) + 2C\,Cos\alpha$$

$$\left\{\begin{aligned} \sin\alpha &= \frac{R+r}{C} = \frac{320+240}{3} = \frac{0.320+0.240}{3} \\ \alpha &= 10.75° = 10°\,45' \\ &= 0.1878\ \text{rad} \end{aligned}\right\}$$

$$L_{cross} = (\pi + 2\times 0.1878)(0.32 + 0.24) + 2\times 3\cos(10.75)$$
$$= 7.865\text{mtr.}$$

For open belt drive,

$$L_{open} = \pi(r_1 + r_2) + 2\alpha(r_1 - r_2) + 2x\cos\alpha$$

$$L_{open} = \pi(R + r) + \frac{(R - r)^2}{C} + 2C$$
$$= \pi(0.32 + 0.24) + \frac{(0.32 - 0.24)^2}{3} + 2\times 3$$
$$= 7.761\ \text{mtr.}$$

$$\left\{\begin{aligned} \sin\alpha &= \frac{r_1 - r_2}{X} \\ &= \frac{0.320 - 0.240}{3} \\ \alpha &= 1°\,31' \end{aligned}\right\}$$

Angle is small approx. relation can be used.

Let the length of belt reduced by,

$$L_{cross} - L_o = 7.865 - 7.761$$
$$= 0.104\ \text{mtr.}$$

Ex. 5.3 **A belt runs over a pulley of 800 mm diameter at a speed of 180 rpm. The angle of the lap is 165° and the maximum tension in the belt is 2kN. Determine the power transmitted if the**

Solution: Given Data:

$$T_1 = 2000\ \text{N}$$
$$d = 0.8\ \text{mtr.}$$
$$N = 180\ \text{rpm}$$

$$\theta = 165°\times\frac{\pi}{180}\ \text{rad} = 2.88\ \text{rad}$$

$\mu = 0.3$

Let,

$$V = \frac{\pi dN}{60} = \frac{\pi \times 0.8 \times 180}{60} = 7.54 \ \ m/sec$$

$$2.3\log_{10}\frac{T_1}{T_2} = \mu\theta$$

$$2.3\log_{10}\left(\frac{T_1}{T_2}\right) = 0.3 \times 2.88$$

$$\therefore \frac{T_1}{T_2} = 2.37$$

$$\left\{\begin{array}{l} T_1 = 2000\,N \\ T_2 = \dfrac{T_1}{2.37} = \dfrac{2000}{2.37} = 843N \end{array}\right\}$$

$$\text{Power} = (T_1 - T_2)v$$

$$= (2000 - 843)7.54$$

$$= 8724 \ W$$

$$P = 8.724\,kW$$

Ex. 5.4 **A casting weights 6 kN and is freely suspended from a rope which makes 2.5 turns round a drum of 200 mm diameter. If the drum rotates at 40 rpm, determine the force required by a man to pull the rope from the other end of the rope. Also, find the power to raise the**

Solution: Given Data:

$W = T_1 = 6000\,N$
$\theta = 2.5 \times 2\pi = 15.70\,rad$
$d = 0.20$ mtr.
$N = 40\,rpm$
$\mu = 0.25$
$T_2 = ?$
$P = ?$

Let,

$$V = \frac{\pi dN}{60} = \frac{\pi \times 0.2 \times 40}{60} = 0.419 \ \ m/sec$$

$$2.3\log_{10}\frac{T_1}{T_2} = \mu\theta = 0.25 \times 15.70$$

$$\therefore \frac{T_1}{T_2} = 50.87 \qquad \therefore T_2 = 117.93\,N$$

$$\text{Power} = (T_1 - T_2)v$$

$$= (6000 - 117.93)0.419$$

$$= 2464 \ \text{Watt}$$

$$P = 2.46 \ kW$$

Ex. 5.5 **A belt drive transmits 8 kW of power from a shaft rotating at 240 rpm to another shaft rotating at 160 rpm. The belt is 8 mm thick. The diameter of smaller pulley is 600 mm and the two shafts are 5 m apart. The coefficient of friction is 0.25. If the maximum stress in the belt is limited to 3 N/mm². Find the width of the belt for (i) open belt drive**

Solution: Given Data:

$P = 8 \times 10^3$ watt
$N_1 = 240$ rpm, $d_1 = 0.600$ m(Smaller pulley)
$N_2 = 160$ rpm
$x = 5$ m
$\mu = 0.25$
$t = 8$ mm
$\sigma = 3\ N/mm^2$

Let,

$$N_1 d_1 = N_2 d_2$$

$$d_2 = \frac{N_1 d_1}{N_2} = \frac{240 \times 0.6}{160} = 0.900 \text{ mtr.(bigger pulley)}$$

(i) Open Belt drive

$$\text{Angle of contact } \theta = (180 - 2\alpha)\frac{\pi}{180} = \left[180 - 2(1.71)\right]\frac{\pi}{180}$$

$$\therefore \theta = 3.08 \text{ rad}$$

$$\left\{ \begin{array}{l} \sin\alpha = \dfrac{r_1 - r_2}{x} = \dfrac{0.450 - 0.300}{5} \\ \alpha = 1.71^\circ \end{array} \right\}$$

Let,

$$2.3 \log_{10}\frac{T_1}{T_2} = \mu\theta = 0.25 \times 3.08$$

$$\therefore \frac{T_1}{T_2} = 2.16$$

$$\text{Power} = (T_1 - T_2)v$$
$$8000 = (T_1 - T_2)7.54$$
$$\therefore T_1 - T_2 = 1061$$

$$\left\{ \begin{array}{l} \text{Velocity } v = \dfrac{\pi d_1 N_1}{60} \\ = \dfrac{\pi \times 0.6 \times 240}{60} \\ v = 7.54 \text{ m/sec} \end{array} \right\}$$

Let,

$$\text{Max tension } T_1 = \sigma b t$$
$$1975 = \sigma b t$$
$$1975 = 3 \times b \times 8$$
$$b = 82.29 \text{ mm}$$

$$\left\{ \begin{array}{l} \dfrac{T_1}{T_2} = 2.16 \ \& \ T_1 - T_2 = 1061 \\ T_1 = 914 \text{ N}, T_2 = 1975 \text{ N} \end{array} \right\}$$

(ii) Cross belt drive

$$\text{Angle of contact } \theta = (180 + 2\alpha)\frac{\pi}{180} = \left[180 + 2(8.62)\right]\frac{\pi}{180}$$

$$\therefore \theta = 3.443 \text{ rad}$$

$$\left\{ \sin\alpha = \frac{r_1 + r_2}{x} = \frac{0.450 + 0.300}{5} \quad \alpha = 8.62 \text{ rad} \right\}$$

$$2.3 \log_{10} \frac{T_1}{T_2} = \mu\theta = 0.25 \times 3.443$$

$$\therefore \frac{T_1}{T_2} = 2.36$$

Let,

$$\text{Max tension } T_1 = \sigma b\, t \qquad \left\{ T_1 - T_2 = 1061 \;\&\; \frac{T_1}{T_2} = 2.36 \quad \therefore T_2 = 780\text{ N}, T_1 = 1840\text{ N} \right\}$$

$$1840 = 3 \times b \times 8$$

$$b = 76.6 \text{ mm}$$

Ex. 5.6 A 100 mm wide and 10 mm thick belt transmits 5 kW of power between two parallel shafts. The distance between the shaft centres is 1.5 m and the diameter of the smaller pulley is 440 mm. The driving and the driven shafts rotate at 60 rpm and 150 rpm respectively. The coefficient of friction is 0.22. Find the stress in the belt if the two pulleys are connected by

Solution: Given Data:

$b = 100 \text{ mm}, t = 10 \text{ mm}$
$P = 5 \times 10^3 \text{ watt}$
$x = 1.5 \text{ m}$
$N_1 = 60 \text{ rpm}$
$N_2 = 150 \text{ rpm}, d_2 = 440 \text{ mm}$
$\mu = 0.22$
$\sigma_{open} = ?$
$\sigma_{cross} = ?$

Let,

$$N_1 d_1 = N_2 d_2$$

$$d_1 = \frac{N_2 d_2}{N_1} = \frac{150 \times 440}{60}$$

$$d_1 = 1100 \text{ mm}$$

(i) Open Belt drive,

Angle of contact $\theta = (180 - 2\alpha)\dfrac{\pi}{180}$

$$= \left[180 - 2(12.7)\right]\frac{\pi}{180}$$

$$\therefore \theta = 2.697 \text{ rad}$$

$$\left\{ \begin{array}{l} \sin\alpha = \dfrac{r_1 - r_2}{X} = \dfrac{550 - 220}{1.5} \\ = \dfrac{0.55 - 0.22}{1.5} \\ \alpha = 12.70 \end{array} \right\}$$

Let,

$$2.3 \log_{10} \frac{T_1}{T_2} = \mu\theta = 0.22 \times 2.697$$

$$\therefore \frac{T_1}{T_2} = 1.81 \quad \_\_\_\_\_\_(1)$$

$$\left\{ \begin{array}{l} v = r \cdot \omega = \left(r + \dfrac{t}{2}\right)\dfrac{2\pi N_2}{60} \\ \text{considering thickness of belt} \\ = 3535 \text{ mm/sec} \\ v = 3.535 \text{ m/sec} \end{array} \right\}$$

$$\text{Power} = (T_1 - T_2)v$$

$$5 \times 10^3 = (T_1 - T_2)3.535$$

$$\therefore T_1 - T_2 = 1414.5\text{N} \quad \_\_\_\_\_\_(2)$$

From equation (1) & (2)

$T_1 = 3160.8$ N

$T_2 = 1746.2$ N

Max tension in the belt $T = \sigma\, b\, t$

$$3160.8 = \sigma \times 100 \times 10$$

$$\sigma = 3.16 \text{ N/mm}^2$$

(ii) Cross Belt drive,

Angle of contact $\theta = (180 + 2\alpha)\dfrac{\pi}{180}$

$$\left\{ \begin{array}{l} \sin\alpha = \dfrac{r_1 + r_2}{X} = \dfrac{0.55 + 0.22}{1.5} \\ \alpha = 30.88 \end{array} \right\}$$

$$= \left[180 + 2(30.88)\right]\frac{\pi}{180}$$

$$\therefore \theta = 4.22$$

Let,

$$2.3 \log_{10} \frac{T_1}{T_2} = \mu\theta = 0.22 \times 4.22$$

$$\therefore \frac{T_1}{T_2} = 2.53$$

$\text{Power} = (T_1 - T_2)v$

$5000 = (T_1 - T_2)3.535$

$$\left\{\begin{array}{l} T_1 - T_2 = 1414.5 \;\&\; \dfrac{T_1}{T_2} = 2.53 \\ \therefore T_2 = 924.5\,N,\; T_1 = 2339\,N \end{array}\right\}$$

$\therefore T_1 - T_2 = 1414.5N$

$\text{Max tension in the belt } T = \sigma\, b\, t$

$2339 = \sigma \times 100 \times 10$

$\sigma = 2.339\ N/mm^2$

Ex. 5.7 **An open belt drive is required to transmit 10 kW of power from a motor running at 600 rpm. The diameter of the driving pulley is 250 mm. Speed of the driven pulley is 220 rpm. The belt is 12 mm thick and has a mass density of 0.001 g/mm². Safe stress in the belt is not to exceed 2.5 N/mm². Two shafts are 1.25 m apart. Take μ = 0.25. Determine the width of**

Solution: Given Data:

$P = 10 \times 10^3$ watt
$N = 600$ rpm
$x = 1.5\,m$
Speed of Driving pulley $N_1 = 600$ rpm
Diameter of driving pulley $d_1 = 250$ mm
Speed of Driven pulley $N_2 = 220$ rpm
$t = 12$ mm

$$\rho = 0.001\,g/mm^3 = \frac{1}{10^3 \times 10^3} kg \times \frac{(10^3)^3}{m^3} = 10^3\,kg/m^3$$

$\sigma = 2.5\,N/mm^2 = 2.5 \times 10^6\,N/m^2$
$x = 1.25\,m$
$\mu = 0.25$
$b = ?$

Let,

$N_1 d_1 = N_2 d_2$

$$d_2 = \frac{N_1 d_1}{N_2} = \frac{600 \times 250}{220}$$

$d_2 = 681.81mm$

Let,

$$v = \left(r + \frac{t}{2}\right) \omega = = \left(r + \frac{t}{2}\right) \frac{2\pi N}{60}$$

$$= \left(125 + \frac{12}{2}\right) \times \frac{2\pi \times 600}{60}$$

$= 8230\ mm/sec$

$v = 8.23\ m/sec$

Let,

$$P = (T_1 - T_2)v$$

$$10 \times 10^3 = (T_1 - T_2)\, 8.23$$

$$\therefore (T_1 - T_2) = 1215 \quad \_\_\_\_\_\_(1)$$

Let,

$$2.3 \log_{10} \frac{T_1}{T_2} = \mu\theta$$

$$= 0.25 \times 2.79$$

$$\therefore \frac{T_1}{T_2} = 2.01 \quad \_\_\_\_\_\_(2)$$

$$\left\{\begin{array}{ll} \text{open belt drive} & \\ \theta = (180 - 2\alpha)\dfrac{\pi}{180}\ \text{rad} & \text{where } \sin\alpha = \dfrac{r_1 - r_2}{X} \\ \theta = \left[180 - 2(9.94)\right]\dfrac{\pi}{180} & = \dfrac{340.90 - 125}{1.25} \\ \theta = 2.79 & \alpha = 9.94\ \text{rad} \end{array}\right\}$$

From equation (1) & (2)

$$T_2 = 1203\,N$$
$$T_1 = 2418\,N$$

Centrifugal Tension (T_c)

$$T_c = m v^2$$
$$= (12b)(8.23)^2$$
$$T_c = 812.8b\,N$$

$$\left\{\begin{array}{l} m = \text{Mass of belt / unit length} \\ \quad = \text{volume per unit length} \times \rho \\ \quad = \text{x sectional area} \times \text{length} \times \rho \\ \quad = \text{width} \times \text{thickness} \times \text{length} \times \rho \\ \quad = b \times 0.012 \times 1 \times 10^3 \\ \quad = 12\,b \end{array}\right\}$$

$$\text{Max tension in the belt } T = \sigma\, b\, t$$

$$= 2.5 \times 10^6 \times b \times 0.012$$

$$= 30{,}000\,b$$

Let,

$$T = T_1 + T_C$$

$$30{,}000b = 2418 + 812.8b$$
$$\therefore b = 0.0828\,m$$
$$b = 82.8\,mm$$

Ex. 5.8 **Two parallel shafts that are 3.5 m apart are connected by two pulleys of 1 m and 400 mm diameter. The larger pulley being the driver runs at 220 rpm. The belt weight 1.2 kg/meter length. The maximum tension in the belt is not to exceed 1.8 kN. The coefficient of friction is 0.28. Owing to slip on one of the pulleys, the velocity of the driven shaft is 520 rpm only. Determine:**

(i) Torque on each shaft
(ii) Power transmitted
(iii) Power lost in friction
(iv) The efficiency of the drive

Solution: Given Data:

$x = 3.5\,m$
$d_1 = 1\,m, \; N_1 = 220\,rpm$
$d_2 = 0.400\,m, \; N_2 = 520\,rpm$
$m = 1.2\,kg/m$
$t = 1.8 \times 10^3\,N$
$\mu = 0.28$

Let,

$$v = \frac{\pi d_1 N_1}{60}$$
$$= \frac{\pi \times 1 \times 220}{60}$$
$$v = 11.52\,m/sec > 10$$

$$T_c = m v^2$$
$$= 1.2\,(11.52)^2$$
$$T_c = 159\,N$$

For open belt drive,

$$\theta = (180 - 2\alpha)\frac{\pi}{180}\,rad$$

$$= [180 - 2\,(4.91)]\frac{\pi}{180}$$
$$\therefore \theta = 2.97\,rad$$

where $\sin\alpha = \dfrac{r_1 - r_2}{X}$

$$= \frac{0.5 - 0.2}{3.5}$$
$$\alpha = 4.91$$

Let,

$$2.3\log_{10}\frac{T_1}{T_2} = \mu\theta = 0.28 \times 2.97$$

$$\left\{\begin{array}{l} \text{Max Tension} \\ T = T_1 + T_C \\ 1.8 \times 10^3 = T_1 + 159 \\ \therefore T_1 = 1641\,N \\ \therefore T_2 = 714\,N \end{array}\right\}$$

$$\therefore \frac{T_1}{T_2} = 2.299$$

1.

$$\text{Torque on larger pulley} = (T_1 - T_2)\, r_1$$
$$= (1641 - 714) \times 0.5$$
$$T_L = 463.5 \ \text{N}\cdot\text{m}$$
$$\text{Torque on smaller pulley} = (T_1 - T_2)\, r_2$$
$$= (1641 - 714) \times 0.200$$
$$T_S = 185.4 \ \text{N}\cdot\text{m}$$

2.

$$\text{Power transmitted } P = (T_1 - T_2)\, v$$
$$= (1641 - 714) \times 11.52$$
$$= 10679 \text{ watt}$$
$$P = 10.679 \text{ kw}$$

3. Power lost in friction,

$$\text{Input Power } P_{in} = \frac{2\,\pi N_1 T_1}{60} = \frac{2\,\pi \times 220 \times 463.5}{60} = 10678 \text{ watt}$$

$$\text{Output Power } P_{out} = \frac{2\,\pi N_2 T_2}{60} = \frac{2\,\pi \times 920 \times 185.4}{60} = 10096 \text{ watt}$$

$$\text{Power Loss} = 10678 - 10096 = 582 \text{ watt}$$

4. The efficiency of the drive (η),

$$\eta = \frac{\text{Output Power}}{\text{Input Power}} = \frac{10096}{10678} = 0.945 = 94.5\,\%$$

Ex. 5.9 **A V- belt drive with the following data transmits power from an electric motor to**

Power transmitted	**= 100 kW**
Speed of the electric motor	**= 750 rpm**
Speed of compressor	**= 300 rpm**
Diameter of compressor pulley	**= 800 mm**
Centre distance between pulleys	**= 1.5 m**
Max speed of the belt	**= 30 m/sec**
Mass of density	**= 900 kg / m^3**
C/s area of belt	**= 350 mm2**
Allowable stress in the belt	**= 2.2 N / mm^2**
Groove angle of pulley	**= 38°= 2β**
Coefficient of friction	**= 0.28**

Determine the number of belts required and length of each

Solution: Let,

$$N_1\, d_1 = N_2\, d_2$$

$$\therefore d_1 = \frac{N_2\, d_2}{N_1} = \frac{300 \times 800}{750} = 320 \text{ mm}$$

Let centrifugal tension (T_C)

$$T_c = m v^2 = 0.315\,(30)^2 = 283.5\,N$$

$$\left\{\begin{aligned} m &= \text{Mass of belt / length} \\ &= \text{Area} \times \text{length} \times \text{density} \\ &= 350 \times 10^{-6} \times 1 \times 900 \\ &= 0.315\,kg/m \\ v &= 30\,m/sec \end{aligned}\right\}$$

Let Maximum Tension in belt,

$$T = \sigma\, b\, t$$

$$= 2.2 \times 350$$

$$T = 770\,N$$

Let,

$$T = T_1 + T_C$$

$$\therefore T_1 = T - T_C = 770 - 283.5$$

$$T_1 = 486.5\,N$$

$$\left\{\begin{aligned} &\text{open belt drive} \\ &\theta = (180 - 2\alpha)\frac{\pi}{180}\,rad && \sin\alpha = \frac{r_1 - r_2}{X} \\ &\theta = \left[180 - 2(9.20)\right]\frac{\pi}{180} && = \frac{400 - 160}{1.5} \\ &\theta = 2.82\,rad && \alpha = 9.20\,rad \end{aligned}\right\}$$

Let,

$$2.3 \log_{10}\frac{T_1}{T_2} = \mu\theta\ \operatorname{cosec}\beta$$

$$= \frac{0.28 \times 2.82}{\sin 19^\circ}$$

$$\therefore \frac{T_1}{T_2} = 11.3 \qquad\qquad \therefore T_2 = 43.1\,N$$

$$\therefore \text{Power} = (T_1 - T_2)v$$

$$= (486.5 - 43.1)\,30$$

$$= 13302\ \text{Watt}$$

$$P = 13.3\,kW$$

$$\text{No. of belt} = \frac{\text{Total power transmitted}}{\text{Power transmitted / belt}} = \frac{100}{13.3} = 7.51 \cong 8\ \text{belts}$$

Length of belt drive (using approximate relation),

$$L_o = \pi(r_1 + r_2) + 2x + \frac{(r_1 - r_2)^2}{x}$$

$$= \pi(0.4 + 0.16) + 2\,(1.5) + \frac{(0.4 - 0.16)^2}{3}$$

$$L_o = 4.79\,m$$

Ex. 5.10 Determine the maximum power transmitted by a V – belt drive having the included v – groove angle of 35°. The belt used is 18 mm deep with 18 mm maximum width and weight 300 g per metre length. The angle of the lap is 145° and the maximum permissible stress is 1.5 N/mm². μ = 0.2.

Solution: Given Data:

V Belt Drive

$2\beta = 35°$

$t = 18\,\text{mm}, w = 18\,\text{mm}$

$m = 0.3\,\text{kg}/\text{m}$

$\theta = 145° \times \frac{\pi}{180}\,\text{rad} = 2.53\,\text{rad}$

$\sigma = 1.5 \times 10^6\,\text{N}/\text{m}^2$

$\mu = 0.2$

$P_{max} = ?$

Let,

$$\text{Max tension in the belt } T = \sigma\, b\, t$$

$$= 1.5 \times 18 \times 18$$

$$= 486\,\text{N}$$

$$\text{Velocity for Max Power } v = \sqrt{\frac{T}{3m}}$$

$$= \sqrt{\frac{486}{3 \times 0.3}}$$

$$v = 23.23\,\text{m}/\text{sec}$$

Now,

$$2.3 \log_{10} \frac{T_1}{T_2} = \mu\theta \operatorname{cosec}\beta$$

$$= \frac{0.2 \times 2.53}{\sin(17.5°)}$$

$$\therefore \frac{T_1}{T_2} = 5.39$$

Let,

$$\text{Centrifugal Tension } T_C = m\,v^2 = 0.3\,(23.23)^2 = 162\,\text{N}$$

$$\left\{ \begin{array}{l} \text{OR} \\ \text{Max Tension condi.} \\ T_C = \frac{T}{3} = \frac{486}{3} = 162\,\text{N} \end{array} \right\}$$

Let,

$$T = T_1 + T_C$$

$$\therefore T_1 = T - T_C = 486 - 162 = 324\ \text{N}$$

$$\therefore \frac{T_1}{T_2} = 5.39 \qquad\qquad \therefore T_2 = 60.2\text{N}$$

$$\text{Max Power } P = (T_1 - T_2)v$$

$$= (324 - 60.2)23.23$$

$$= 6128.074\ \text{Watt}$$

$$P = 6.12\ \text{kW}$$

Ex. 5.11 **The grooves on the pulleys of a multiple rope drive have an angle of 50° and accommodate rope of 22 mm diameter having a mass of 0.8 kg/metre length for which a safe operating tension of 1200 N has been laid down. The two pulleys are of equal size. The drive is designed for max power conditions. Speeds of both pulleys are 180 rpm. Assuming μ = 0.25. Determine the diameter of pulleys and no. of ropes when power is transmitted 150 kW.**

Solution: Given Data:

Rope Drive

$2\beta = 50^\circ$

$d = \text{dia. of rope} = 22\ \text{mm}$

$m = 0.8\ \text{kg/m}$

$T = 1200\ \text{N}$

$d_1 = d_2 = \text{dia. of pulley} \therefore \theta = 180^\circ$

Max Power conditions

$N_1 = N_2 = 180\ \text{rpm}$

$\mu = 0.25$

$P = 150 \times 10^3\ \text{Watt}$

$d_1 = d_2 = ?$

No. of Ropes = ?

For Max Power Conditions....

$$T_1 = \frac{2}{3}T = \frac{2}{3} \times 1200 = 800\ \text{N}$$

$$T = T_1 + T_C$$

$$\therefore T_C = T - T_1 = 1200 - 800 = 400\ \text{N}$$

$$T_C = m v^2$$

$$400 = 0.8\, v^2$$

$$\therefore v = 22.36\ \text{m/sec}$$

$$\left\{ \begin{array}{l} \text{OR} \\ v = \sqrt{\dfrac{T}{3m}} = \sqrt{\dfrac{1200}{3 \times 0.8}} \\ = 22.36\ \text{m/sec} \end{array} \right\}$$

Let,

$$\text{Velocity } v = \frac{\pi D_1 N_1}{60}$$

$$23.36 = \frac{\pi D_1 \times 180}{60} \qquad\qquad \therefore D_1 = 2.37\text{m}$$

Let,

$$2.3\log_{10}\frac{T_1}{T_2}=\mu\theta\ \text{cosec}\beta$$

$$=0.25\times\left(180\times\frac{\pi}{180}\right)\frac{1}{\sin 25°}$$

$$\therefore\frac{T_1}{T_2}=6.42 \qquad\qquad \therefore T_2=124.61\,\text{N}$$

$$\text{Power } P=(T_1-T_2)v$$

$$=(800-124.61)22.36$$

$$P=15.10\,\text{kW / Rope}$$

$$\text{No. of Ropes}=\frac{\text{Total Power}}{\text{Power / Rope}}=\frac{150\,\text{kW}}{15.10\,\text{kW}}=9.93\cong 10\,\text{Ropes}$$

Ex. 5.12 **The following data relate to a rope**
Power = 20kW
transmitted = 480 mm
Diameter of = 80 rpm
Angle of lap on smaller pulley =
160° No. of Ropes = 8
Mass of Rope / m length = 48 G2
kg Limiting working tension =
132 G^2 Coefficient of friction =
0.3
Angle of groove = 44°

Solution: Given Data:

$P = 20\times 10^3$ Watt
d = dia. of pulley = 0.48 m
N = 80 rpm
$\theta = 160°\times\frac{\pi}{180}$ rad
n = No. of Ropes = 8
$m = 48\,G^2$ kg / m
$T = 132\,G^2$ kN
$\mu = 0.3$
$2\beta = 44°$
$T_0 = ?$
dia. of Rope = ?

Let,

$$\text{Power transmitted / Rope}=\frac{20\times 10^3}{8}=2500\,\text{Watt}$$

$$\text{Velocity of Rope}=\frac{\pi d N}{60}=\frac{\pi\times 0.48\times 80}{60}=2.01\,\text{m/sec}$$

Let,

$$\text{Power } P = (T_1 - T_2)v$$

$$2500 = (T_1 - T_2)2.01$$

$$T_1 - T_2 = 1244\,N$$

Let,

$$2.3\log_{10}\frac{T_1}{T_2} = \mu\theta\ \text{cosec}\beta$$

$$= 0.3 \times \left(160 \times \frac{\pi}{180}\right)\frac{1}{\sin 22^\circ}$$

$$\therefore \frac{T_1}{T_2} = 9.38 \qquad \left\{\begin{array}{l}\text{Solving equation}\\ T_1 = 1392.44\\ T_2 = 148.44\end{array}\right\}$$

1.

$$\text{Initial Tension } T_0 = \frac{T_1 + T_2}{2} = 770.44\,N$$

2. Diameter of Rope (D)

$$\text{Total Tension } T = T_1 + T_C$$
$$132 \times 10^3 \times G^2 = 1392.44 + 48\,G^2\,(2.01)^2$$
$$G = 0.1028$$

$$\text{Now Girth(Circumference) of Rope} = \pi D$$
$$0.1028 = \pi D \qquad \therefore D = 0.032\,m$$

Ex. 5.13 **2.5 kW power is transmitted by an open belt drive. The linear velocity of the belt is 2.5 m/sec. The angle of the lap on the smaller pulley is 165°. The coefficient of friction is 0.3.**

Determine the effect on power transmission in the following cases:

(i) Initial tension in the belt is increased by 8%.
(ii) Initial tension in the belt is decreased by 8%.
(iii) The angle of the lap is increased by 8% by the use of an idler pulley, for the same speed and the tension on the tight side.
(iv) Coefficient of friction is increased by 8% by suitable dressing to the friction surface of the belt.

Solution: Let,

$$\text{Power } P = (T_1 - T_2)v$$

$$2500 = (T_1 - T_2)2.5$$

$$\therefore T_1 - T_2 = 1000\,N \_\_\_\_\_\_(1)$$

Also,

$$2.3\log_{10}\frac{T_1}{T_2} = \mu\theta = 0.3\times 165^\circ \times \frac{\pi}{180}$$

$$\therefore \frac{T_1}{T_2} = 2.37 \quad \_\_\_\_\_(2)$$

By Solving,

$T_2 = 729.9\,\text{N}$
$T_1 = 1729.92\,\text{N}$

$$\text{Initial Tension } T_0 = \frac{T_1+T_2}{2} = 1229.9\,\text{N}$$

Initial Tension increased by 8%,

$$T_0 = 1229.9\times 1.08 = 1328.30\,\text{N}$$

$$\text{Or} \quad T_0 = \frac{T_1+T_2}{2}$$

$$1328.30 = \frac{T_1+T_2}{2} \quad \rightarrow T_1 + T_2 = 2656.60\,\text{N} \quad \_\_\_\_\_(3)$$

As μ and θ remain unchanged,

$$\therefore \frac{T_1}{T_2} = 2.37 \quad \_\_\_\_\_(4)$$

By solving equation (3) & (4),

$T_1 = 1868.3\,\text{N}$
$T_2 = 788.30\,\text{N}$

Let,

$$\text{Power } P = (T_1 - T_2)v$$

$$= (1868.3 - 788.30)2.5$$

$$P = 2.7\,\text{kW}$$

$$\therefore \text{increase in power} = \frac{2.7-2.5}{2.5} = 0.08 = 8\%$$

Initial Tension decreased by 8%,

$$T_0 = 1229.9\times(1-0.08) = 1131.50\,\text{N}$$

$$\text{Or} \quad T_0 = \frac{T_1+T_2}{2}$$

$$1131.50 = \frac{T_1+T_2}{2} \quad \rightarrow T_1 + T_2 = 2263\,\text{N} \quad \_\_\_\_\_(5)$$

As μ and θ remain unchanged,

$$\therefore \frac{T_1}{T_2} = 2.37 \quad \_\_\_\_\_(6)$$

By solving equation (5) & (6),

$T_1 = 1591.5N$
$T_2 = 671.5N$

Let,

$$\text{Power } P = (T_1 - T_2)v$$

$$= (1591.5 - 671.5)2.5$$
$$P = 2.3kW$$

$$\therefore \text{decrease in power} = \frac{2.5 - 2.3}{2.5} = 0.08 = 8\%$$

The angle of Lap (θ) is increased by 8% with the same speed and T_1,

Let,

$$2.3\log_{10}\frac{T_1}{T_2} = \mu\theta = 0.3 \times \left(165° \times \frac{\pi}{180} \times 1.08\right)$$

$$\therefore \frac{T_1}{T_2} = 2.54$$

T_1 is the same.

$T_1 = 1729.92\,N$
$T_2 = 681.07\,N$

$$\text{Power } P = (T_1 - T_2)v$$

$$= (1729.92 - 681.07)2.5$$
$$P = 2.62\,kW$$

$$\therefore \text{increase in power} = \frac{2.62 - 2.5}{2.5} = 0.048 = 4.8\%$$

Coefficient of friction is increased by 8%,

Let,

$$2.3\log_{10}\frac{T_1}{T_2} = \mu\theta$$
$$= (0.3 \times 1.08) \times 165 \times \frac{\pi}{180}$$

$$\therefore \frac{T_1}{T_2} = 2.54 \quad \_\_\_\_\_(7)$$

Let,

$$T_0 = \frac{T_1 + T_2}{2} = 1229.9$$

$$\therefore T_1 + T_2 = 2459.8\,N \quad \_\_\_\_\_(8)$$

By solving equation (7) & (8),

$$T_2 = 694.9\,N$$
$$T_1 = 1764.9\,N$$

Let,

$$\text{Power } P = (T_1 - T_2)v$$

$$= (1764.9 - 694.9)2.5$$
$$P = 2.67\,kW$$

$$\therefore \text{increase in power} = \frac{2.67 - 2.5}{2.5} = 0.07 = 7\%$$

Ex. 5.14 In a belt-drive, the mass of the belt is 1 kg/m length and its speed is 6 m/sec. The drive transmits 9.6 kW of power. Determine the initial tension in the belt and strength of the belt. The coefficient of friction is 0.25 and the angle of the lap is 220°.

Solution: Given Data:

$$m = 1\,kg/m$$
$$v = 6\,m/sec$$
$$P = 9.6 \times 10^3 \text{ Watt}$$
$$T_0 = ?$$
$$\text{Strength} = ?$$
$$\mu = 0.25$$
$$\theta = 220° \times \frac{\pi}{180} \text{ rad}$$

Let,

$$\text{Power } P = (T_1 - T_2)v$$

$$9600 = (T_1 - T_2)6 \qquad \therefore T_1 - T_2 = 1600\,N \quad \_\_\_\_\_\_(1)$$

Let,

$$2.3 \log_{10} \frac{T_1}{T_2} = \mu\theta$$
$$= 0.25 \times \left(220 \times \frac{\pi}{180}\right)$$

$$\therefore \frac{T_1}{T_2} = 2.31 \quad \_\_\_\_\_(2)$$

Solving equation (1) & (2),

$$T_1 = 2594\,N$$
$$T_2 = 994\,N$$

Let,

Centrifugal Tension $T_C = m\,v^2$

$= 1\,(6)^2 = 36\,N$

Initial Tension $T_0 = \frac{T_1 + T_2}{2} + T_C = \frac{2594 + 994}{2} + 36 = 1830\,N$

Strength of the belt = Total Tension on tight side

$= T_1 + T_C$

$= 2594 + 36 = 2630\,N$

Ex. 5.15 In an open bet drive, the diameters of the larger and smaller pulley are 1.2 m & 0.8 m respectively. The smaller pulley rotates at 320 rpm. The centre distance between the shafts is 4 m. When stationary, the initial tension in the belt is 2.8 kN. The mass of the belt is 1.8 kg/m and μ = 0.25. Determine the power transmitted.

Solution: Given Data:

$d_1 = 1.2\,m,\ N_1 = ?$
$d_2 = 0.8\,m,\ N_2 = 320\,rpm$
$x = 4\,m$
$T_0 = 2800\,N$
$m = 1.8\,kg/m$
$\mu = 0.25$
Power = ?

Velocity of Belt $= \frac{\pi d_2 N_2}{60} = \frac{\pi \times 0.8 \times 320}{60} = 13.40\,m/sec$

Centrifugal Tension $T_C = m\,v^2$

$= 1.8\,(13.4)^2 = 323.4\,N$

Initial Tension $T_0 = \frac{T_1 + T_2}{2} + T_C$

$2800 = \frac{T_1 + T_2}{2} + 323.4$

$\therefore T_1 + T_2 = 4953N$ ______(1)

For open belt drive,

Angle of contact $\theta = (180 - 2\alpha)\frac{\pi}{180}$ $\left\{\begin{array}{l}\sin\alpha = \frac{r_1 - r_2}{X} = \frac{0.6 - 0.4}{4} \\ \alpha = 2.86\end{array}\right\}$

$= \left[180 - 2(2.86)\right]\frac{\pi}{180}$

$\therefore \theta = 3.042\,rad$

Let,

$2.3\log_{10}\frac{T_1}{T_2} = \mu\theta$

$= (0.25 \times 3.042)$

$$\therefore \frac{T_1}{T_2} = 2.14 \quad \_\_\_\_\_(2)$$

By solving equation (1) & (2),

$T_1 = 3376\,N$
$T_2 = 1577\,N$

$$\text{Power } P = (T_1 - T_2)v$$
$$= (3376 - 1577)13.4 = 24106 \text{ Watt} = 24.10\,kW$$

Ex. 5.16 The initial tension in a belt drive is found to be 600 N and the ratio of friction tension is 1.8.

The mass of the belt is 0.8 kg/m length. Determine:

(i) The velocity of the belt for maximum power transmission
(ii) Tension on the tight side of the belt when it is started
(iii) Tension on the tight side of the belt when running at maximum speed

Solution: (i) Velocity of the belt (v)

Let max power condition for initial tension,

$$v = \sqrt{\frac{T_0}{3m}} = \sqrt{\frac{600}{3 \times 0.8}} = 15.81\,m/sec$$

(ii) Tension on tight side when belt is started v = 0, $T_C = 0$

$$T_1 = \frac{2kT_0}{k+1} = \frac{2(1.8)600}{1.8+1} = 771.4\,N$$

(iii) Tension on the tight side when the belt is running at max speed

$$T_1 = \frac{2k(T_0 - T_C)}{k+1}$$
$$= \frac{2(1.8)[600 - 199.7]}{1.8+1}$$
$$T_1 = 514.6\,N \qquad \left\{ \begin{aligned} T_C &= m\,v^2 \\ &= 0.8(15.8)^2 = 199.7\,N \end{aligned} \right\}$$

Or

$$T_0 = \frac{T_1 + T_2}{2} + T_C$$ … .... with this equation you can solve the problem.

$$T_0 = \frac{T_1 + T_2}{2} + T_C$$
$$600 = \frac{T_1 + \left(\frac{T_1}{1.8}\right)}{2} + 199.7$$
$$\therefore T_1 = 514.6\,N$$

Ex. 5.17 The driving pulley of an open belt drive is 800 mm diameter and rotates at 320 rpm while transmitting power to a driven pulley of 250 mm diameter. The Young's modulus of elasticity of the belt material is 110 N/mm². Determine the speed lost by the driven pulley due to creep if the stresses in the tight and slack sides of the belt are found to be 0.8 N/mm² and 0.32 N/mm² respectively.

Solution: Given Data:

Open Belt Drive

$D_1 = 0.8\,m,\ N_1 = 320\,rpm$

$D_2 = 0.250\ m$

$E = 110\ N/mm^2$

$\sigma_1 = 0.8\,N/mm^2$

$\sigma_2 = 0.32\,N/mm^2$

Let,

$$\frac{N_2}{N_1} = \frac{D_1}{D_2}\left[\frac{E + \sqrt{\sigma_2}}{E + \sqrt{\sigma_1}}\right] \text{ (Velocity Ratio with creep)}$$

$$N_2 = 320 \times \frac{800}{250}\left[\frac{110 + \sqrt{0.32}}{110 + \sqrt{0.8}}\right] = 1021\,rpm$$

Let velocity ratio without creep,

$$\frac{N_2}{N_1} = \frac{D_1}{D_2}$$

$$N_2 = 320 \times \frac{800}{250} = 1024\,rpm$$

Speed lost due to creep $= 1024 - 1021 = 3\,rpm$

Ex. 5.18 The centre to centre distance between two sprockets of a chain drive is 600 mm. The chain drive is used to reduce the speed from 180 rpm to 90 rpm on the driving sprocket has 18 teeth and a pitch circle diameter of 480 mm. Determine:

(i) No. of teeth on the driven sprocket

(ii) Pitch and the length of chain

Solution: Given Data:

$C = \text{Centre distance between sprocket} = 600\,mm$

$N_1 = 180\,rpm\ ,T_1 = 18\text{ teeth}$

$N_2 = 90\ rpm\ ,T_2 = ?$

1.

$$\frac{N_2}{N_1} = \frac{T_1}{T_2} \qquad \therefore T_2 = T_1 \times \frac{N_1}{N_2}$$

$$= 18 \times \frac{180}{90} = 36\text{ Teeth}$$

2. Pitch of the chain (p)

$$p = 2R\sin\left(\frac{180^\circ}{T}\right) = 2 \times 0.240 \times \sin\frac{180^\circ}{36} = 0.04183\ \text{m} = 41.83\ \text{mm}$$

3. Length of chain (L)

$$L = p\left[\frac{T+t}{2} + \frac{\left[\operatorname{cosec}\frac{180^\circ}{T} - \operatorname{cosec}\frac{180^\circ}{t}\right]^2}{4k} + 2k\right]$$

$$L = 0.04183\left[\frac{36+18}{2} + \frac{\left[\operatorname{cosec}\frac{180^\circ}{36} - \operatorname{cosec}\frac{180^\circ}{18}\right]^2}{4\times 14.343} + 2\times 14.343\right]$$

$$\left\{\begin{array}{l} C = \text{centre dist. between sprocket} = k\cdot p \\ \therefore k = \dfrac{C}{p} = \dfrac{0.600}{0.04183} = 14.343 \end{array}\right\}$$

Therefore, $L = 2.351\text{m}$

<u>References</u>:

1. Theory of Machines, Rattan S S, Tata McGraw-Hill
2. Theory of Machines, Khurmi R. S., Gupta J. K., S. Chand Publication

6.1 Introduction

When a body slides over another, the motion is resisted by a force called "***Force of Friction***".

- It is impossible to produce a perfectly smooth surface. A block is placed on a surface, the interlocking of projecting particles takes place due to surface roughness of the surfaces.
- The force of friction on a body is parallel to the sliding surface and acts in a direction opposite to that of the sliding body.

The case I (To be reduced force of friction)

- In the case of lathe machine, journal bearing, etc. where the power transmitted is reduced due to friction. It has to be increased by the use of lubricated surfaces.

Case II (To be increased force of friction)

- In the process, where the power is transmitted through friction attempts are made to increase it to transmit more power. E.g. Friction clutch & Belt drive. Even the toughness of a nut and bolt is dependent mainly on the force of friction.

6.1.1 Types of Friction

6.1.1.1 Dry Friction

It occurs when there is relative motion between two completely unlubricated surfaces. They are divided into two types:

(a) Solid Friction: When the two surfaces have a sliding motion relative to each other, it is called solid friction.

(b) Rolling Friction: Friction due to rolling of one surface over another is called rolling friction. E.g. ball and roller bearings.

6.1.1.2 Skin or Greasy Friction

When the two surfaces in contact have a minute thin layer of lubricant between them, it is known as skin or greasy friction. Higher spots on the surface break through the lubricant and come in contact with the other surface.

6.1.1.3 Film Friction

When the two surfaces in contact are completely separated by a lubricant, friction will occur due to the shearing of different layers of the lubricant. This is known as film friction or viscous friction.

6.1.2 Laws of Friction

Experiments have shown that the force of solid friction

- is directly proportional to the normal reaction between the two surfaces s opposes the motion between the surfaces.
- depends upon the materials of the two surfaces.
- is independent of the area of contact.
- is independent of the velocity of sliding.

6.1.3 Coefficient of Friction and Limiting angle of Friction

Let a body of weight *W* rests on a smooth and dry plane surface. Under the circumstances, the plane surface also exerts a reaction force R_n on the body which is normal to the plane surface. If the plane surface considered is horizontal, R would be equal and opposite to W.

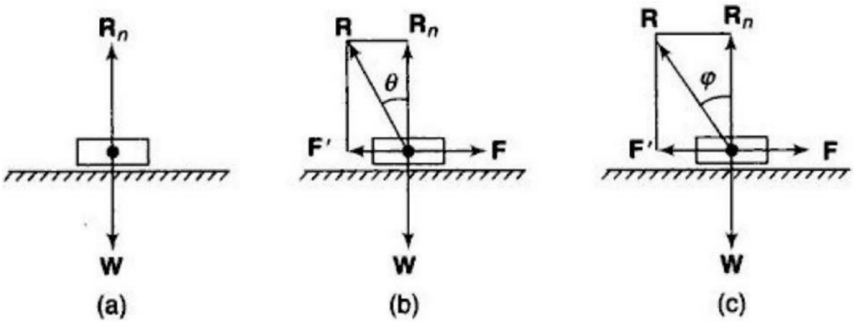


Fig.6.1 - Coefficient of Friction

Let a small horizontal force F be applied to the body to move it on the surface. So long the body is unable to move, the equilibrium of the body provides,

$$R_n = W \text{ and}$$

$$F = F'$$

where F'= Horizontal force resisting the motion of the body. As force F is increased, the relative force F' also increased accordingly.

F' and R_n, the friction and the normal reaction force can be combined into a single reaction force R with angle θ.

$$R\cos\theta = W \text{ and}$$

$$R\sin\theta = F$$

At a moment, when the force F would just move the body, the value of F' or R sinθ (equal to F) is called the static force of friction. Angle θ attains the value φ and the body is in equilibrium under the action of three forces as follow:

1. F, in the horizontal direction
2. W, in the vertically downward direction and
3. R, at an angle φ with the normal (inclined towards the force of friction).

Let,

$$F' \propto R_n$$
$$= \mu R_n$$

where μ = coefficient of friction

$$\therefore \mu = \frac{F'}{R_n}$$

In *Fig.6.1* (c)

$$\tan\varphi = \frac{F'}{R_n} = \frac{\mu R_n}{R_n}$$

Hence,

$$\tan\varphi = \mu$$

The angle φ is known as "limiting angle of friction" or simply "angle of friction".

6.1.4 Angle of Repose

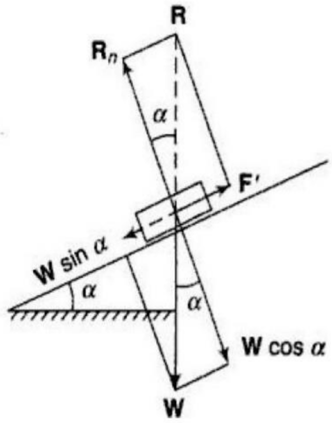


Fig.6.2 - Angle of Repose

When a body is at rest on an inclined plane making an angle α with the horizontal, the forces acting on the body are:

W, the weight of the body in a downward direction

R_n, normal reaction

F', the force resisting the motion of the body.

From equilibrium conditions,

$$W \sin\alpha = F' \text{ and } W \cos\alpha = R_n$$

If the angle of inclination of the plane is increased, the body will just slide down the plane of its own when

$$W \sin\alpha = F' = \mu R_n$$

$$W \sin\alpha = \mu (W \cos\alpha)$$

$$\tan\alpha = \mu = \tan\varphi$$

This maximum value of angle of inclination of the plane with the horizontal when the body starts sliding of its own is known as the angle of repose or limiting angle of friction.

6.1.5 Motion up the plane

Consider a body moving up an inclined plane under the action of force F shown in *Fig.6.3* (a). As the motion is up the plane, the friction force $F' = \mu R_n$ would act downward along the plane. The body is in equilibrium under the action of three forces: Weight of body (W), External force (F) and Resultant reaction (R).

Applying Lami's theorem, we get

$$\frac{F}{\sin\left[180-(\alpha+\varphi)\right]} = \frac{W}{\sin\left[180\{\theta-(\alpha+\varphi)\}\right]}$$

$$\frac{F}{\sin(\alpha+\varphi)} = \frac{W}{\sin\left[\theta-(\alpha+\varphi)\right]}$$

$$\therefore\ F = \frac{W\sin(\alpha+\varphi)}{\sin\left[\theta-(\alpha+\varphi)\right]}$$

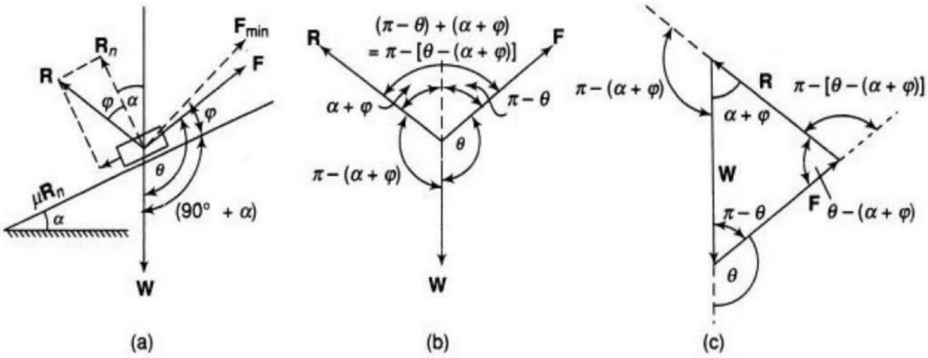


Fig.6.3 - Motion up the plane

Case 1: If the force applied is horizontal, θ=90°

$$F = \frac{W\sin(\alpha+\varphi)}{\sin\left[90° - (\alpha+\varphi)\right]} = \frac{W\sin(\alpha+\varphi)}{\cos(\alpha+\varphi)} = W\tan(\alpha+\varphi)$$

Case 2: If the force applied is parallel to the plane, θ=90° + α

$$\therefore F = \frac{W\sin(\alpha+\varphi)}{\sin\left[\theta - (\alpha+\varphi)\right]}$$

$$= \frac{W\sin(\alpha+\varphi)}{\sin\left[(90°+\alpha) - (\alpha+\varphi)\right]}$$

$$= \frac{W\sin(\alpha+\varphi)}{\sin(90° - \varphi)}$$

$$= \frac{W\sin(\alpha+\varphi)}{\cos\varphi}$$

$$= \frac{W(\sin\alpha\cos\varphi + \cos\alpha\sin\varphi)}{\cos\varphi}$$

$$= W(\sin\alpha + \cos\alpha\tan\varphi)$$

$$= W(\sin\alpha + \mu\cos\alpha) \qquad (\because \tan\varphi = \mu)$$

Case 3: F will be minimum if the denominator on the right-hand side is maximum,

$$F = \frac{W\sin(\alpha+\varphi)}{\sin\left[\theta - (\alpha+\varphi)\right]}$$

$$\sin\left[\theta - (\alpha+\varphi)\right] = 1$$

$$\theta - (\alpha+\varphi) = 90°$$

$$\theta - (90° + \alpha) = \varphi$$

The angle between F and the inclined plane should be equal to the angle of friction. In that case,

$$F_{min} = W\sin(\alpha+\varphi)$$

Efficiency: It is defined as the ratio of the forces required to move the body without consideration and with consideration of the force of friction. $\eta = F_0/F$

Let F_0 = force required to move the body up the plane without friction.

In the absence of friction, φ = 0.

$$F_0 = \frac{W\sin(\alpha+\varphi)}{\sin[\theta-(\alpha+\varphi)]} \qquad \{\text{No friction } \therefore \varphi = 0\}$$

$$= \frac{W\sin\alpha}{\sin(\theta-\alpha)}$$

$$\text{Let} \quad \eta = \frac{F_0}{F} \quad \begin{Bmatrix} F_0 = \text{Force without friction} \\ F = \text{Force with friction} \end{Bmatrix}$$

$$= \frac{\dfrac{W\sin\alpha}{\sin(\theta-\alpha)}}{\dfrac{W\sin(\alpha+\varphi)}{\sin[\theta-(\alpha+\varphi)]}}$$

$$= \frac{W\sin\alpha}{\sin(\theta-\alpha)} \frac{\sin[\theta-(\alpha+\varphi)]}{W\sin(\alpha+\varphi)}$$

$$= \frac{\sin\alpha}{\sin(\theta-\alpha)} \frac{\sin\theta\cos(\alpha+\varphi)-\cos\theta\sin(\alpha+\varphi)}{\sin\theta\cos\alpha-\cos\theta\sin\alpha}$$

$$= \frac{\sin\alpha}{\sin(\theta-\alpha)} \frac{\sin\theta\sin(\alpha+\varphi)\left[\dfrac{\cos(\alpha+\varphi)}{\sin(\alpha+\varphi)} - \dfrac{\cos\theta}{\sin\theta}\right]}{\sin\theta\sin\alpha\left[\dfrac{\cos\alpha}{\sin\alpha} - \dfrac{\cos\theta}{\sin\theta}\right]}$$

$$= \frac{\cot(\alpha+\varphi)-\cot\theta}{\cot\alpha-\cot\theta}$$

$$\text{If } \theta = 90°$$

$$\eta = \frac{\cot(\alpha+\varphi)-\cot 90°}{\cot\alpha-\cot 90°}$$

$$= \frac{\cot(\alpha+\varphi)-\cot 90°}{\cot\alpha-\cot 90°}$$

$$\eta = \frac{\tan\alpha}{\tan(\alpha+\varphi)}$$

6.1.6 Motion down the plane

When the body moves the down plane, the force of friction F' = μ R_n acts in the upward direction and reaction R (i.e. combination of R= and F') is inclined.

Assume that F acts downwards. Applying Lami's theorem **Error! Reference source not found.** (b),

$$\frac{F}{\sin[\pi-(\varphi-\alpha)]} = \frac{W}{\sin[\theta+(\varphi-\alpha)]} \qquad \textit{Eq. (6.1)}$$

$$\therefore \; F = \frac{W\sin(\varphi-\alpha)}{\sin[\theta+(\varphi-\alpha)]}$$

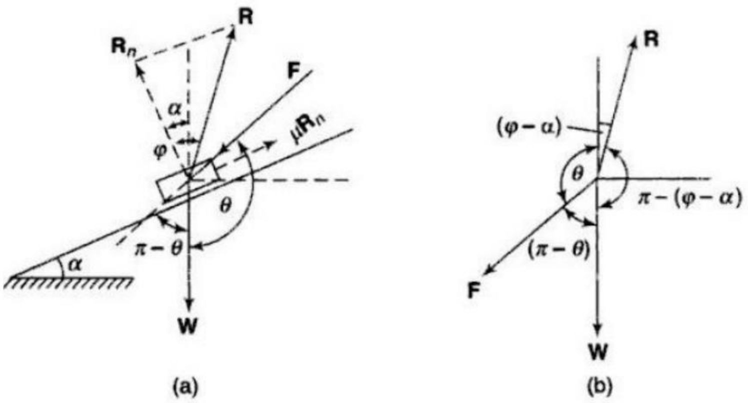


Fig.6.4 - Motion down the plane

In *Eq. (6.1)*, F is positive only for $\varphi > \alpha$ and when $\varphi = \alpha$, the force required to slide the body down is zero or body is on the point of moving down under its own weight W.

When $\varphi<\alpha$, i.e. the angle of friction < the angle of the inclined plane, F will be negative meaning that a force equal to F is to be applied in the opposite direction to resist the motion.

For a given value of α, F is minimum when the denominator of *Eq. (6.1)* is maximum.

$$F_{min} = W\sin(\varphi-\alpha)$$

$$F_0 = \frac{W\sin(-\alpha)}{\sin(\theta-\alpha)} \qquad \{\text{No friction } \therefore \varphi = 0\}$$

$$= \frac{-W\sin\alpha}{\sin(\theta-\alpha)}$$

Note: The force is negative indicating that in the absence of the force of friction.

Efficiency: It is defined as the ratio of the forces required to move the body with and without the consideration of force of friction.

$$\eta = \frac{F}{F_0} = \frac{W\sin(\varphi-\alpha)}{\sin[\theta+(\varphi-\alpha)]}\frac{\sin(\theta-\alpha)}{W\sin\alpha}$$

$$= \frac{\sin(\varphi-\alpha)}{\sin\theta\cos(\varphi-\alpha)+\cos\theta\sin(\varphi-\alpha)}\frac{\sin\theta\cos\alpha-\cos\theta\sin\alpha}{\sin\alpha}$$

$$= \frac{\sin(\varphi-\alpha)}{\sin\theta\sin(\varphi-\alpha)\left[\frac{\cos(\varphi-\alpha)}{\sin(\varphi-\alpha)}+\frac{\cos\theta}{\sin\theta}\right]}\frac{\sin\theta\sin\alpha\left[\frac{\cos\alpha}{\sin\alpha}-\frac{\cos\theta}{\sin\theta}\right]}{\sin\alpha}$$

$$\eta = \frac{\cot\alpha-\cot\theta}{\cot(\varphi-\alpha)+\cot\theta}$$

When $\theta = 90°$ or the force applied is horizontal.

$$\eta = \frac{\cot\alpha}{\cot(\varphi-\alpha)} = \frac{\tan(\varphi-\alpha)}{\tan\alpha}$$

6.2 Screw Threads

A screw thread is obtained when the hypotenuse of a right-angle triangle is wrapped around the circumference of a cylinder.

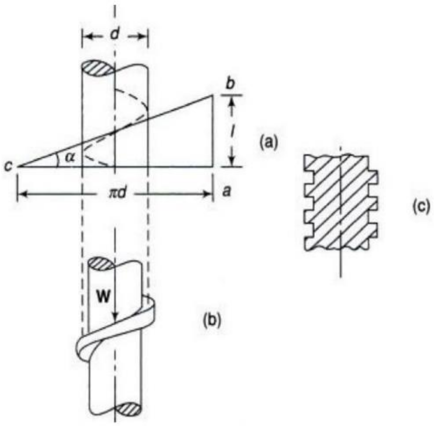


Fig.6.5 - Screw Threads

In *Fig.6.5*, *d* = diameter of cylinder and *l* = lead (or p = pitch for single start thread)

Length of base = circumference of the cylinder of screw thread = πd

$$\therefore \quad \tan\alpha = \frac{l}{\pi d} = \frac{p}{\pi d}$$

6.2.1 Square Threads

A square-threaded screw used as a jack to raise a load W. Faces of the square threads in the sectional view is normal to the axis of the spindle. Force F acting horizontally is the force of the screw thread required to slide the load W up the inclined plane.

$$F = \frac{W\sin(\alpha+\varphi)}{\sin\left[90^\circ-(\alpha+\varphi)\right]} \qquad (\text{Taken } \theta = 90^\circ)$$

$$= \frac{W\sin(\alpha+\varphi)}{\cos(\alpha+\varphi)}$$

$$= W\tan(\alpha+\varphi)$$

$$= W\frac{\tan\alpha+\tan\varphi}{1-\tan\alpha\tan\varphi}$$

$$= W\frac{\frac{l}{\pi d}+\mu}{1-\frac{l}{\pi d}\mu} \qquad \left\{\begin{array}{l}\tan\alpha = \dfrac{l}{\pi d}\\ \tan\varphi = \mu\end{array}\right\}$$

$$= W\frac{l+\mu\pi d}{\pi d-\mu l}$$

A bar is usually fixed to the screw head to use as a lever for the application of force.

Let, f = force applied at the end of the bar of length L

Then

$$fL = F\frac{d}{2} = Fr$$

$$\therefore\ f = \frac{Fr}{L} = \frac{Wr}{L}\tan(\alpha+\varphi)$$

If weight to be lowered,

$$F = \frac{W\sin(\varphi-\alpha)}{\sin[\theta-(\varphi-\alpha)]} \qquad (\text{Taken } \theta = 90°)$$

$$= \frac{W\sin(\varphi-\alpha)}{\sin[90°-(\varphi-\alpha)]}$$

$$= \frac{W\sin(\varphi-\alpha)}{\cos(\varphi-\alpha)}$$

$$= W\tan(\varphi-\alpha)$$

$$\therefore\ f = \frac{Fr}{L} = \frac{Wr}{L}\tan(\varphi-\alpha)$$

6.2.1.1 Screw Efficiency

$$\eta = \frac{\text{work done in lifting load / rev}}{\text{work done by applied load / rev}}$$

$$= \frac{W\times l}{F\times \pi d}$$

$$= \frac{W}{W\tan(\alpha+\varphi)}\tan\alpha$$

$$= \frac{\tan\alpha}{\tan(\alpha+\varphi)}$$

This is maximum, when $\frac{d\eta}{d\alpha} = 0$.

$$\therefore\ \frac{d}{d\eta} = \left[\frac{\tan\alpha}{\tan(\alpha+\varphi)}\right] = 0$$

$$\frac{\sec^2\alpha\tan(\alpha+\varphi) - \sec^2(\alpha+\varphi)\tan\alpha}{\tan^2(\alpha+\varphi)} = 0$$

$$\sec^2\alpha\tan(\alpha+\varphi) - \sec^2(\alpha+\varphi)\tan\alpha = 0$$

$$\frac{\tan(\alpha+\varphi)}{\sec^2(\alpha+\varphi)} = \frac{\tan\alpha}{\sec^2\alpha}$$

$$\frac{\sin(\alpha+\varphi)}{\cos(\alpha+\varphi)}\cos^2(\alpha+\varphi) = \frac{\sin\alpha}{\cos\alpha}\cos^2\alpha$$

$$\sin(\alpha+\varphi)\cos(\alpha+\varphi) = \sin\alpha\cos\alpha$$

$$2\sin(\alpha+\varphi)\cos(\alpha+\varphi) = 2\sin\alpha\cos\alpha$$

$$\sin 2(\alpha+\varphi) = \sin 2\alpha$$

This is possible if $(\alpha+\varphi)=\alpha$, i.e. $\varphi=0$ (or no friction).

$$\sin 2(\alpha+\varphi)=\sin(\pi-2\alpha)$$
$$2(\alpha+\varphi)=\pi-2\alpha$$
$$4\alpha+2\varphi=\pi$$
$$\alpha=\frac{\pi-2\varphi}{4}=45^\circ-\frac{\varphi}{2}$$

Thus, the necessary condition for the maximum efficiency is

$$\alpha=45^\circ-\frac{\varphi}{2}$$

$$\therefore\ \eta_{max}=\frac{\tan\alpha}{\tan(\alpha+\varphi)}$$

$$=\frac{\tan\left(45^\circ-\frac{\varphi}{2}\right)}{\tan\left(45^\circ-\frac{\varphi}{2}+\varphi\right)}$$

$$=\tan\left(45^\circ-\frac{\varphi}{2}\right)\frac{1}{\tan\left(45^\circ+\frac{\varphi}{2}\right)}$$

$$=\left(\frac{\tan 45^\circ-\tan\frac{\varphi}{2}}{1+\tan 45^\circ\tan\frac{\varphi}{2}}\right)\left(\frac{1-\tan 45^\circ\tan\frac{\varphi}{2}}{\tan 45^\circ+\tan\frac{\varphi}{2}}\right)$$

$$=\frac{\left(1-\tan\frac{\varphi}{2}\right)\left(1-\tan\frac{\varphi}{2}\right)}{\left(1+\tan\frac{\varphi}{2}\right)\left(1+\tan\frac{\varphi}{2}\right)}$$

$$=\frac{\left(1-\tan\frac{\varphi}{2}\right)^2}{\left(1+\tan\frac{\varphi}{2}\right)^2}$$

$$=\frac{\left(1-\frac{\sin\varphi/2}{\cos\varphi/2}\right)^2}{\left(1+\frac{\sin\varphi/2}{\cos\varphi/2}\right)^2}$$

$$=\frac{\left(\cos\frac{\varphi}{2}-\sin\frac{\varphi}{2}\right)^2}{\left(\cos\frac{\varphi}{2}+\sin\frac{\varphi}{2}\right)^2}$$

$$=\frac{\cos^2\frac{\varphi}{2}+\sin^2\frac{\varphi}{2}-2\cos\frac{\varphi}{2}\sin\frac{\varphi}{2}}{\cos^2\frac{\varphi}{2}+\sin^2\frac{\varphi}{2}+2\cos\frac{\varphi}{2}\sin\frac{\varphi}{2}}$$

$$=\frac{1-2\cos\frac{\varphi}{2}\sin\frac{\varphi}{2}}{1+2\cos\frac{\varphi}{2}\sin\frac{\varphi}{2}}$$

$$\therefore \eta_{\max} = \frac{1 - \sin\varphi}{1 + \sin\varphi}$$

$$\therefore \text{ Mechanical advantage} = \frac{\text{weight lifted}}{\text{force applied}} = \frac{W}{f} = \frac{W}{\frac{Wr}{L}\tan(\alpha+\varphi)} = \frac{L}{r}\cot(\alpha+\varphi)$$

$$\text{Also Velocity ratio} = \frac{\text{distance moved by force / rev}}{\text{distance moved by load / rev}}$$

$$VR = \frac{2\pi L}{l} = \frac{L}{\frac{l}{\pi d}\frac{d}{2}} = \frac{L}{r\tan\alpha}$$

6.2.1.2 *Overhauling and self-locking screw*

We know that the tangential force required to lower the load at the circumference of the screw is F = W tan(φ-α).

Torque, $T = F\,r = = W\tan(\varphi - \alpha)\,r$

If φ<α, T will be negative. That means the load will start moving down without applying torque. Such a condition is called ***"overhauling of the screw"***.

If φ> α, T will be positive. That means that some torque is required to lower the load and such a screw is called ***"self-locking screw"***.

6.2.1.3 *The efficiency of self-locking screw*

$$\begin{aligned}
\eta &= \frac{\tan\alpha}{\tan(\alpha+\varphi)} \\
&\le \frac{\tan\varphi}{\tan(\alpha+\varphi)} \qquad \text{(For self – locking screw } \varphi > \alpha) \\
&\le \frac{\tan\varphi}{\tan 2\varphi} \\
&\le \frac{\tan\varphi}{\left(\frac{2\tan\varphi}{1-\tan^2\varphi}\right)} \\
&\le \frac{1}{2} - \frac{\tan^2\varphi}{2}
\end{aligned}$$

Note: The efficiency of the self-locking screw is less than 50%. If it is greater than 50% then the screw is overhauling screw.

6.2.2 V Threads

In the case of V-threads, the faces are inclined to the axis of the spindle even if the helix angle is neglected. *Fig.6.6* shows a section through a V-thread in which 2β is the angle between the faces of the thread (α has not been considered Thus, not shown).

If R_n, is the normal reaction then clearly the axial component of R_n, must be equal to W, i.e.,

$$W = R_n \cos\beta$$

$$R_n = \frac{W}{\cos\beta}$$

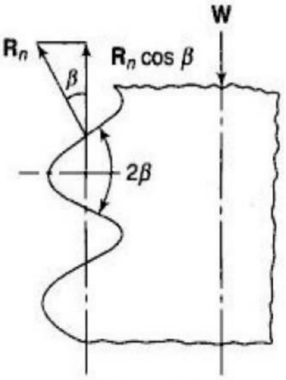


Fig.6.6 - Screw Threads

Friction force on the surface = μR_n

$$= \mu \frac{W}{\cos\beta}$$

$$= \frac{\mu}{\cos\beta} W \qquad \left\{\mu' = \frac{\mu}{\cos\beta}\right\}$$

$$= \mu' \cos\beta$$

6.3 Introduction about pivot & collar friction

It frequently happens that a rotating shaft is subjected to axial thrust. In order to bear this thrust exerted on the shaft, it is necessary to provide the thrust bearing at shaft end. This bearing not only takes up the axial thrust but in additional help to preserve the shaft in their correct axial position.

The relative motion between the contact surfaces of a thrust bearing and the shaft is resisted by friction between their surfaces.

Assumptions:

a) **Uniform pressure theory**
If the bearings are new and the fit between the shaft and bearing surface is perfect, it is safe to assume that the load is well distributed over the entire area of contact.

or

The intensity of pressure distributed over the area of contact is constant.

P = constant

b) **Uniform wear theory**
When the bearings become old, the bearing surface is worn out. The wear of surface at the different point of contact depends upon the product of the intensity of pressure and velocity of rubbing at point of contact (PV).
The velocity of rubbing is also different since it depends upon the radius from the axis of rotation for a given uniform angular speed of the shaft.
So, P V = constant
P r = constant

6.3.1 Types of pivot bearing

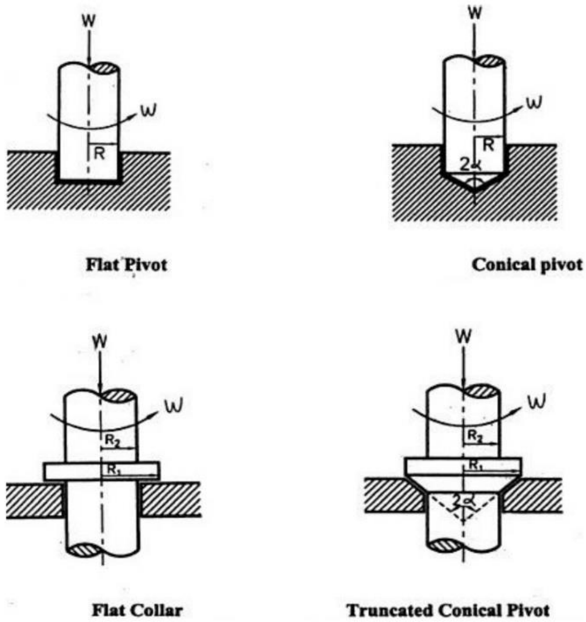


Fig.6.7 - Types of pivot bearing

1) Flat pivot bearing
2) Flat collar pivot bearing
3) Conical pivot bearing
4) Trapezoidal or truncated pivot bearing

6.3.1.1 Flat pivot bearing

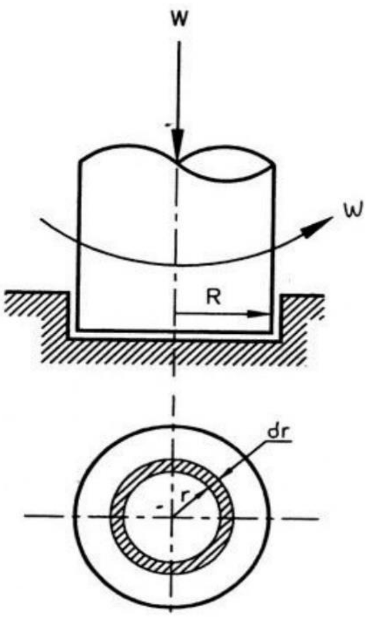


Fig.6.8 - Flat pivot bearing

Consider a shaft supported in a flat pivot bearing and rotating with uniform velocity ω.

Let, W = Axial load on bearing (N)

R = Radius of the shaft (radius of bearing) (mm)

P = Intensity of pressure per unit area (N/mm^2)

μ = Coefficient of friction between the shaft and bearing surface

a) Uniform pressure

Assume the intensity of pressure over the entire bearing surface is constant.

P = constant

$$P = \frac{\text{Load}}{\text{Bearing surface area}} = \frac{W}{\pi R^2}$$

Consider an elemental ring of radius r and thickness dr

So the area of the ring,

$$\delta A = 2\pi r\, dr$$

Load acting on the ring,

$$\delta W = \text{pressure} \times \text{area of ring}$$

$$\delta W = P\, 2\pi r dr$$

Frictional torque on the ring,

$$T_r = F_r\, r$$

$$= \left(P\, 2\pi r dr\right) \mu\, r$$

$$= 2\pi\mu\, P r^2\, dr$$

Total torque on the bearing,

$$T = \int_0^R T_r\, dr$$

$$= \int_0^R 2\pi\mu\, P r^2 dr$$

$$= 2\pi\mu\, P \left[\frac{r^3}{3}\right]_0^R$$

$$= 2\pi\mu\, P \frac{R^3}{3}$$

$$= 2\pi\mu \frac{W}{\pi R^2} \frac{R^3}{3} \qquad \left\{P = \frac{W}{\pi R^2}\right\}$$

$$T = \frac{2}{3}\mu W R$$

b) Uniform wear

Assuming that wear is uniform over the entire bearing surface, i.e. Pr = constant = C. Hence, P=C/r.

Load acting on the ring,

$$\delta W = P\,dA$$
$$= P(2\pi r dr)$$
$$= \frac{C}{r}(2\pi r dr)$$
$$\delta W = 2\pi C dr$$

The total load transmitted,

$$W = \int_0^R 2\pi C\,dr$$
$$= 2\pi C[r]_0^R$$
$$W = 2\pi CR \qquad \left\{\therefore\ C = \frac{W}{2\pi R}\right\}$$

Frictional torque at the ring,

$$T_r = 2\pi\mu\,Pr^2\,dr$$
$$= 2\pi\mu\frac{C}{r}r^2 dr$$
$$= 2\pi\mu Crdr$$

Total torque,

$$T = \int_0^R T_r dr$$
$$= \int_0^R 2\pi\mu Crdr$$
$$= 2\pi\mu C\left[\frac{r^2}{2}\right]_0^R$$
$$= 2\pi\mu\frac{W}{2\pi R}\frac{R^2}{2}$$
$$T = \frac{1}{2}\mu WR$$

6.3.1.2 Flat collar bearing

Consider a shaft supported in a flat collar bearing.

Let, W = Total axial load on bearing

R_1 = External radius of collar

R_2 = Internal radius of collar

P = Intensity of pressure per unit area

μ = Coefficient of friction between the shaft and bearing surface

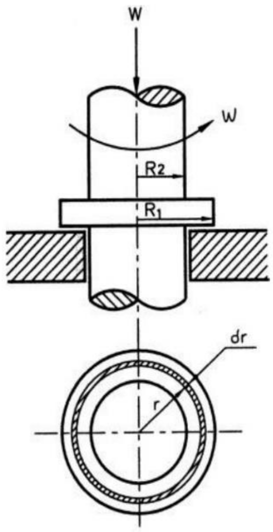


Fig.6.9 - Flat collar bearing

a) Uniform pressure

The pressure is uniformly distributed over the bearing surface area.

$$P = \frac{\text{Load}}{\text{Area}} = \frac{W}{\pi\left(R_1^2 - R_2^2\right)}$$

For flat pivot bearing torque on the ring,

$$T_r = 2\pi\mu\, Pr^2\, dr$$

Total torque,

$$T = \int_{R_2}^{R_1} T_r\, dr$$

$$= \int_{R_2}^{R_1} 2\pi\mu\, Pr^2\, dr$$

$$= 2\pi\mu P\left[\frac{r^3}{3}\right]_{R_2}^{R_1} \qquad \{P = \text{constant}\}$$

$$= 2\pi\mu P\frac{R_1^3 - R_2^3}{3}$$

$$= 2\pi\mu\frac{W}{\pi\left(R_1^2 - R_2^2\right)}\frac{R_1^3 - R_2^3}{3}$$

$$T = \frac{2}{3}\mu W\frac{R_1^3 - R_2^3}{R_1^2 - R_2^2}$$

b) Uniform wear

Assuming that wear is uniform over the entire bearing surface, i.e. Pr = constant = C. Hence P=C/r.

Load acting on the ring,

$$\delta W = P\,dA$$
$$= P(2\pi r dr)$$
$$= \frac{C}{r}(2\pi r dr)$$
$$\delta W = 2\pi C dr$$

The total load transmitted,

$$W = \int_{R_2}^{R_1} 2\pi C\,dr$$
$$= 2\pi C[r]_{R_2}^{R_1}$$
$$W = 2\pi C(R_1 - R_2) \qquad \left\{\therefore\ C = \frac{W}{2\pi(R_1 - R_2)}\right\}$$

We know that for flat pivot bearing, $T_r = 2\pi\mu C r dr$

Total torque,

$$T = \int_{R_2}^{R_1} T_r dr$$
$$= \int_{R_2}^{R_1} 2\pi\mu C r dr$$
$$= 2\pi\mu C\left[\frac{r^2}{2}\right]_{R_2}^{R_1}$$
$$= 2\pi\mu \frac{W}{2\pi(R_1 - R_2)} \frac{(R_1^2 - R_2^2)}{2}$$
$$T = \frac{1}{2}\mu W(R_1 - R_2)$$

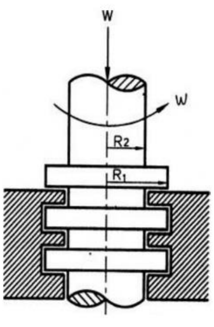


Fig.6.10 - Multiple flat collar bearing

For multiple flat collar bearing,

$$P = \frac{\text{Load}}{\text{Area}} = \frac{\text{Load}}{\text{no. of collar} \times \text{bearing surface area}} = \frac{W}{n\,\pi\left(R_1^2 - R_2^2\right)}$$

6.3.1.3 Conical pivot bearing

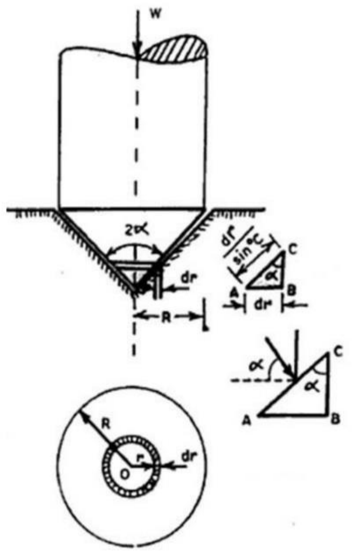


Fig.6.11 – Conical pivot bearing

Consider a shaft supported in a conical pivot bearing.

Let, W = Total axial load on bearing

R = Radius of the collar

P_n = Intensity of pressure normal to the cone

A = Semi angle of the cone

μ = Coefficient of friction between the shaft and bearing surface

Consider a small ring of radius r and thickness dr.

Length of the ring,

$$\delta l = dr\ \text{cosec}\alpha$$

Area of the ring,

$$\delta A = 2\pi r\ \delta l$$

$$= 2\pi r\ dr\,\text{cosec}\alpha$$

a) Uniform pressure

Normal load acting on the ring,

$$\delta W_n = \text{normal pressure} \times \text{area}$$
$$= P_n\ 2\pi r dr\ \text{cosec}\alpha$$

Vertical load acting on the ring,

$$\delta W = \text{vertical component of } P_n \times \text{area}$$
$$= P_n \sin\alpha \; 2\pi r dr \operatorname{cosec}\alpha$$
$$= P_n \; 2\pi r dr$$

The total load transmitted,

$$W = \int_0^R \delta W$$
$$= \int_0^R P_n 2\pi r \, dr$$
$$= 2\pi P_n \left[\frac{r^2}{2}\right]_0^R$$
$$= 2\pi P_n \frac{R^2}{2}$$
$$W = \pi R^2 P_n \qquad \left\{P_n = \frac{W}{\pi R^2} \text{ i.e. pressure is uniformly distributed}\right\}$$

Frictional resistance at radius r,

$$F_r = \mu \, \delta W_n$$
$$= \mu \, P_n \; 2\pi r dr \operatorname{cosec}\alpha \;\; 2\pi\mu \, P_n \operatorname{cosec}\alpha \; rdr$$

Frictional torque,

$$T_r = F_r r$$
$$= (2\pi\mu \, P_n \operatorname{cosec}\alpha \; rdr)\, r$$
$$= 2\pi\mu \, P_n \operatorname{cosec}\alpha \; r^2 dr$$

Total frictional torque,

$$T = \int_0^R 2\pi\mu \, P_n \operatorname{cosec}\alpha \; r^2 dr$$
$$= 2\pi\mu \, P_n \operatorname{cosec}\alpha \left[\frac{r^3}{3}\right]_0^R$$
$$= 2\pi\mu \frac{W}{\pi R^2} \operatorname{cosec}\alpha \frac{R^3}{3}$$
$$T = \frac{2}{3}\mu W R \operatorname{cosec}\alpha$$

b) Uniform wear

Assuming that wear is uniform over the entire bearing surface, i.e. P_n r = constant = C. So P_n = C/r.

Vertical load on the ring,

$$\delta W = P_n \; 2\pi r \delta r$$

$$= \frac{C}{r} 2\pi r dr$$
$$= 2\pi C dr$$

The total load transmitted,

$$W = \int_0^R 2\pi C dr$$
$$= 2\pi C [r]_0^R$$
$$W = 2\pi CR \qquad \left\{\therefore\ C = \frac{W}{2\pi R}\right\}$$

We know that frictional torque on the ring,

$$T_r = 2\pi\mu\ P_n \operatorname{cosec}\alpha\ r^2 dr$$
$$= 2\pi\mu \frac{C}{r} \operatorname{cosec}\alpha\ r^2 dr$$
$$= 2\pi\mu\ C \operatorname{cosec}\alpha\ r dr$$

Total frictional torque T,

$$T = \int_0^R T_r dr$$
$$= \int_0^R 2\pi\mu\ C \operatorname{cosec}\alpha\ r dr$$
$$= 2\pi\mu\ C \operatorname{cosec}\alpha \left[\frac{r^2}{2}\right]_0^R$$
$$= 2\pi\mu \frac{W}{2\pi R} \operatorname{cosec}\alpha \frac{R^2}{2}$$
$$T = \frac{1}{2}\mu WR \operatorname{cosec}\alpha$$

6.3.1.4 Trapezoidal or truncated conical pivot bearing

Let, W = Total axial load on bearing

R_1 = External radius of collar

R_2 = Internal radius of collar

P_n = Intensity of pressure normal to the cone

2α= Angle of cone

μ = Coefficient of friction between the shaft and bearing surface

a) Uniform pressure

The pressure is uniformly distributed over the bearing surface area.

$$P = \frac{\text{Load}}{\text{Area}} = \frac{W}{\pi\left(R_1^2 - R_2^2\right)}$$

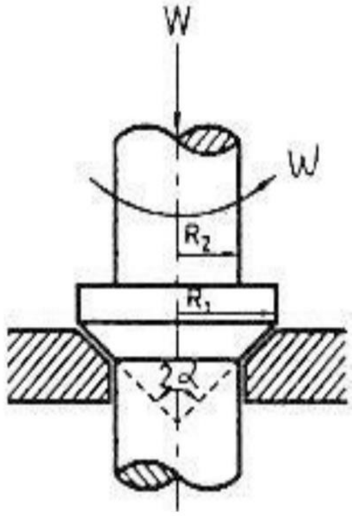


Fig.6.12 - Trapezoidal or truncated conical pivot bearing

Consider a small ring of radius r, thickness dr and *truncated* length dl. We have derived the equation for frictional conical pivot bearing torque,

$$T_r = 2\pi\mu\, P_n \operatorname{cosec}\alpha\, r^2 dr$$

Total frictional torque,

$$T = \int_{R_2}^{R_1} 2\pi\mu\, P_n \operatorname{cosec}\alpha\, r^2 dr$$

$$= 2\pi\mu\, P_n \operatorname{cosec}\alpha \left[\frac{r^3}{3}\right]_{R_2}^{R_1}$$

$$= 2\pi\mu \frac{W}{\pi\left(R_1^2 - R_2^2\right)} \operatorname{cosec}\alpha \frac{\left(R_1^3 - R_2^3\right)}{3}$$

$$T = \frac{2}{3}\mu W \frac{\left(R_1^3 - R_2^3\right)}{\left(R_1^2 - R_2^2\right)} \operatorname{cosec}\alpha$$

b) Uniform wear

Assuming that wear is uniform over the entire bearing surface, i.e. $P_n r$ = constant = C. So P_n=C/r.

Vertical load on the ring,

$$\delta W = P_n\, 2\pi r \delta r$$

$$= \frac{C}{r} 2\pi r dr$$

$$= 2\pi C dr$$

The total load transmitted,

$$W = \int_{R_2}^{R_1} 2\pi C dr$$

$$= 2\pi C [r]_{R_1}^{R_2}$$

$$W = 2\pi C(R_1 - R_2) \qquad \left\{\therefore\ C = \frac{W}{2\pi(R_1 - R_2)}\right\}$$

As we know that frictional torque on the ring,

$$T_r = 2\pi\mu\, C \operatorname{cosec}\alpha\ rdr$$

Total frictional torque T,

$$T = \int_{R_2}^{R_1} 2\pi\mu\, C \operatorname{cosec}\alpha\ rdr$$

$$= 2\pi\mu\, C \operatorname{cosec}\alpha \left[\frac{r^2}{2}\right]_{R_2}^{R_1}$$

$$= 2\pi\mu \frac{W}{2\pi(R_1 - R_2)} \operatorname{cosec}\alpha \frac{(R_1^2 - R_2^2)}{2}$$

$$T = \frac{1}{2}\mu W(R_1 + R_2)\operatorname{cosec}\alpha$$

6.4 Friction clutches

A clutch is a device used to transmit the rotary motion of one shaft to another when desired. The axes of the two shafts are coincident.

6.4.1 Disc clutch

A disc clutch consists of a clutch plate attached to a splined hub that is free to slide axially on splines cut on the driven shaft. The clutch plate is made of steel and has a ring of friction lining on each side. The engine shaft supports a rigidly fixed flywheel.

A spring-loaded pressure plate presses the clutch plate firmly against the flywheel when the clutch is engaged.

When disengaged, the springs press against a cover attached to the flywheel. Thus, both the flywheel and the pressure plate rotate with the input shaft. The movement of the clutch pedal is transferred to the pressure plate through a thrust bearing.

Fig.6.13 shows the pressure plate pulled back by the release levers and the friction linings on the clutch plate are no longer in contact with the pressure plate or the flywheel. The flywheel rotates without driving the clutch plate and thus, the driven shaft.

When the foot is taken off the clutch pedal, the pressure on the thrust bearing is released. As a result, the springs become free to move the pressure plate to bring it in contact with the clutch plate. The clutch plate slides on the splined hub and is tightly gripped between the pressure plate and the flywheel. The friction between the linings on the clutch plate, and the flywheel on one side and the pressure plate on the other, cause the clutch plate and hence, the driven shaft to rotate.

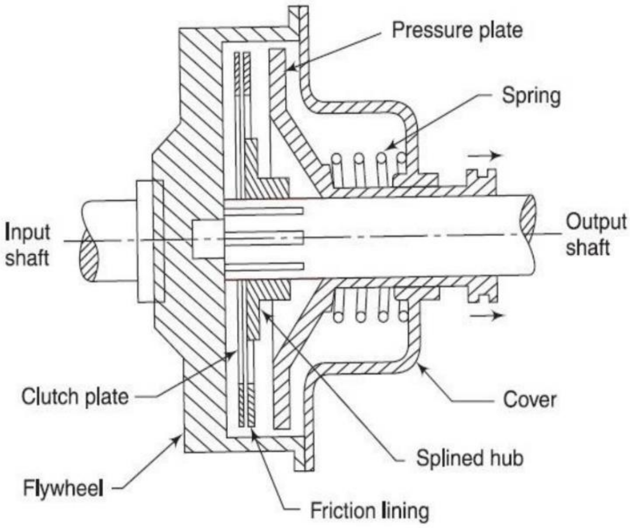


Fig.6.13 - Disc clutch

6.4.2 Multi-plate clutch

In a multi-plate clutch, the number of frictional linings and the metal plates is increased which increases the capacity of the clutch to transmit torque. *Fig.6.14* shows a simplified diagram of a multi-plate clutch.

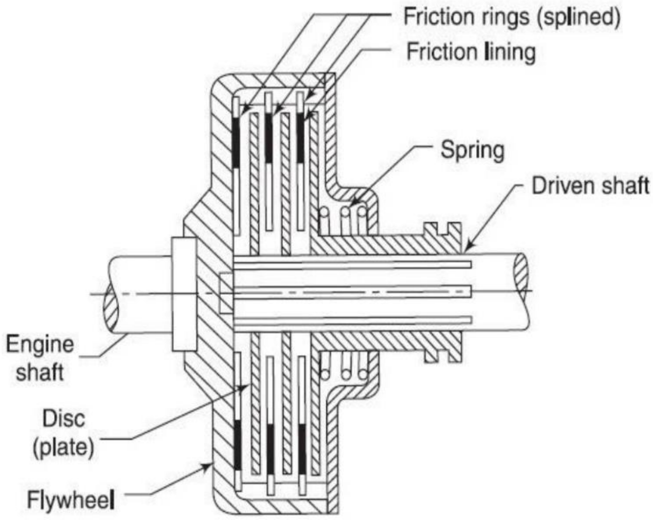


Fig.6.14 - Multi-plate clutch

The friction rings are splined on their outer circumference and engage with corresponding splines on the flywheel. They are free to slide axially. The friction material thus rotates with the flywheel and the engine shaft. The number of friction rings depends upon the torque to be transmitted.

The driven shaft also supports discs on the splines which rotate with the driven shaft and can slide axially. If the actuating force on the pedal is removed, a spring presses the discs into contact with the friction rings and the torque is transmitted between the engine shaft and the driven shaft.

If n is the total number of plates both on the driving and the driven members, the number of active surfaces will be n – 1.

6.4.3 Cone clutch

In a cone clutch (*Fig.6.15*), the contact surfaces are in the form of cones. In the engaged position, the friction surfaces of the two cones A and B are in complete contact due to spring pressure that keeps one cone pressed against the other all the time.

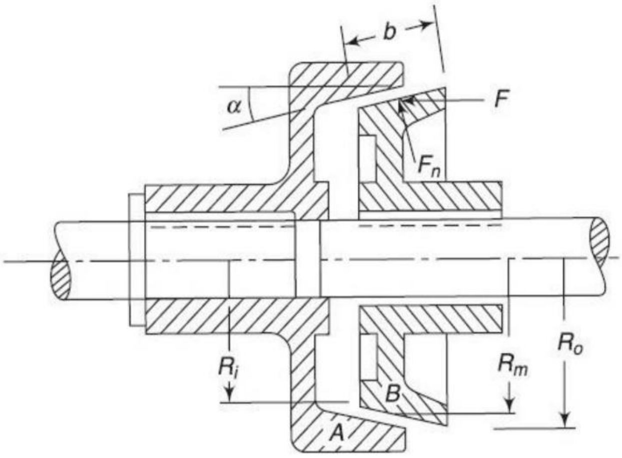


Fig.6.15 - Cone clutch

When the clutch is engaged, the torque is transmitted from the driving shaft to the driven shaft through the flywheel and the friction cones. For disengaging the clutch, the cone B is pulled back through a lever system against the force of the spring. The advantage of a cone clutch is that the normal force on the contact surfaces is increased.

If F= Axial force

F_n= Normal force

α= Semi angle of the cone

Applying uniform wear theory,

$$F_n = \frac{F}{\sin\alpha} \qquad \textit{Eq. (6.2)}$$

Axial force, $$F = \int_{R_i}^{Ro} \text{Axial force on elemetnt area}$$

$$= \int_{R_i}^{Ro} \text{Pressure on element} \times \text{Area}$$

$$= \int_{R_i}^{Ro} P\,2\pi r\,dr$$

$$= \int_{R_i}^{Ro} \frac{C}{r} 2\pi r\,dr \qquad \left\{\begin{aligned} Pr &= C \\ &= \text{Constant} \end{aligned}\right\}$$

$$= \int_{R_i}^{Ro} 2\pi C\,dr$$

$$= 2\pi C\left[r\right]_{R_i}^{R_o}$$

$$= 2\pi\,Pr\left(R_o - R_i\right)$$

The intensity of pressure P at radius r of the collar,

$$P = \frac{F}{2\pi r\left(R_o - R_i\right)}$$

Putting in *Eq. (6.9)*,

$$F_n = \frac{F}{\sin\alpha}$$

$$= \frac{2\pi Pr\left(R_o - R_i\right)}{\sin\alpha} \qquad \left\{\begin{aligned} &\sin\alpha = \frac{\left(R_o - R_i\right)}{b} \\ &b = \text{width of cone face}\end{aligned}\right\}$$

$$F_n = 2\pi Prb$$

Let,

$$T = \frac{1}{2}\mu F\frac{\left(R_o + R_i\right)}{\sin\alpha} \qquad \left\{T = \frac{1}{2}\mu W\left(R_o + R_i\right)\operatorname{cosec}\alpha\right\}$$

$$= \mu\frac{F_n \sin\alpha}{\sin\alpha}\frac{\left(R_o + R_i\right)}{2} \qquad \left\{\begin{aligned} \text{Here } R_m &= \text{Mean radius} \\ &= \frac{\left(R_o + R_i\right)}{2}\end{aligned}\right\}$$

$$T = \mu F_n R_m$$

However, cone clutches have become obsolete as small cone angles and exposure to dust and dirt tend to bind the two cones and it becomes difficult to disengage them.

6.4.4 Centrifugal clutch

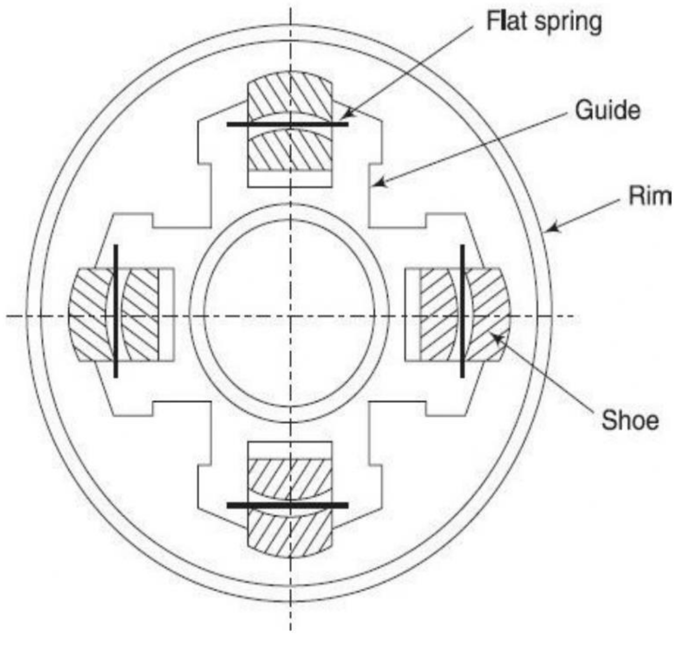


Fig.6.16 - Centrifugal clutch

Centrifugal clutches are being increasingly used in automobiles and machines. A centrifugal clutch has a driving member consisting of four sliding blocks (*Fig.6.16*).

These blocks are kept in position by means of flat springs provided for the purpose. As the speed of the shaft increases, the centrifugal force on the shoe increases.

When the centrifugal force exceeds the resisting force of the springs, the shoes move forward and press against the inside of the rim and thus, torque is transmitted to the rim.

In this way, the clutch is engaged only when the motor gains sufficient speed to take up the load in an effective manner. The outer surfaces of the shoes are lined with some friction material.

Let, m = mass of each shoe

R = inner radius of the pulley rim

r = distance of the center of mass of each shoe from the shaft axis

n = number of shoes

ω= normal speed of the shaft in rad/s

ω' = speed at which the shoe moves forward

μ= coefficient of friction between the shoe and the rim.

The centrifugal force exerted by each shoe at the time of engagement with the rim = $mr\omega'^2$ = resisting force of the spring.

The centrifugal force exerted by each shoe at normal speed = $mr\omega^2$

Net normal force exerted by each shoe on the rim $= mr\omega^2 - mr\omega'^2$

$= mr\left(\omega^2 - \omega'^2\right)$

The frictional force acting tangentially on each shoe $= \mu\, mr\left(\omega^2 - \omega'^2\right)$

Frictional torque acting on each shoe $= \mu\, mr\left(\omega^2 - \omega'^2\right)R$

Total frictional torque acting $= \mu\, mr\left(\omega^2 - \omega'^2\right)R\, n$

If p is the maximum pressure intensity exerted on the shoe, then

$$mr\left(\omega^2 - \omega'^2\right) = p\, l b \qquad \begin{Bmatrix} l = \text{Length of shoe} \\ b = \text{Width of shoe} \end{Bmatrix}$$

Note: Usually, the clearance between the shoe and the rim is very small and is neglected.

6.5 Brakes

A brake is defined as a mechanical device that is used to absorb the energy possessed by a moving system or mechanism by the friction. The primary purpose of the brake is to slow down or completely stop the motion of moving systems such as rotating drums, machines or vehicles. It is also used to hold the parts of the system in position at rest.

An automobile brake is used either to reduce the speed of the car or bring it to rest. It also used to keep the car stationary on the downhill road. The energy absorbed by the brake can be either kinetic or potential or both. In automobile applications, the brake absorbs the kinetic energy of moving vehicles.

In hoists and elevators, the brake absorbs the potential energy released by the object during the braking period. The energy absorbs by the brake converts into heat energy and dissipated to the surrounding. Heat dissipation is a serious problem in brake application.

6.6 Classification of brakes

Brakes are classified into the following three groups:

(a) **Mechanical brakes** are operated by mechanical means such as levers, springs, and pedals. Depending upon the shape of the friction material, the mechanical brakes are classified as **Block brakes, Band Brakes, Block and Band Brake and internal or external shoe brakes.**

(b) **Hydraulic brakes and pneumatic brakes** which are operated by fluid pressure such as oil pressure or air pressure.

(c) **Electrical brakes** are operated by magnetic force and which include magnetic particle brakes, hysteresis brakes, and eddy current brakes.

Brake capacity depends upon the following factor.

- The unit pressure between the braking surface.
- The contacting area of the braking surface.
- The radius of the brake drum
- The coefficient of friction
- The ability of the brakes to dissipate heat that is equivalent to the energy being absorbed.

6.6.1 Energy Equations

The first step in the design of a mechanical brake is to determine the braking-torque capacity for the given application. The braking-torque depends upon the amount of energy absorbed by the brake. When a mechanical system of mass m moving with a velocity v_1 is slowed down to the velocity $v_2$during the period of braking, the kinetic energy absorbed by the brake is given by

$$KE = \frac{1}{2}m\left(v_1^2 - v_2^2\right)$$

Where KE = kinetic energy absorbed by the brake (J)

m = mass of the system (kg)

v_1 and v_2 = initial and final velocities of the system (m/s)

Similarly, the kinetic energy of the rotating body is given by

$$KE = \frac{1}{2}I\left(\omega_1^2 - \omega_2^2\right)$$

$$KE = \frac{1}{2}mk^2\left(\omega_1^2 - \omega_2^2\right)$$

where I =mass moment of inertia of the rotating body (kg-m^2)

k = radius of gyration of the body (m)

ω_1, ω_2 = initial and final angular velocities of the body (rad/s)

In certain applications like hoists, the brake absorbs the potential energy released by the moving weight during the braking period. When a body of mass *m* falls through a distance h, the potential energy absorbed by the brake during the braking period is given by

$$PE = mgh$$

where, g =gravitational constant (9.81 m/s^2)

Depending upon the type of application, the total energy absorbed by the brake is determined by adding the respective quantities of energy discussed above. This energy is equated to the work done by the brake.

Therefore,

$$E = T_b \theta$$

where E = total energy absorbed by the brake (J)

T_b = braking torque (N-m)

θ = angle through which the brake drum rotates during the braking period (rad)

6.6.2 Block or Shoe Brake

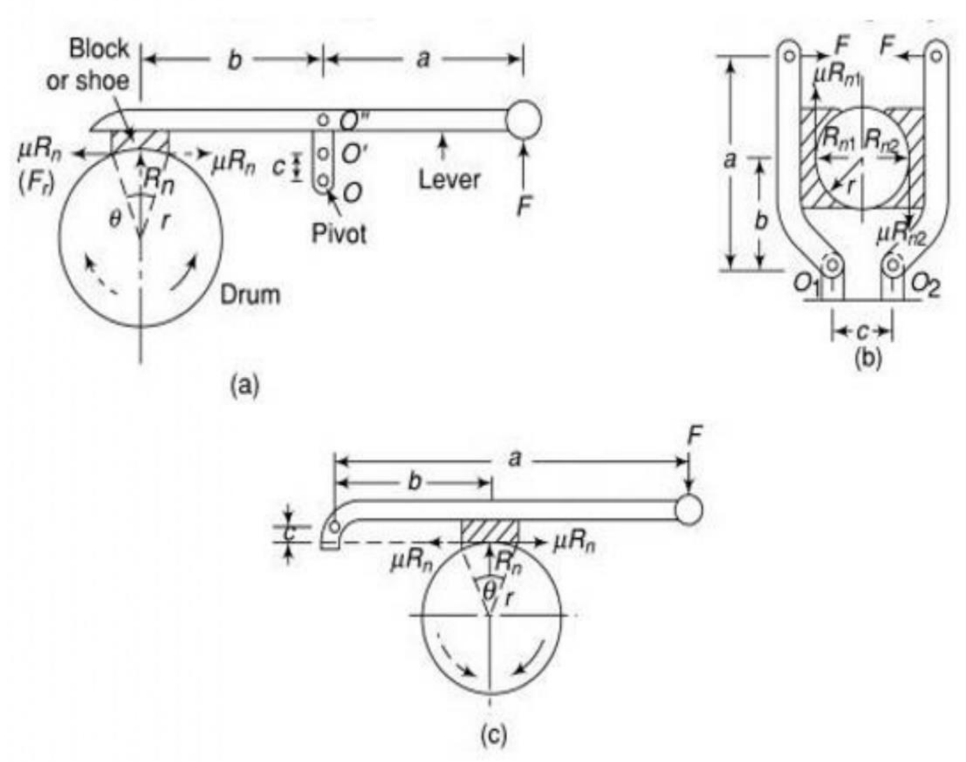


Fig.6.17 - Block or Shoe Brake

A block or shoe brake consists of a block or shoe which is pressed against a rotating drum. The force on the drum is increased by using a lever [*Fig.6.17* (a)].

If only one block is used for the purpose, aside from thrust on the bearing of the shaft supporting the drum will act. This can be prevented by using two blocks on the two sides of the drum [*Fig.6.17* (b)]. This also doubles the braking torque.

A material softer than that of the drum or the rim of the wheel is used to make the blocks so that these can be replaced easily on wearing. Wood and rubber are used for light and slow vehicles and cast steel for heavy and fast ones.

Let,

r = Radius of drum

μ = Co – efficient of friction

F_r = Radial force applied on the drum

R_n = Normal reaction on the block

F = Force applied at the lever end

F_f = Frictional force $= \mu R_n$

Assuming that normal reaction R_n and frictional force F_f act at the midpoint of the block.

$$\text{Breaking Torque} = \text{Frictional force} \times \text{Radius}$$
$$= \mu R_n \times r$$

The direction of the frictional force on the drum is to be opposite to that of its rotation while on the block it is in the same direction. Taking moments about the pivot O [Fig. 2.10(a)],

$$F \times a - R_n \times b + \mu R_n \times c = 0$$
$$R_n = \frac{F \cdot a}{b - \mu c}$$
$$F = R_n \cdot \frac{b - \mu c}{a}$$

When $b = \mu c$, F = 0 which implies that the force needed to apply the brake is virtually zero, or that once contact is made between the block and the drum, the brake is applied itself. Such a brake is known as a self – locking brake.

As the moment of the force F_f about O is in the same direction as that of the applied force F, F_f aids in applying the brake. Such a brake is known as a self – energized brake.

If the rotation of the drum is reversed, i.e., it is made clockwise,

$$F = R_n\left[\left(b + \mu c\right)/a\right]$$

which shows that the required force F will be far greater than what it would be when the drum rotates counterclockwise.

If the pivot lies on the line of action of E_f, i.e., at O', c = 0 and $F = R_n \frac{a}{b}$,

which is valid for clockwise as well as for counterclockwise rotation.

If c is made negative, i.e., if the pivot is at O",

$$F = R_n\left(\frac{b + \mu c}{a}\right) \quad \text{for counter – clockwise rotation}$$

and

$$F = R_n\left(\frac{b - \mu c}{a}\right) \quad \text{for clockwise rotation}$$

In case the pivot is provided on the same side of the applied force and the block as shown in *Fig.6.17* (c), the equilibrium condition can be considered accordingly.

In the above treatment, it is assumed that the normal reaction and the frictional force act at the midpoint of the block. However, this is true only for small angles of contact. When the angles of contact are more than 40°, the normal pressure is less at the ends than at the center.

In that case, μ has to be replaced by an equivalent coefficient of friction μ' given by

$$\mu' = \mu\left(\frac{4\sin\left(\frac{\theta}{2}\right)}{\theta + \sin\theta}\right)$$

6.6.3 Band Brake

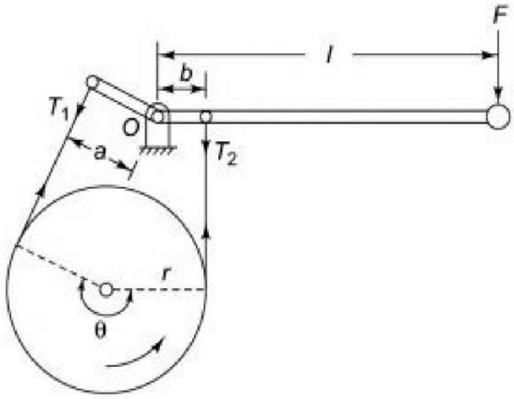


Fig.6.18 - Band Brake

It consists of a rope, belt or flexible steel band (lined with friction material) which is pressed against the external surface of a cylindrical drum when the brake is applied. The force is applied at the free end of a lever (*Fig.6.18*).

Brake torque on the drum = $(T_1 - T_2) r$

where r is the effective radius of the drum.

The ratio of the tight and slack side tensions is given by $T_1 / T_2 = e^{\mu\theta}$ on the assumption that the band is on the point of slipping on the drum.

The effectiveness of the force F depends upon the

- the direction of rotation of the drum
- the ratio of length a and b
- the direction of the applied force F.

To apply the brake to the rotating drum, the band has to be tightened on the drum. This is possible if

1. F is applied in the downward direction when a>b
2. F is applied in the upward direction when a<b

If the force applied is not as above, the band is further loosened on the drum which means no braking effect is possible.

6.6.3.1 a > b, F Downwards

(a) Rotation Counter – Clockwise

For counterclockwise rotation of the drum, the tight and the slack sides of the band will be as shown in *Fig.6.18*. Considering the forces acting on the lever and taking moments about the pivot,

$$F\,l - T_1\,a + T_2\,b = 0$$

$$\therefore\ F = \frac{T_1 a - T_2\, b}{l}$$

As $T_1 > T_2$ and $a > b$ under all conditions, the effectiveness of the brake will depend upon the force F.

(b) Rotation Clockwise

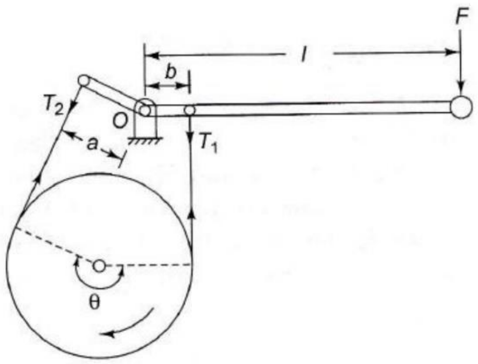


Fig.6.19 - Rotation Clockwise

In this case, the tight and the slack sides are reversed as shown in *Fig.6.19*.

Now,

$$F\,l - T_2\,a + T_1\,b = 0$$

$$\therefore\; F = \frac{T_2 a - T_1\, b}{l}$$

As $T_2 < T_1$ and $a > b$, the brake will be effective as long as $T_2.\,a$ is greater than $T_1.\,b$

$$T_2\,a > T_1\,b \quad \text{or} \quad \frac{T_2}{T_1} > \frac{b}{a}$$

Or

i.e., as long as the ratio of T_2 to T_1 is greater than the ratio b/a.

When $\frac{T_2}{T_1} \le \frac{a}{b}$ F is zero or negative, i.e., the brake becomes self – locking as no force is needed to apply the brake. Once the brake has been engaged, no further force is required to stop the rotation of the drum.

6.6.3.2 a < b, F upwards

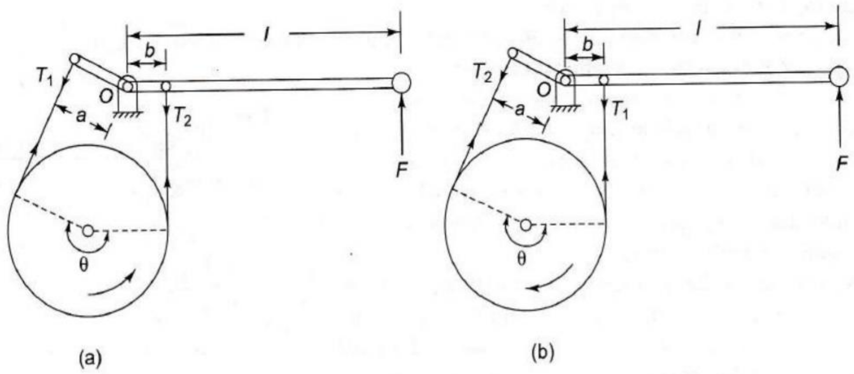


Fig.6.20 - a < b, F upwards

(a) Rotation Counter – Clockwise

The tight and the slack sides will be as shown in *Fig.6.20* (a).

Therefore,

$$F \cdot l + T_1 a - T_2 b = 0$$

or

$$F = \frac{T_2 b - T_1 a}{l}$$

As $T_2 < T_1$ and $b > a$ the brake is operative only as long as

$$T_2 b > T_1 a \quad \text{or} \quad \frac{T_2}{T_1} > \frac{a}{b}$$

Once T_2 / T_1 becomes equal to a/b, F required is zero and the brake becomes self – locking.

(b) Rotation Clockwise

The tight and slack sides are shown in *Fig.6.20* (b).

From *Fig.6.20* (b),

$$F \cdot l + T_1 b - T_2 a = 0 \quad \text{or} \quad F = \frac{T_1 b - T_2 a}{l}$$

As $T_1 > T_2$ and b> a, under all conditions, the effectiveness of the brake will depend upon the force F. When a = b, the band cannot be tightened and thus, the brake cannot be applied.

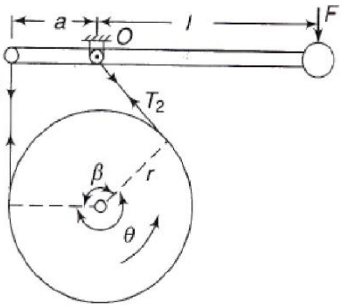


Fig.6.21 b = 0 and F downwards

The band brake just discussed is known as a differential band brake. However, if either a or b is made zero, a simple band brake is obtained. If b = 0 (*Fig.6.21*) and F downwards,

$$F \cdot l - T_1 a = 0$$

or

$$F = T_1 \frac{a}{l}$$

Similarly, the force can be found in other cases.

- Note that such a brake can neither have self – energizing properties nor it can be self – locked.

The brake is said to be more effective when maximum braking force is applied with the least effort F. For case (i), when a > b and F are downwards, the force (effort) F required is less when the rotation is clockwise assuming that the brake is effective.

For case (ii), when a < b and F are upwards, F required is less when the rotation is counterclockwise assuming that the brake is effective.

Thus, for the given arrangement of the differential brake, it is more effective when

(a) a > b, F downwards, rotation clockwise

(b) a > b, F upwards, rotation counterclockwise.

The advantages of self – locking is taken in hoists and conveyers where motion is permissible in only one direction. If the motion gets reversed somehow, the self – locking is engaged which can be released only by reversing the applied force.

It is seen that a differential band brake is more effective only in one direction of rotation of the drum.

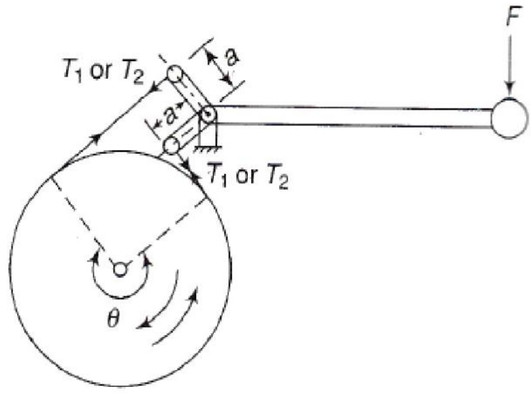


Fig.6.22 – Two-way band brake

However, a two-way band brake can also be designed which is equally effective for both the directions of rotation of the drum (*Fig.6.22*). In such a brake, the two lever arms are made equal.
For both directions of rotation of the drum,

$$F \cdot l - T_1 a - T_2 a = 0$$

or

$$F = (T_1 + T_2)\frac{a}{l}$$

Band brake offers the following advantages:

(i) Band brake has a simple construction. It has a small number of parts. These features reduce the cost of a band brake.
(ii) Most equipment manufacturers can easily produce band brake without requiring specialized facilities like a foundry or forging shop. The friction lining is the only part that must be purchased from outside agencies.
(iii) A band brake is more reliable due to the small number of parts.
(iv) Band brake requires little maintenance.

The disadvantages of band brake are as follows:

(i) The heat dissipation capacity of a band brake is poor.
(ii) The wear of friction lining is uneven from one end to the other.

Band brakes are used in applications like bucket conveyors, hoists and chain saws. They are more popular as back-stop devices.

6.6.3.3 *Band and Block Brake*

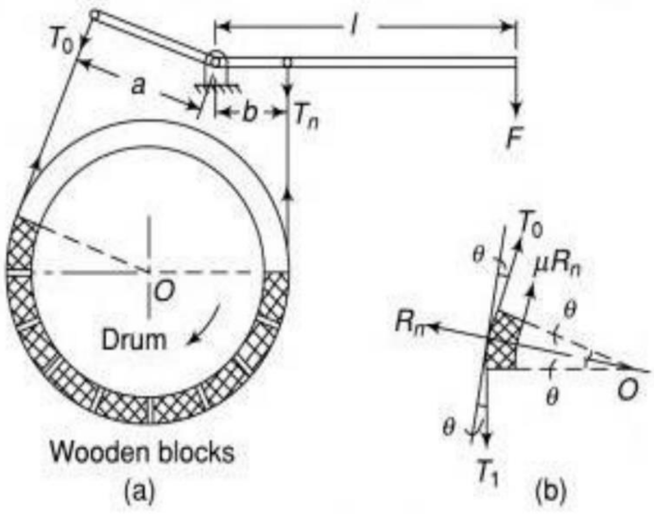


Fig.6.23 - Band and Block Brake

A band and block brake consists of a number of wooden blocks secured inside a flexible steel band. When the brake is applied, the blocks are pressed against the drum.

Two sides of the band become tight and slack as usual. Wooden blocks have a higher coefficient of friction. Thus, increasing the effectiveness of the brake. Also, such blocks can be easily replaced on being worn out [*Fig.6.23* (a)].

Each block subtends a small angle of 2θ at the center of the drum. The frictional force on the blocks acts in the direction of rotation of the drum. For n blocks on the brake,

Let,

T_0 = Tension on the slack side.

T_1 = Tension on the tight side after one block.

T_2 = Tension on the tight side after two blocks.

$\vdots$

T_n = Tension on the tight side after n blocks.

μ = Coefficient of friction.

R_n = Normal reaction on the block.

Resolving the forces horizontally and vertically

$$(T_1 - T_0)\cos\theta = \mu R_n$$
$$(T_1 + T_0)\sin\theta = R_n$$
$$\therefore \frac{T_1 - T_0 \cos\theta}{T_1 + T_0 \sin\theta} = \frac{\mu R_n}{R_n}$$

$$\therefore \frac{(T_1 - T_0)}{T_1 + T_0} = \frac{\mu \tan\theta}{1}$$

$$\therefore \frac{(T_1 - T_0) + (T_1 + T_0)}{(T_1 - T_0) - (T_1 + T_0)} = \frac{\mu \tan\theta + 1}{\mu \tan\theta - 1}$$

$$-\frac{2T_1}{2T_0} = -\frac{1+\mu\tan\theta}{1-\mu\tan\theta}$$

$$\frac{T_1}{T_0} = \frac{1+\mu\tan\theta}{1-\mu\tan\theta}$$

Similarly,

$$\frac{T_2}{T_1} = \frac{1+\mu\tan\theta}{1-\mu\tan\theta} \text{ and so on}$$

$$\vdots$$

$$\frac{T_n}{T_{n-1}} = \frac{1+\mu\tan\theta}{1-\mu\tan\theta}$$

$$\frac{T_n}{T_0} = \frac{T_n}{T_{n-1}} \cdot \frac{T_{n-1}}{T_{n-2}} \cdots\cdots \frac{T_2}{T_1} \cdot \frac{T_1}{T_0}$$

$$= \left[\frac{1+\mu\tan\theta}{1-\mu\tan\theta}\right]^n$$

6.6.3.4 Internal Expanding Brake

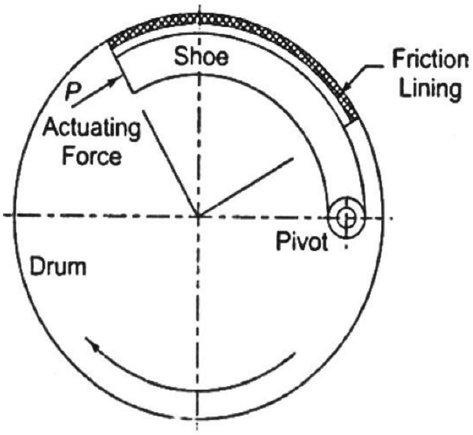


Fig.6.24 - Internal Expanding Brake

The construction of an internal expanding brake is shown in *Fig.6.24*. It consists of a shoe, which is pivoted at one end and subjected to an actuating force P at the other end. A friction lining is fixed on the shoe and the complete assembly of shoe lining and pivot is placed inside the brake drum.

Internal shoe brakes, with two symmetrical shoes, are used on all automobile vehicles. The actuating force is usually provided by means of a hydraulic cylinder or a cam mechanism. The analysis of the internal shoe brake is based on the following assumptions:

- The intensity of normal pressure between the friction lining and the brake drum at any point is proportional to its vertical distance from the pivot.
- The brake drum and the shoe are rigid.
- The centrifugal force acting on the shoe is negligible.
- The coefficient of friction is constant.

6.7 Braking of Vehicle

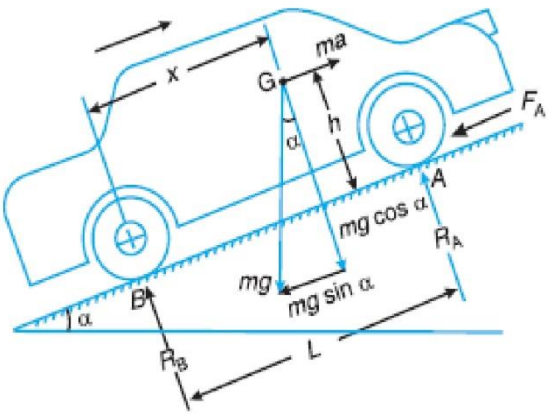


Fig.6.25 - A vehicle moving up an inclined plane

In a four-wheeled moving vehicle, the brakes may be applied to

1. The rear wheels only,
2. The front wheels only, and
3. All the four wheels.

In all the above mentioned three types of braking, it is required to determine the retardation of the vehicle when brakes are applied. Since the vehicle retards, therefore it is a problem of dynamics. But it may be reduced to an equivalent problem of statics by including the inertia force in the system of forces actually applied to the vehicle. The inertia force is equal and opposite to the braking force causing retardation.

Consider a vehicle moving up an inclined plane shown in *Fig.6.25*.

Let,

$$\alpha = \text{Angle of inclination of the plane to the horizontal.}$$
$$\text{m} = \text{mass of vehicle (Such that its weight mg in Newton).}$$
$$\text{h} = \text{Height of C.G. of the vehicle above the road surface (mtr).}$$
$$\text{x} = \text{Perpendicular distance of C.G. from the rear axle (mtr).}$$
$$\text{L} = \text{Distance between centre of rear and front wheel (wheel Base)(mtr).}$$
$$R_A, R_B = \text{Reactions of the ground on the front \& rear wheel (N).}$$
$$\mu = \text{Coefficient of friction between tyre and road surface.}$$
$$\text{a} = \text{Retardation of the vehicle } (m/s^2).$$

6.7.1 Brakes applied to Rear wheel only

It is a common way of braking a vehicle in which the braking force acts at the rear wheel.

Resolving the forces,

$$F_B + m\,g\,\text{Sin}\,\alpha = m\,a \qquad \textit{Eq. (6.3)}$$

$$R_A + R_B = m\,g\,\text{Sin}\alpha \qquad \textit{Eq. (6.4)}$$

$$\left\{ \begin{array}{l} \because F_B = \text{Total breaking force acting at} \\ \qquad \text{the rear wheel due to application of brakes} \\ \quad = \mu R_B \end{array} \right\}$$

Taking moment about G,

$$F_B \times h + R_B \times x = R_A (L - x) \qquad \left\{ \begin{aligned} \because F_B &= \mu R_B \\ R_A &= m g \cos\alpha - R_B \end{aligned} \right\}$$

$$\mu R_B \cdot h + R_B \times x = (m g \cos\alpha - R_B)(L - x)$$

$$R_B (\mu h + x) = m g \cos\alpha (L - x) - R_B (L - x)$$

$$R_B (\mu h + x + L - x) = m g \cos\alpha (L - x)$$

$$\therefore R_B = \frac{m g \cos\alpha (L - x)}{L + \mu h}$$

and

$$\begin{aligned} R_A &= m g \cos\alpha - R_B \\ &= m g \cos\alpha - \frac{m g \cos\alpha (L - x)}{L + \mu h} \\ R_A &= \frac{m g \cos\alpha (x + \mu h)}{L + \mu h} \end{aligned}$$

From *Eq. (6.3)*

$$F_B + m g \sin\alpha = m$$

$$\begin{aligned} \therefore a &= \frac{F_B + m g \sin\alpha}{m} \\ &= \frac{F_B}{m} + g \sin\alpha \\ a &= \frac{\mu R_B}{m} + g \sin\alpha \quad \{\text{Putting value of } R_B \text{ in equation}\} \end{aligned}$$

$$\therefore a = \mu \frac{\left(\dfrac{m g \cos\alpha (L - x)}{L + \mu h} \right)}{m} + g \sin\alpha$$

$$\therefore a = \frac{\mu g \cos\alpha (L - x)}{L + \mu h} + g \sin\alpha$$

Notes: (1) On Road level α = 0, therefore,

$$a = \frac{\mu g (L - x)}{L + \mu h}$$

(2) When a vehicle moves down a plane, *Eq. (6.3)* becomes $F_B - m g \sin\alpha = m a$

$$\begin{aligned} \therefore a &= \frac{F_B}{m} - g \sin\alpha \\ &= \frac{\mu R_B}{m} - g \sin\alpha \\ a &= \frac{\mu g \cos\alpha (L - x)}{L + \mu h} - g \sin\alpha \end{aligned}$$

6.7.2 Brake applied to Front wheels only

It is a very rare way of braking the vehicle, in which the braking force acts at the front wheels only.

Let,

$$F_A = \text{Braking force acting at the front wheel.}$$
$$= \mu R_A$$

Resolving the forces horizontally and vertically,

$$F_A + m\,g\,\text{Sin}\,\alpha = m\,a \qquad \textit{Eq. (6.5)}$$
$$R_A + R_B = m\,g\,\text{Cos}\alpha \qquad \textit{Eq. (6.6)}$$

Taking moments about G,

$$F_A \cdot h + R_B \cdot x = R_A\,(L - x) \qquad \left\{ \begin{array}{l} F_A = \mu R_A \\ R_B = m\,g\cos\alpha - R_A \\ \text{Putting values....} \end{array} \right\}$$

$$\mu R_A \cdot h + (m\,g\cos\alpha - R_A)\,x = R_A\,(L - x)$$

$$\mu R_A \cdot h + m\,g\cos\alpha \cdot x = R_A \cdot L$$

$$R_A\,(L - \mu h) = m\,g\cos\alpha \cdot x$$

$$R_A = \frac{m\,g\cos\alpha \cdot x}{L - \mu h}$$

Eq. (6.9)

$$R_A + R_B = m\,g\cos\alpha$$

$$R_B = m\,g\cos\alpha - R_A$$

$$= m\,g\cos\alpha - \frac{m\,g\cos\alpha \cdot x}{L - \mu h}$$

$$= m\,g\cos\alpha\left[1 - \frac{x}{L - \mu h}\right]$$

$$R_B = m\,g\cos\alpha\left[\frac{L - \mu h - x}{L - \mu h}\right]$$

Eq. (6.5)

$$F_A + m\,g\sin\alpha = m\,a$$

$$a = \frac{F_A + m\,g\sin\alpha}{m}$$

$$= \frac{\mu R_A + m\,g\sin\alpha}{m} \qquad \{\because \text{Putting Value of } R_A\}$$

$$= \frac{\mu\left[\dfrac{m\,g\cos\alpha \cdot x}{(L - \mu h)}\right] + m\,g\sin\alpha}{m}$$

$$a = \frac{\mu\, g \cos\alpha \cdot x}{L - \mu h} + g \sin\alpha$$

Notes: (1) On a Road level α = 0,

$$\therefore R_A = \frac{m\, g \cos\alpha \cdot x}{L - \mu h} = \frac{m\, g\, x}{L - \mu h} \qquad \{\cos 0^\circ = 1\}$$

$$R_B = m\, g \cos\alpha \left[\frac{L - \mu h - x}{L - \mu h}\right] = \frac{m\, g\,(L - \mu h - x)}{L - \mu h}$$

$$a = \frac{\mu\, g \cos\alpha \cdot x}{L - \mu h} + g \sin\alpha \qquad \left\{\begin{aligned} \because \cos 0^\circ &= 1 \\ \sin 0^\circ &= 0 \end{aligned}\right\}$$

$$= \frac{\mu\, g \cdot x}{L - \mu h}$$

(2) When a vehicle moves down the plane,

$$F_A - m\, g \sin\alpha = m\, a$$

$$a = \frac{F_A}{m} - \frac{m\, g \sin\alpha}{m}$$

$$= \frac{\mu R_A}{m} - g \sin\alpha$$

$$= \frac{\mu\, g \cos\alpha \cdot x}{L - \mu h} - g \sin\alpha$$

6.7.3 Brakes applied to all four wheels

This is the most common way of braking the vehicle, in which the braking force acts on both the rear and front wheels.

$$F_A = \text{Braking force for front wheels} = \mu\, R_A$$
$$F_B = \text{Braking force for rear wheels} = \mu\, R_B$$

Note: A little consideration will show that when the brakes are applied to all the four wheels, the braking distance (*i.e.* the distance in which the vehicle is brought to rest after applying the brakes) will be the least. It is due to this reason that the brakes are applied to all the four wheels.

Resolving the forces,

$$F_A + F_B + m\, g \,\mathrm{Sin}\, \alpha = m\, a \qquad \textit{Eq. (6.7)}$$

$$R_A + R_B = m\, g \,\mathrm{Cos}\alpha \qquad \textit{Eq. (6.8)}$$

Taking moment about G,

$$(F_A + F_B)\, h + R_B \cdot x = R_A(L - x) \qquad \textit{Eq. (6.9)}$$

$$\left\{\begin{aligned} F_A &= \mu\, R_A \\ F_B &= \mu\, R_B \\ R_B &= m\, g \cos\alpha - R_A \\ &\text{Putting values in eq.(6.9)} \end{aligned}\right\}$$

$$\mu(R_A + R_B)\, h + (m\, g \cos\alpha - R_A)\, x = R_A\,(L - x)$$

$$\mu\left(R_A + m\,g\cos\alpha - R_A\right)h \; + \; \left(m\,g\cos\alpha - R_A\right)x = R_A\,(L - x)$$

$$R_A = \frac{m\,g\cos\alpha\,(\mu h + x)}{L}$$

$$\therefore R_B = m\,g\cos\alpha - R_A$$

$$= m\,g\cos\alpha - m\,g\cos\alpha\left(\frac{\mu h + x}{L}\right)$$

$$= m\,g\cos\alpha\left(\frac{L - \mu h - x}{L}\right)$$

Eq. (6.7)

$$F_A + F_B + m\,g\sin\alpha = m\,a$$

$$\mu\left(R_A + R_B\right) + m\,g\sin\alpha = m\,a$$

$$\mu\,m\,g\cos\alpha + m\,g\sin\alpha = m\,a \qquad \left\{\because R_A + R_B = m\,g\cos\alpha\right\}$$

$$a = g\left(\mu\cos\alpha + \sin\alpha\right)$$

Notes: (1) On a Road level α = 0,

$$R_A = \frac{m\,g\cos\alpha\,(\mu h + x)}{L} = \frac{mg(\mu h + x)}{L}$$

$$R_B = m\,g\left(\frac{L - \mu h - x}{L}\right)$$

$$a = g\;\mu$$

(2) If the vehicle moves down the plane, then it may be written as,

$$F_A + F_B - m\,g\sin\alpha = m\,a$$

$$\mu\left(R_A + R_B\right) - m\,g\sin\alpha = m\,a$$

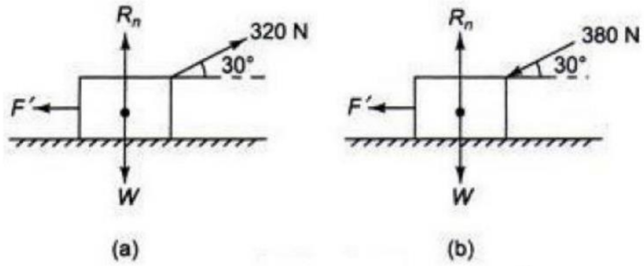
$$\mu\,m\,g\cos\alpha - m\,g\sin\alpha = m\,a \qquad \left\{\because R_A + R_B = m\,g\cos\alpha\right\}$$

$$a = g\left(\mu\cos\alpha - \sin\alpha\right)$$

6.8 Problems

Ex. 6.1 **The force required just to move a body on a rough horizontal surface by pulling is 320 N inclined at 30° and by pushing 380 N at the same angle. Find the weight of the body and the coefficient of friction.**

Solution:



(a) Consider the pull Fig. (a)

Resolve forces horizontally $F' = 320\cos 30°$

$$\mu R_n = 277 \qquad \therefore R_n = \frac{277}{\mu} \qquad \ldots\ldots(1)$$

Resolve forces horizontally $R_n + 320\sin 30° = W$

$$\frac{277}{\mu} + 160 = W \qquad \therefore \mu = \frac{277}{W-160} \qquad \ldots\ldots(2)$$

(b) Consider the push Fig. (b)

Resolve forces horizontally $F' = 380\cos 30°$

$$\mu R_n = 329 \qquad \therefore R_n = \frac{329}{\mu} \qquad \ldots\ldots(3)$$

Resolve forces horizontally $R_n = W + 380\sin 30°$

$$\frac{329}{\mu} = W + 190 \qquad \therefore \mu = \frac{329}{W+190} \qquad \ldots\ldots(4)$$

Comparing equations (3) & (4),

$$\frac{277}{W-160} = \frac{329}{W+190} \Rightarrow \boxed{W = 2024.4\text{ N}}$$

$$\boxed{\therefore\ \mu = 0.1486}$$

Ex. 6.2 **A body is to be moved up an inclined plane by applying a force parallel to the plane surface. It is found that a force of 3 KN is required to just move it up the plane when the angle of inclination is 10° whereas the force needed increase to 4 KN when the angle of inclination is increased to 15°. Determine the weight of the body and the coefficient of friction.**

Solution: When the force applied is parallel to the plane surface,

$$F = W(\sin\alpha + \mu\cos\alpha)$$

$$3000 = W(\sin 10° + \mu\cos 10°) \qquad \ldots\ldots(1)$$

$$\& \ 4000 = W(\sin 15° + \mu\cos 15°) \qquad \ldots\ldots(2)$$

Dividing equation (2) by (1),

$$\therefore \frac{4000}{3000} = \frac{W(\sin 15° + \mu \cos 15°)}{W(\sin 10° + \mu \cos 10°)}$$

$$\therefore \sin 10° + \mu \cos 10° = 0.75(\sin 15° + \mu \cos 15°)$$

$$\therefore \mu(\cos 10° - 0.75 \cos 15°) = 0.75 \sin 15° - \sin 10°$$

$$\therefore 0.2604\,\mu = 0.0205$$

$$\boxed{\therefore \mu = 0.0786} \text{ and } \boxed{W = 11.95 \text{ kN}}$$

Ex. 6.3 **A square threaded bolt with a core diameter of 25 mm and a pitch of 10 mm is tightened by screwing a nut. The mean diameter of the bearing surface of the nut is 60 mm. The coefficient of friction for the nut and the bolt is 0.12 and for the nut and the bearing surface, it is 0.15. Determine the force required at the end of a 400 mm long spanner if the load on the bolt is 12 KN.**

Solution: Given data:

d_c = 25 mm, p = 10 mm, μ = 0.12, μ_1= 0.16, l = 400 mm, W = 12 X 10^3 N

To be Calculated:

F (force required)

$$\text{Let } d(\text{mean dia.}) = d_c + \frac{p}{2} = 25 + \frac{10}{2} = 30\text{mm}$$

$$\text{Also } \tan\alpha = \frac{p}{\pi d} = \frac{10}{\pi \times 30} = 0.1061 \qquad \therefore \alpha = 6.05°$$

$$\& \ \mu = \tan\varphi \Rightarrow \therefore \varphi = \tan^{-1}(0.12) = 6.84°$$

$$\text{Let torque } T_1 = F \times r \qquad \{\text{Force is applied horizontally}\}$$

$$= W \tan(\alpha + \varphi) r$$

$$= 12000 \tan(6.05 + 6.84)\frac{30}{2}$$

$$= 41192.44 \text{ N.mm}$$

$$\text{Total torque } T = T_1 + T_2$$

$$= F \times \frac{d}{2} + \mu_1 WR$$

$$= 41192.44 + 0.16 \times 12 \times 10^3 \times 30$$

$$= 98792.44 \text{ N.mm}$$

$$\text{Now Torque } T = \text{Force} \times \text{Distance}$$

$$98792.44 = F' \times 400$$

$$\boxed{F' = 246.98 \text{ N}}$$

Ex. 6.4 **The cutting speed of a broaching machine is 9 m per minute. The cutter of the machine is pulled by a square threaded screw with a notational diameter of 60 mm and a pitch 12 mm. The operating nut takes an axial load of 500 N on a flat surface of 80 mm external diameter and 48 mm internal diameter. Determine the power required to rotate the operating nut. Take μ = 0.14 for all contact surfaces on the nut.**

Solution: Given data:

d_o = 60 mm, p = 12 mm, W = 500 N, R_1= 80/2= 40 mm, R_2 = 48/2= 24 mm, μ = tan φ = 0.14, μ_1= 0.14, l = 400 mm,

To be Calculated:

P (power required)

$$\text{Let } d(\text{mean dia.}) = d_o - \frac{p}{2} = 60 - \frac{12}{2} = 54\text{mm} \qquad \text{Also } \tan\alpha = \frac{p}{\pi d} = \frac{12}{\pi \times 54} = 0.0707 \quad \therefore \alpha = 4.046°$$

$$\& \ \mu = \tan\varphi \Rightarrow \therefore \ \varphi = \tan^{-1}(0.14) = 7.97°$$

$$\begin{aligned}\text{Let force } F &= W\tan(\alpha+\varphi) \\ &= 500\tan(4.046+7.97) \\ &= 106.42 \text{ N}\end{aligned}$$

$$\begin{aligned}\text{Total torque } T &= T_1 + T_2 \\ &= F \times \frac{d}{2} + \mu_1 W\left(\frac{R_1+R_2}{2}\right) \\ &= 106.42 \times \frac{54}{2} + 0.14 \times 500 \times \left(\frac{40+24}{2}\right) \\ T &= 5113.34 \text{ N.mm} = 5.11 \text{ N.m}\end{aligned}$$

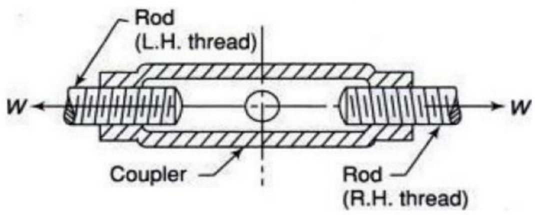
$$\text{Speed} = \frac{\text{Cutting speed}}{\text{Pitch}} = \frac{9 \text{ m/min}}{0.012 \text{ m}} = 750 \text{ rpm}$$

$$\text{Power } P = \frac{2\pi NT}{60} = \frac{2\pi \times 750 \times 5.11}{60}$$

$$\boxed{P = 401.33 \text{ W}}$$

Ex. 6.5 **The railway coaches with the help of two tie rods of a turnbuckle with right and left-handed threads having single start square threads. The pitch and mean diameter of the thread are 8 mm and 30 mm respectively. What will be the work done in bringing the two coaches closer through a distance of 160 mm against a steady load of 2 KN? Take μ =0.12.**

Solution:



Given data:

p = 8 mm, d = 30 mm, μ = 0.12, W = 2000 N

To be Calculated:

W (work done)

$$\tan\alpha = \frac{p}{\pi d} = \frac{8}{\pi \times 30} = 0.0848 \;\therefore \alpha = 4.85° \quad \& \;\mu = \tan\varphi \Rightarrow \therefore\; \varphi = \tan^{-1}(0.12) = 6.84°$$

$$\begin{aligned}\text{Torque on each rod } T &= F \times r \\ &= W\tan(\alpha+\varphi)r \\ &= 2000\tan(4.85+6.84)\frac{30}{2} \\ &= 6207.24 \text{ N.mm} \\ &= 6.20 \text{ N.m}\end{aligned}$$

$$\text{Total torque on coupling nut} = 2\times T = 2\times 6.20 = 12.4 \text{ N.m}$$

In one complete revolution of the road, each coach will move through a distance equal to pitch.

Number of turns required to move the coaches through a distance by 160 mm,

$$= 160/(2\times p) = 160/(2\times 8) = 10$$

$$\begin{aligned}\text{Work done } W &= T.\theta \\ &= 12.4\times 2\pi \times 10\end{aligned}$$

$$\boxed{W = 780.4 \text{ N.m}}$$

$$\text{Efficiency } \eta = \frac{\tan\alpha}{\tan(\alpha+\varphi)} = \frac{\tan 4.85}{\tan(4.85+6.84)}$$

$$= 0.41$$

$$\boxed{\eta = 41\%}$$

Ex. 6.6 **A Whitworth bolt with an angle of V-threads as 55° has a pitch of 6 mm and a mean diameter of 32 mm. the mean radius of the bearing surface where the nut is tightened is 20 mm. Determine the force required at the end of a 400 mm long spanner when the load on the bolt is 8 KN. The coefficient of friction for the nut and the bolt is 0.1 and for the nut and the bearing surface is 0.15.**

Solution: Given data:

V-threads, 2β = 55°, p = 6 mm, d = 32 mm, R = 20 mm, W = 8 X 10^3N, μ = 0.1 = tan φ, μ_1= 0.15

To be Calculated:

F' (force required at the end of a 400 mm long spanner)

$$\mu' = \frac{\mu}{\cos\beta} = \frac{0.1}{\cos 27.5} = 0.113 \quad \& \quad \tan\alpha = \frac{p}{\pi d} = \frac{6}{\pi\times 32} = 0.0707 \quad \therefore \alpha = 3.42°$$

$$\text{Also } \mu' = \tan\varphi \Rightarrow \therefore\; \varphi = \tan^{-1}(0.113) = 6.48°$$

$$\begin{aligned}\text{Let force } F &= W\tan(\alpha+\varphi) \\ &= 8\times 10^3 \tan(3.42+6.45) \\ &= 1391.90 \text{ N}\end{aligned}$$

$$\text{Total torque } T = T_1 + T_2$$
$$= F \times \frac{d}{2} + \mu_1 WR$$
$$= 1391.90 \times \frac{32}{2} + 0.15 \times 8 \times 10^3 \times 20$$
$$T = 46270.4 \text{ N.mm}$$

$$\text{Let Torque } T = \text{Force} \times \text{Length}$$
$$46270.4 = F' \times 400$$
$$\boxed{F' = 115.67 \text{ N}}$$

Ex. 6.7 **A load of 15 KN is raised by means of a screw jack. The mean diameter of the square threaded screw is 42 mm and the pitch is 10 mm. A force of 120 N is applied at the end of a lever to raise the load. Determine the length of the lever to be used and the mechanical advantage obtained. Is the screw self-locking? Take μ=0.12.**

Solution: Given data:

$W = 15 \times 10^3$ N, d = 42 mm, p = 10 mm ,F = 120 N, $\mu = 0.12 = \tan\varphi$

To be Calculated:

l & MA (length of the lever to be used and the mechanical advantage)

$$\tan\alpha = \frac{p}{\pi d} = \frac{10}{\pi \times 42} = 0.0758 \;\therefore \alpha = 4.335^\circ \quad \& \;\; \mu = \tan\varphi \Rightarrow \therefore\; \varphi = \tan^{-1}(0.12) = 6.843^\circ$$

$$\text{Let torque } T = F \times r$$
$$= W \tan(\alpha + \varphi) r$$
$$= 15 \times 10^3 \tan(4.335 + 6.843)\frac{42}{2}$$
$$= 62245.99 \text{ N.mm}$$

$$\text{Let Torque } T = \text{Force} \times \text{Length}$$
$$62245.99 = 120 \times l$$
$$\boxed{l = 518.7 \text{ mm}}$$

$$\text{Mechanical Advatage (MA)} = \frac{W}{F} = \frac{15 \times 10^3}{120}$$
$$\boxed{MA = 125}$$

$$\text{Efficiency } \eta = \frac{\tan\alpha}{\tan(\alpha + \varphi)} = \frac{\tan 4.335}{\tan(4.335 + 6.843)}$$
$$= 0.3836$$
$$= 38.36\% < 50\%$$

So screw is self-locking.

Ex. 6.8 **The following data relate to a screw jack:**

Pitch of the threaded screw =8 mm

The diameter of the threaded screw =40 mm

Coefficient of friction between screw and nut =0.1
Load =20 KN

Assuming that the load rotates with the screw, determine the

i. The ratio of torque required to raise and lower the load.
ii. The efficiency of the machine.

Solution: Given data:

p = 8 mm, d = 40 mm, $\mu = 0.1 = \tan\varphi$, W = 20 X 10^3 N

To be Calculated:

Ratio of torque required to raise and lower the load and Efficiency of the machine

(1) Torque to raise the load (T_1),

$$\begin{aligned} T_1 &= F\times r = W\tan(\alpha+\varphi)r \\ &= 20\times10^3 \tan(3.64+5.71)20 \\ &= 65860 \text{ N.mm} \\ &= 65.86 \text{ N.m} \end{aligned}$$

Torque to lower the load (T_2),

$$\begin{aligned} T_2 &= F\times r = W\tan(\varphi-\alpha)r \\ &= 20\times10^3 \tan(5.71-3.64)20 \\ &= 14457 \text{ N.mm} \\ &= 14.46 \text{ N.m} \end{aligned}$$

$$\text{Ratio} = \frac{T_1}{T_2} = \frac{65.86}{14.46} = \boxed{4.56}$$

(2) The efficiency of the machine,

$$\begin{aligned} \text{Efficiency } \eta &= \frac{\tan\alpha}{\tan(\alpha+\varphi)} \\ &= \frac{\tan 3.64}{\tan(3.64+5.71)} \\ &= 0.3863 \end{aligned}$$

$$\boxed{\eta = 38.63\%}$$

Ex. 6.9 **In a screw jack, the diameter of the threaded screw is 40 mm and the pitch is 8 mm. The load is 20 KN and it does not rotate with the screw but is carried on a swivel head having a bearing diameter of 70 mm. The coefficient of friction between the swivel head and the spindle is 0.08 and between the screw and nut is 0.1. Determine the total torque required to raise the load and efficiency.**

Solution: Given data:

d = 40mm, p = 8mm, W = 20KN, D = 70mm, $\mu = 0.1 = \tan\varphi$, $\mu_1 = 0.08$

To be Calculated:

The total torque required to raise the load and the efficiency

$$\tan\alpha = \frac{p}{\pi d} = \frac{8}{\pi \times 40} = 0.0637 \ \therefore \alpha = 3.64° \quad \& \ \mu = \tan\varphi \Rightarrow \therefore \ \varphi = \tan^{-1}(0.1) = 5.71°$$

$$\text{Let force } F = W\tan(\alpha+\varphi)$$
$$= 20\times10^3 \tan(3.64+5.71)$$
$$= 3293.049 \text{ N}$$

$$\text{Total torque } T = T_1 + T_2$$
$$= \text{Torque to raise the load} + \text{Torque due to collar friction}$$
$$= F\times\frac{d}{2} + \mu_1 WR$$
$$= 3293.049\times\frac{40}{2} + 0.08\times20\times10^3\times35$$
$$= 121860.98 \text{ N.mm}$$
$$\boxed{T = 121.86 \text{ N.m}}$$

$$\text{Efficiency } \eta = \frac{\tan\alpha}{\tan(\alpha+\varphi)} \quad \text{or} \quad \eta = \frac{\text{W.D. in lifting load / rev}}{\text{W.D. by applied force / rev}}$$
$$= \frac{\tan 3.64}{\tan(3.64+5.71)} \quad\quad = \frac{W\times P}{F\times\pi d} = \frac{W}{F}\tan\alpha = \frac{20\times10^3 \tan 3.64}{3293.049}$$
$$= 0.3863 \quad\quad = 0.3863$$
$$\boxed{\eta = 38.63\%} \quad\quad \boxed{\eta = 38.63\%}$$

Ex. 6.10 **A screw jack raises a load of 16 KN through a distance of 150 mm. The mean diameter and the pitch of the screw are 56 mm and 10 mm respectively. Determine the work done and the efficiency of the screw jack when the**

i. Load rotates with the screw

ii. Loose head on which the load rests does not rotate with the screw and the outside and the inside diameters of the bearing surface of the loose head are 50 mm and 10 mm respectively.

Take μ =0.11.

Solution: Given data:

d = 56 mm, p = 10 mm, W = 16 KN, h = 150 mm, μ = 0.11= tan φ, μ_1= 0.08

To be Calculated:

Work done and the efficiency of the screw jack

$$\tan\alpha = \frac{p}{\pi d} = \frac{10}{\pi\times56} = 0.0568 \ \therefore \alpha = 3.25° \quad \& \ \mu = \tan\varphi \Rightarrow \therefore \ \varphi = \tan^{-1}(0.11) = 6.28°$$

$$\text{Torque to raise the load } T_1 = F\times r$$
$$= W\tan(\alpha+\varphi)r$$
$$= 16\times10^3 \tan(3.25+6.28)\frac{0.056}{2}$$
$$T_1 = 75.21 \text{ N.m}$$

In one complete revolution distance moved by screw = pitch = 10 mm

$$\therefore \text{ Number of revolutions by screw} = \frac{150\ (\text{dist.})}{10\ (\text{pitch})} = 15$$

(1) When the load rotates with screw

Work done in raising the load/rev = $T.2\pi$

$$\therefore \text{ Total work done in raising the load} = T.2\pi.N$$
$$= 75.21 \times 2\pi \times 15$$
$$= \boxed{7088\ \text{N.m}}$$

$$\text{Efficiency } \eta = \frac{\tan\alpha}{\tan(\alpha+\varphi)}$$
$$= \frac{\tan 3.25}{\tan(3.25+6.28)}$$
$$= 0.3382$$
$$\boxed{\eta = 33.82\%}$$

(2) When the load does not rotate with the screw

$$\text{Mean diameter of bearing surface R} = \frac{1}{2}\left(\frac{50+10}{2}\right) = 15\ \text{mm}$$

$$\text{Torque due to collar friction } T_2 = \mu WR$$
$$= 0.11 \times 16 \times 10^3 \times 0.015$$
$$T_2 = 26.4\ \text{N.m}$$

Total friction torque required to raise the load,

$$T = T_1 + T_2$$
$$= 75.21 + 26.4$$
$$T = 101.61\ \text{N.m}$$

$$\therefore \text{ Total work done in raising the load} = T.2\pi.N$$
$$= 101.61 \times 2\pi \times 15$$
$$= \boxed{9577\ \text{N.m}}$$

$$\text{Efficiency } \eta = \frac{\text{W.D. in lifting load / rev}}{\text{W.D. by applied force / rev}}$$
$$= \frac{W \times P}{F \times \pi d} = \frac{W}{F}\tan\alpha = \frac{16 \times 10^3 \tan 3.25}{3629}$$
$$= 0.25$$
$$\boxed{\eta = 25\%}$$

Ex. 6.11 **In a thrust bearing, the external and the internal diameters of the contacting surface are 320 mm and 200 mm respectively. The total axial load is 80 KN and the intensity of pressure is 350 KN/m^2. The shaft rotates at 400 RPM. Taking the μ=0.06. Calculate the**

power lost in overcoming the friction. Also, find the number of collars required for the bearing.

Solution: Given data:

d_1 = 320mm, d_2 = 200mm, W = 80KN, P = 350KN/m^2 N = 400rpm, μ = 0.06

To be Calculated:

Power lost in overcoming the friction and the number of collars required for the bearing

Using uniform pressure theory torque,

$$T = \frac{2}{3}\mu W\left(\frac{R_1^3 - R_2^3}{R_1^2 - R_2^2}\right)$$

$$= \frac{2}{3}\times 0.06\times 80\times 10^3\left[\frac{(0.16)^3 - (0.10)^3}{(0.16)^2 - (0.10)^2}\right]$$

$$= \frac{3200\times 3.096\times 10^{-3}}{0.0156}$$

$$T = 635.07\ \text{N.m}$$

Power lost in friction (P_f),

$$P_f = T.\omega$$

$$= T\times\frac{2\pi N}{60}$$

$$= 635.07\times\frac{2\pi\times 400}{60}$$

$$= 26602.03\ \text{W}$$

$$\boxed{P_f = 26.60\ \text{kW}}$$

Number of collars required (n),

$$P = \frac{W}{n\pi\left(R_1^2 - R_2^2\right)}$$

$$\therefore\ n = \frac{W}{P\pi\left(R_1^2 - R_2^2\right)}$$

$$= \frac{80\times 10^3}{350\times 10^3\pi\left[(0.16)^2 - (0.10)^2\right]}$$

$$= 4.66$$

$$\boxed{n = 5}$$

Ex. 6.12 **A conical pivot with the angle of the cone as 100° supports a load of 18 KN. The external radius is 2.5 times the internal radius. The shaft rotates at 150 RPM. If the intensity of pressure is to be 300 KN/m^2 and coefficient of friction as 0.05, what is the power lost in working against the friction?**

Solution: Given data:

2α=100°, W = 18KN, R_1 = 2.5R_2, N = 150rpm, P = 300KN/m^2, μ = 0.05

To be Calculated:

Power lost in overcoming the friction

$$\text{Let } P = \frac{W}{\pi\left(R_1^2 - R_2^2\right)}$$

$$300\times10^3 = \frac{18\times10^3}{\pi\left[\left(2.5R_2^2\right) - R_2^2\right]}$$

$$\therefore\ R_1 = 0.0603 \text{ m}$$
$$\therefore\ R_2 = 0.1508 \text{ m}$$

$$\text{Let } T = \frac{2}{3}\mu W \operatorname{cosec}\alpha\left(\frac{R_1^3 - R_2^3}{R_1^2 - R_2^2}\right)$$

$$= \frac{2}{3}\times0.05\times18\times10^3\times\operatorname{cosec}50^\circ\left[\frac{(0.1508)^3 - (0.0603)^3}{(0.1508)^2 - (0.0603)^2}\right]$$

$$= \frac{783.244\times3.2100\times10^{-3}}{0.01910}$$

$$T = 131.63 \text{ N.m}$$

Power lost in friction (P_f),

$$P_f = T.\omega$$
$$= T\times\frac{2\pi N}{60}$$
$$= 131.63\times\frac{2\pi\times150}{60}$$
$$= 2067 \text{ W}$$
$$\boxed{P_f = 2.067 \text{ kW}}$$

Ex. 6.13 **A thrust bearing of a propeller shaft consists of a number of collars. The shaft is of 400 mm diameter and rotates at a speed of 90 rpm. The thrust on the shaft is 300 KN. If the intensity of pressure is to be 200 KN/m^2 and the coefficient of friction is 0.06. Determine the external diameter of the collar and the number of collars. The power lost in friction is not to exceed 48 KW.**

Solution: Given data:

R_2 = 200 mm, N = 90 rpm, W = 300 KN, P = 200 KN/m^2 = 0.2 N/mm^2, μ = 0.06, P_f = 48 kW

To be Calculated:

External diameter of the collar and the number of collars

$$\text{Let } P_f = \frac{2\pi NT}{60}$$

$$48\times10^3 = \frac{2\pi\times90\times T}{60} \Rightarrow \therefore T = 5093 \text{ N.m}$$

$$\text{Let } T = \frac{2}{3}\mu W\left(\frac{R_1^3 - R_2^3}{R_1^2 - R_2^2}\right)$$

$$= \frac{2}{3}\mu W\left[\frac{(R_1 - R_2)(R_1^2 + R_2^2 + R_1R_2)}{(R_1 - R_2)(R_1 + R_2)}\right]$$

$$5093\times10^3 = \frac{2}{3}\times0.06\times300\times10^3\left[\frac{(R_1^2 + 200^2 + 200R_1)}{(R_1 + 200)}\right]$$

$$R_1 = 352 \text{ mm} \quad \text{(Taking positive sign only)}$$

$$P = \frac{W}{n\pi(R_1^2 - R_2^2)}$$

$$\therefore\ n = \frac{W}{P\pi(R_1^2 - R_2^2)}$$

$$= \frac{300\times10^3}{0.2\pi\left[352^2 - 200^2\right]}$$

$$= 5.69$$

$$\boxed{n = 6}$$

Ex. 6.14 **The inner and the outer radii of a plate clutch are 40 mm and 80 mm respectively. Determine the maximum, minimum and the average pressure when the axial force is 3 KN.**

Solution: Maximum pressure will be at inner radius,

$$F = 2\pi P_i R_i (R_o - R_i) \qquad \{F = 2\pi Pr(R_o - R_i)\}$$

$$3000 = 2\pi P_i\, 0.04(0.08 - 0.04)$$

$$P_{i(max)} = 298.4\times10^3 \text{ N/m}^2$$

$$P_{i(max)} = 298.4 \text{ kN/m}^2$$

Minimum pressure will be at outer radius,

$$F = 2\pi P_o R_o (R_o - R_i)$$

$$3000 = 2\pi P_o\, 0.08(0.08 - 0.04)$$

$$P_{o(min)} = 149.2\times10^3 \text{ N/m}^2$$

$$P_{o(min)} = 149.2 \text{ kN/m}^2$$

$$\text{Average pressure} = \frac{\text{Total Normal Force}}{\text{C/S Area}} = \frac{3000}{\pi(0.08^2 - 0.04^2)} = 189.9\times10^3 \text{ N/m}^2$$

Ex. 6.15 **A single plate clutch is required to transmit 8 KW at 1000 RPM. The axial pressure is limited to 70 KN/m². The mean radius of the plate is 4.5 times the radial width of the friction surface. If both sides of the plate are effective and the coefficient of friction is 0.25. find the inner and the outer radii of the plate and the mean radius and width of the friction lining.**

Solution: Given data:

$P = 8KW$, $N = 1000$ rpm, $P_i = 70\ KN/m^2$, $\mu = 0.25 = \tan\varphi$

To be Calculated:

Inner and the outer radii of the plate and the mean radius, Width of the friction lining

$$\text{Let } R_m = \frac{R_o + R_i}{2} = 4.5(R_o - R_i) = 4.5b$$

$$R_o + R_i = 9(R_o - R_i)$$

$$8R_o = 10R_i$$

$$R_o = 1.25R_i$$

In the clutch, it is safer to apply uniform wear theory and maximum pressure is at the inner radius, i.e., $P_i = 70\ KN/m^2$

$$P = T\omega$$

$$8\times10^3 = T\times\frac{2\pi\times1000}{60}$$

$$\therefore \quad T = 76.39\ \text{N.m}$$

$$\text{Torque, } T = \frac{1}{2}\mu F(R_o + R_i)n \qquad \begin{Bmatrix} n = \text{Number of surfaces} = 2 \\ F = 2\pi P_i R_i (R_o - R_i) \end{Bmatrix}$$

$$= \frac{\mu}{2}\left[2\pi P_i R_i (R_o - R_i)\right](R_o + R_i)2$$

$$76.39 = \mu\left[2\pi\times70\times10^3\times R_i(1.25R_i - R_i)\right](1.25R_i + R_i)$$

$$76.39 = 0.25\times2\pi\times70\times10^3\times0.5625R_i^3$$

$$\boxed{\therefore\ R_i = 0.1073\ \text{m}}$$

$$\boxed{\therefore\ R_o = 0.1341\ \text{m}}$$

$$R_m = 4.5(R_o - R_i)$$

$$= 4.5(0.1341 - 0.1073)$$

$$\boxed{\therefore\ R_m = 0.1207\ \text{m}}$$

$$\text{Width } w = \frac{R_m}{4.5} = \frac{0.1207}{4.5}$$

$$= 0.0268\ \text{m}$$

$$\boxed{w = 26.8\ \text{mm}}$$

Ex. 6.16 **A single plate clutch transmits 25 KW at 900 RPM. The maximum pressure intensity between the plates is 85 KN/m². The outer diameter of the plate is 360 mm. Both sides of the plate are effective and the coefficient of friction is 0.25. Determine**

i. The inner diameter of the plate

ii. Axial force to engage the clutch.

Solution: Given data:

$P = 25$ KW, $N = 900$ rpm, $P_i = 85\ KN/m^2$, $\mu = 0.25 = \tan\varphi$, $R_o = 0.180$ m

To be Calculated:

Inner diameter of the plate and Axial force to engage the clutch

Let $P = T\omega$

$$25 \times 10^3 = T \times \frac{2\pi \times 900}{60}$$
$$\therefore \quad T = 265.25 \text{ N.m}$$

$$\text{Torque, } T = \frac{1}{2}\mu F (R_o + R_i) n \qquad \begin{Bmatrix} n = \text{Number of surfaces} = 2 \\ F = 2\pi P_i R_i (R_o - R_i) \end{Bmatrix}$$

$$= \frac{\mu}{2}\left[2\pi P_i R_i (R_o - R_i)\right](R_o + R_i) n$$
$$265.25 = 0.25 \times \pi \times 85 \times 10^3 \times R_i (0.18 - R_i)(0.18 + R_i) 2$$
$$R_i = 0.1315 \text{ m}$$
$$\boxed{\therefore\ R_i = 131.5 \text{ mm}}$$

$$\text{Axial force, } F = 2\pi P_i R_i (R_o - R_i) n$$
$$= 2\pi \times 85 \times 10^3 \times 0.1315 (0.18 - 0.1315) 2$$
$$= 6812 \text{ N}$$
$$\boxed{F = 6.812 \text{ kN}}$$

Ex. 6.17 **A friction clutch is used to rotate a machine from a shaft rotating at a uniform speed of 250 rpm. The disc type clutch has both of its sides effective, the coefficient of friction is 0.3. The outer and the inner diameters of the friction plates are 200 mm and 120 mm respectively. Assuming uniform wear of the clutch, the intensity of pressure is not to be more than 100 KN/m^2. If the moment of inertia of the rotating parts of the machine is 6.5 kg.m^2. Determine the time to attain the full speed by the machine and the energy loss in the slipping of the clutch. What will be the intensity of pressure if the condition of uniform pressure of the clutch is considered? Also, determine the ratio of power transmitted with uniform wear to that with uniform pressure.**

Solution: Given data:

N = 250 rpm, μ = 0.3, R_o = 0.100 m, R_i = 0.060 m, P_i = 100 KN/m^2, R_o = 0.180 m, I =65 Kg.m^2

(i) Time to attain full speed in uniform wear condition

$$F = 2\pi P_i R_i (R_o - R_i)$$
$$= 2\pi \times 100 \times 10^3 \times 0.06 (0.1 - 0.06)$$
$$= 1508 \text{ N}$$

$$\text{Torque } T = \frac{\mu F}{2}(R_o - R_i) n$$
$$= \frac{0.3 \times 1508}{2}(0.1 - 0.06) 2$$
$$= 72.38 \text{ N.m}$$

$$\text{Power} = T.\omega = T\times\frac{2\pi N}{60} = 72.38\times\frac{2\pi\times 250}{60} = 1895 \text{ W}$$

$$\text{Also Torque } \; T = I.\alpha \qquad \{\alpha = \text{Angulr acceleration}\}$$

$$72.38 = 6.5\times\alpha$$

$$\therefore\; \alpha = 11.135 \text{ rad}/\text{s}^2$$

$$\text{Let } \alpha = \frac{\omega}{t} = \frac{2\pi N}{60t}$$

$$11.135 = \frac{2\pi 2\times 50}{60\times t}$$

$$\boxed{\therefore\; t = 2.35 \text{ s}}$$

(ii) The energy lost during the slipping period

Angle turn by driving shaft (θ_1),

$$\theta_1 = \omega t = \frac{2\pi N}{60} t = \frac{2\pi\times 250}{60}\times 2.35 = 61.5 \text{ rad}$$

Angle turn by driving shaft (θ_2),

$$\theta_2 = \omega_o t + \frac{1}{2}\alpha t^2 = 0 + \frac{1}{2}\times 11.135\times 2.35^2 = 30.75 \text{ rad}$$

The energy lost during the slipping period $= T\left(\theta_1 - \theta_2\right)$

$$= 72.38\left(61.5 - 30.75\right)$$
$$= 2226 \text{ N.m}$$
$$= 2.226 \text{ kN.m}$$

(ii) Pressure in uniform pressure condition and ratio

$$P = \frac{F}{\pi\left(R_o^2 - R_i^2\right)} = \frac{1508}{\pi\left(0.1^2 - 0.06^2\right)} = 75000 \text{ N}/\text{m}^2 = 75 \text{ kN}/\text{m}^2$$

$$\text{Also } T = \frac{2}{3}\mu F\left(\frac{R_o^3 - R_i^3}{R_o^2 - R_i^2}\right) n$$

$$= \frac{2}{3}\times 0.3\times 1508\left(\frac{0.1^3 - 0.06^3}{0.1^2 - 0.06^2}\right)\times 2 \qquad \{n = \text{Number of surface} = 2\}$$

$$= 73.89 \text{ N.m}$$

$$\text{Power} = T.\omega = T\times\frac{2\pi N}{60} = 73.89\times\frac{2\pi\times 250}{60} = 1934 \text{ W}$$

$$\text{Now } \frac{\text{Power with uniform wear}}{\text{Power with uniform pressure}} = \frac{1895}{1934} = \boxed{0.98}$$

Ex. 6.18 **A multi-plate disc clutch transmits 55 KW of power at 1800 RPM. The coefficient of friction for the friction surface is 0.1. The axial intensity of the pressure is not to exceed 160**

	KN/m², the internal radius is 80 mm and is 0.7 times the external radius. Find the number of the plates needed to transmit the required torque.

Solution: Given data:

P = 55 KW, N = 1800 rpm, μ = 0.1, P_i = 160 KN/m², R_i = 0.080 m, $R_i = 0.7\ R_o$, $\therefore R_o$ = 0.1143 m

To be Calculated:

Number of the plates

Assuming uniform wear condition,

$$\begin{aligned} F &= 2\pi P_i R_i (R_o - R_i) \\ &= 2\pi \times 160 \times 10^3 \times 0.08(0.1143 - 0.08) \\ &= 2759\ \text{N} \end{aligned}$$

$$\begin{aligned} T &= \frac{1}{2}\mu F(R_o + R_i) \\ &= \frac{1}{2} \times 0.1 \times 2759(0.1143 + 0.08) \\ &= 26.78\ \text{N.m / surface} \end{aligned}$$

Total torque transmitted,

$$T = \frac{P}{\omega} = \frac{55 \times 10^3}{\frac{2\pi \times 1800}{60}} = 291.8\ \text{N.m}$$

Number of friction surface required $n = \frac{291.8}{26.78} = 10.9 \cong \boxed{11}$

Note: There will be 12 plates. 6 plates (rings) revolve with the driving or engine shaft and the other 6 with the driven shaft.

Ex. 6.19	**A multi-plate disc clutch transmits 30 KW of power at 1800 RPM. It has four discs on the driving shaft and three discs on the driving shaft and three discs on the driven shaft providing six pairs of contact surfaces. The external and internal diameters of the contact surface are 200 mm and 100 mm respectively.** **Assuming the clutch to be new, find the total spring load pressing the plates together. The coefficient of friction is 0.3.** **Also, determine the maximum power transmitted when the contact surface has worn away by 0.4 mm. There are 8 springs and the stiffness of each spring is 15 KN/m.**

Solution: Given data:

P = 30KW, N = 1800rpmμ = 0.3, contact surface = 6 pair = 12, R_o = 100mm, R_i = 50mm,

8 springs with stiffness k = 15KN/m = 15N/mm

To be Calculated:

The total spring load and the maximum power transmitted

New clutch has uniform pressure distribution

$$\text{Let } P = T\omega$$

$$30 \times 10^3 = T \times \frac{2\pi \times 1800}{60} \quad \Rightarrow \quad \therefore\ T = 159.15 \text{ N.m}$$

Torque transmitted by a new clutch,

$$T = \frac{2}{3}\mu F\left(\frac{R_o^3 - R_i^3}{R_o^2 - R_i^2}\right)n$$

$$159.15 = \frac{2}{3} \times 0.3 \times F\left(\frac{100^3 - 50^3}{100^2 - 50^2}\right)$$

$$\boxed{\therefore\ F = 1137 \text{ N}}$$

When the surfaces are worn out,

Total wear = Number of surfaces x wear on each surface

= 12 x 0.4 = 4.8 mm

Total spring stiffness = 8 x 15 = 120 N/mm

So reduction in spring force = 120 x 4.8 = 576 N

New axial load = 1137–576 = 561 N

Let torque (wear on clutch and uniform wear theory)

$$T = \frac{1}{2}\mu F(R_o + R_i)n = \frac{1}{2} \times 0.3 \times 561(0.1 + 0.05)6$$
$$= 75.735 \text{ N.m}$$

Maximum power transmitted,

$$P_{max} = T.\omega$$
$$= 75.735 \times \frac{2\pi \times 1800}{60}$$
$$\boxed{P_{max} = 14.275 \text{ kW}}$$

Ex. 6.20 **A torque of 350 N.m is transmitted through a cone clutch having a mean diameter of 300 mm and a semi cone angle of 15°. The maximum normal pressure at the mean radius is 150 KN/m². The coefficient of friction 0.3. Calculate the width of the contact surface. Also, find the axial force to engage the clutch.**

Solution: Given data:

T = 350 Nm, D_m = 300mm ∴ R_m =150 mm, α = 15°, P_{max} = 150 KN/m², μ = 0.3

To be Calculated:

The width of the contact surface and the axial force to engage the clutch

For cone clutch,

$$T = \mu F_n R_m$$
$$350 = 0.3 \times F_n \times 0.150$$
$$\therefore\ F_n = 7778 \text{ N}$$

$$\text{and } F_n = 2\pi P_{max} R_m b$$

$$7778 = 2\pi \times 150 \times 10^3 \times 0.150 \times b$$

$$\therefore\ b = 0.055 \text{ m}$$

$$\boxed{\therefore\ b = 55 \text{ mm}}$$

Axial force,

$$F = F_n \sin\alpha$$

$$= 7778 \times \sin 15°$$

$$\boxed{F = 2012.4 \text{ N}}$$

Ex. 6.21 **The semi cone angle of a cone clutch is 12.5° and the contact surfaces have a mean diameter of 80 mm. coefficient of friction 0.32. What is the minimum torque required to produce slipping of the clutch for an axial force of 200 N? If the clutch is used to connect an electric motor with a stationary flywheel, determine the time needed to attain the full speed and the energy lost during slipping. Motor speed is 900 rpm and the moment of inertia of the flywheel is 0.4 KG.m².**

Solution: Given data:

α = 12.5°, R_m =0.04 m, μ = 0.32, F = 200 N, N = 900 rpm, I = 0.4 KG.m^2

To be Calculated:

Minimum torque required and time needed to attain the full speed

Let Torque

$$T = I.\alpha_a \qquad \{\alpha_a = \text{Angular acceleration}\}$$

$$\mu F_n R_m = I.\alpha_a$$

$$\mu \frac{F}{\sin\alpha} R_m = I.\alpha_a$$

$$0.32 \times \frac{200}{\sin 12.5°} \times 0.04 = 0.4 \times \alpha_a$$

$$\therefore\ \alpha_a = 29.57 \text{ rad/s}^2$$

$$\text{Also Torque } T = I.\alpha_a$$

$$= 0.4 \times 29.57$$

$$\boxed{T = 11.828 \text{ N.m}}$$

$$\alpha_a = \frac{\omega}{t}$$

$$29.57 = \frac{\frac{2\pi \times 900}{60}}{t} \Rightarrow \boxed{\therefore t = 3.187 \text{ s}}$$

During slipping period

Angle turned by the driving shaft, $\theta_1 = \omega.t = \frac{2\pi \times 900}{60} \times 3.187 = 300.4 \text{ rad}$

Angle turned by the driven shaft, $\theta_2 = \omega_0 t + \frac{1}{2}\alpha_a t^2 = 0 + \frac{1}{2} \times 29.57 \times (3.187)^2 = 150.2 \text{ rad}$

The energy lost in friction $= T(\theta_1 - \theta_2) = 11.828(300.4 - 150.2) = \boxed{1776.5 \text{ N.m}}$

Ex. 6.22	**A cone clutch with a semi cone angle of 15° transmits 10 KW at 600 RPM. The normal pressure intensity between the surface in contact is not to exceed 100 KN/m². The width of the friction surface is half of the mean diameter. Assume μ=0.25. Determine the** **i. Outer and inner diameters of the plate** **ii. Width of the cone face** **iii. Axial force to engage the clutch**

Solution: Given data:

α = 15°, P=10 KW, N = 600 rpm, P_i = 100 KN/m², b = $D_m/2$ = R_m, μ = 0.25

To be Calculated:

R_o, R_i, b and F

(i) Let b = R_m

$$\frac{R_o - R_i}{\sin\alpha} = \frac{R_o + R_i}{2}$$

$$\therefore\ R_o - R_i = \sin 15° \left(\frac{R_o + R_i}{2}\right)$$

$$\therefore\ R_o - R_i = 0.129R_o + 0.129R_i$$

$$\therefore\ R_o = 0.129R_o + 0.129R_i$$

Now Power, P = T.ω

$$10 \times 10^3 = T \times \frac{2\pi \times 600}{60} \quad \Rightarrow \therefore T = 159 \text{ N.m}$$

Applying uniform wear theory (Intensity of pressure is maximum at inner radius)

$$T = \mu F_n \frac{(R_o + R_i)}{2}$$

$$= \mu \frac{F}{\sin\alpha} \frac{(R_o + R_i)}{2}$$

$$= \frac{\mu}{2\sin\alpha}\left[2\pi.P_i.R_i(R_o - R_i)\right](R_o + R_i)$$

$$= \frac{\mu\pi P_i}{\sin\alpha} R_i\left(R_o^2 - R_i^2\right)$$

$$159 = \frac{0.25 \times \pi \times 100 \times 10^3}{\sin 15°} R_i\left[(1.296R_i)^2 - R_i^2\right]$$

$$\boxed{\therefore R_i = 0.0917 \text{ m} = 91.7 \text{ mm}}$$

$$\boxed{\therefore R_o = 91.7 \times 1.296 = 118.8 \text{ mm}}$$

(ii) Width of cone face (b),

$$b = \frac{R_o - R_i}{\sin\alpha} = \frac{118.8 - 91.7}{\sin 15°} = 105 \text{ mm}$$

(iii) Axial force (F),

$$F = 2\pi P_i R_i (R_o - R_i)$$
$$= 2\pi \times 100 \times 10^3 \times 0.0917(0.1188 - 0.0917)$$
$$\therefore\ F = 1561\ \text{N}$$

Ex. 6.23	**A centrifugal clutch transmits 20 KW of the power at 750 RPM. The engagement of the clutch commences at 70% of the running speed. The inside diameter of the drum is 200 mm and the distance of the center of mass of each shoe is 40 mm from the contact surface. Assume μ=0.25 and spring stiffness 150 KN/m. Determine:** **i. Mass of each shoe** **ii. Net force exerted by each shoe on the drum surface** **iii. Power transmitted when the shoe is worn 2 mm and is not readjusted.**

Solution: Given data:

n = 4, P=10 KW, N = 750 rpm, P_i = 100 KN/m^2, μ = 0.25, R = 200 mm, r = 0.2–0.04 = 0.16 m

To be Calculated:

m, F and P

(i) Mass of shoe (m)

$$\omega = \frac{2\pi \times 750}{60} = 78.53\ \text{rad/s} \Rightarrow \omega' = 0.7 \times 78.5 = 55\ \text{rad/s}$$
$$P = T.\omega$$
$$20 \times 10^3 = T \times 78.53 \quad \Rightarrow \quad T = 254.8\ \text{N.m}$$

The total frictional torque acting $= \mu m r (\omega^2 - \omega'^2).R.n$

$$254.8 = 0.25 \times m \times 0.16(78.5^2 - 55^2)0.2 \times 4$$
$$\boxed{\therefore m = 2.538\ \text{kg}}$$

(ii) Net force exerted by each shoe on the drum surface

$$= mr(\omega^2 - \omega'^2)$$
$$= 2.538 \times 0.16(78.5^2 - 55^2)$$
$$= 1274\ \text{N}$$

(iii) Spring force exerted by each spring = 2.538 x 0.16 x 55^2

= 1228.4 N

When the shoe wears 2 mm, each shoe has to move forward by 2 mm. This increases the distance of the center of mass of the shoe from 160 mm to 162 mm. Also, the spring force is increased due to its additional displacement of 2 mm.

Additional spring force = Stiffness x Displacement

= 150 x 10^3 x 0.002

= 300 N

Total spring force = 1228.4 + 300 = 1528.4 N

Net force exerted by each shoe on the drum surface

$$= mr\omega^2 - 1528.4$$
$$= 2.538 \times 0.162 \times 78.5^2 - 1528.4$$
$$= 1005.2 \text{ N}$$

Total frictional torque acting $= \mu F.R.n$

$$T = 0.25 \times 1005.2 \times 0.2 \times 4 = 201.04 \text{ N.m}$$

$$P = T\omega$$
$$= 201.04 \times 78.5$$
$$= 15782 \text{ W}$$
$$\boxed{P = 15.782 \text{ kW}}$$

Ex. 6.24 **A brake as shown in the below figure is fitted with a C.I. brake shoe.**

The braking torsional moment = 360 N.m

The drum diameter = 300 mm

The coefficient of friction = 0.3

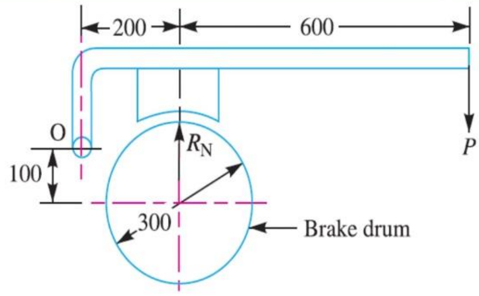
Find:

(i) force P for counter-clockwise rotation

(ii) force P for clockwise rotation

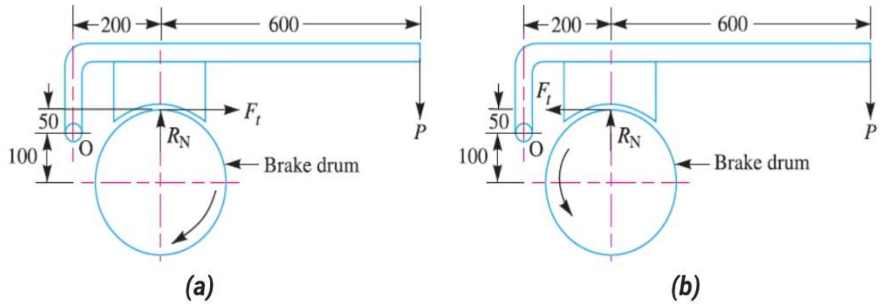
(iii) where must pivot be placed to make brake self-locking with clockwise rotation

Solution:



Given data:

T_b = 360 N.m, d = 300 mm, μ = 0.3



For the clockwise rotation of the brake drum, the frictional force or the tangential force (F_t) acting at the contact surfaces is shown in Fig. (a)

Braking torque, $T_b = F_t \times r$

$= \mu R_N \times r$

$360 = 0.3 \times R_N \times 0.15$

Normal force, R_N = 8000 N

Now taking moments about the fulcrum O,

$$P\ (600 + 200) + F_t \times 50 = R_N \times 200$$

$$P \times 800 + (0.3 \times 8000)\ 50 = 8000 \times 200$$

$$P \times 800 = 1480 \times 10^3$$

$$P = 1850 \text{ N}$$

For the counterclockwise rotation of the brake drum, the frictional force or the tangential force (F_t) acting at the contact surfaces is shown in Fig. (b)

Now taking moments about the fulcrum O,

$$P\ (600 + 200) = F_t \times 50 + R_N \times 200$$

$$P \times 800 = (0.3 \times 8000)\ 50 + 8000 \times 200$$

$$P \times 800 = 1720 \times 10^3$$

$$P = 2150 \text{ N}$$

Location of the pivot or fulcrum to make the brake self-locking

The clockwise rotation of the brake drum is shown in Fig. (a). Let x be the distance of the pivot or fulcrum O from the line of action of the tangential force (F_t). Taking moments about the fulcrum O,

$$P\ (600 + 200) + F_t\ (x) - R_N \times 200 = 0$$

In order to make the brake self-locking, $F_t\ (x)$ must be equal to $R_N \times 200$, so that the force P is zero.

$$F_t\ (x) = R_N \times 200$$

$$2400\ (x) = 8000 \times 200$$

$$x = 667 \text{ mm}$$

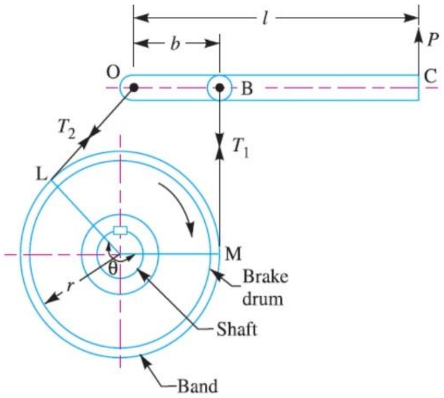
Ex. 6.25 **A simple band brake is shown in Figure below is applied to a shaft carrying a flywheel of mass 400 kg. The radius of gyration of the flywheel is 450 mm and runs at 300 RPM. The coefficient of friction is 0.2 and the brake drum diameter is 240 mm. Take b = 120 mm, l = 420 mm, θ = 210°, then find out the followings:**

(i) The torque applied due to hand load of 100 N

(ii) The number of turns of the flywheel before it is brought to rest

The time required to bring it to rest from the moment of the application of the brake.

Solution:



Given data:

$m = 400$ kg, $k = 450$ mm, $N = 300$ rpm, $\mu = 0.2$, $d = 240$ mm, $\theta = 210° = 3.665$ rad

Taking moments about the fulcrum O,

$$T_1 \times b = P \times l$$

$$T_1 \times 0.12 = P \times 0.42$$

$$T_1 \times 0.12 = 100 \times 0.42$$

$$T_1 = 350 \text{ N}$$

Now,

$$\frac{T_1}{T_2} = e^{\mu\theta} = e^{0.2 \times 3.665}$$

$$\frac{T_1}{T_2} = 2.081$$

$$T_2 = \frac{T_1}{2.081} = \frac{350}{2.081}$$

$$T_2 = 168.188 \text{ N}$$

Braking torque,

$$T_b = F_t \times r$$

$$= (T_1 - T_2) \times r$$

$$= (350 - 168.188) \times 0.12$$

$$= 21.817 \text{ N.m}$$

Work done against friction due to absorption of K.E.

$$F_t \times S = \frac{1}{2} m v^2$$

$$F_t \times S = \frac{1}{2} m\,(r\omega)^2$$

$$S = \frac{m}{F_t \times 2}\left(\frac{\pi d N}{60}\right)^2$$

$$S = \frac{400}{181.81 \times 2}\left(\frac{\pi \times 0.24 \times 300}{60}\right)^2$$

$$= 15.63 \text{ m}$$

$$S = \pi D \times n$$

$$\text{No. of turns, } n = \frac{S}{\pi D}$$

$$= \frac{15.63}{\pi \times 0.24}$$

$= 20.73$

$T_b = I\,\alpha$

$= (mk^2)\,\alpha$

$21.817 = (400 \times 0.450^2)\,\alpha$

$\alpha = 0.269$

$\omega = \omega_o + \alpha t$

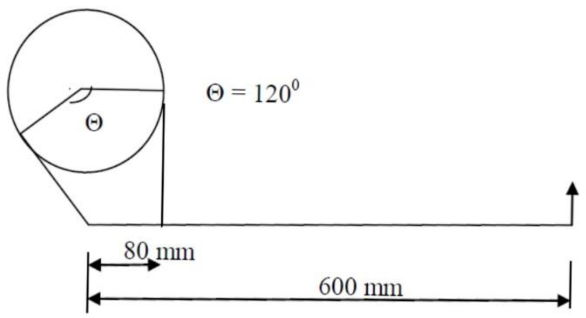
$$0 = \left(\frac{2\pi N}{60}\right) + \alpha t$$

$$0 = \left(\frac{2\pi \times 300}{60}\right) + 0.269 \times t$$

$t = 116.78$ sec

Ex. 6.26 **A band brake shown in the Figure below is used to balance a torque of 980 N-m at the drum shaft. The drum diameter is 400 mm (rotating in the clockwise direction) and the allowable pressure between lining and drum is 0.5 MPa. The coefficient of friction is 0.25. Design the steel band, shaft, brake lever, and fulcrum pin, if all these elements are made from steel having permissible tensile stress 70 MPa and shear stress 50 MPa.**

Solution:



Given data:

$T_b = 980$ N.m, $d = 400$ mm, $p = 0.5$ MPa, $\mu = 0.25$, $\sigma_t = 70$ MPa, $\tau = 50$ MPa,

$\theta = 120° = 2.094$ rad

Braking torque,

$T_b = F_t \times r$

$= (T_1 - T_2) \times r$

$980 \times 10^3 = (T_1 - T_2) \times 200$

$T_1 - T_2 = 4.9 \times 10^3$

Now,

$$\frac{T_1}{T_2} = e^{\mu\theta} = e^{0.25 \times 2.094}$$

$$\frac{T_1}{T_2} = 1.688$$

$1.688\ T_2 - T_2 = 4.9 \times 10^3$

$$T_2 = 7122.09 \text{ N}$$

$$T_1 = 12022.09 \text{ N}$$

Taking moments about the fulcrum O,

$$T_2 \times 80 = P \times 600$$

$$7122.09 \times 80 = P \times 600$$

$$P = 949.612 \text{ N}$$

Design of Shaft

Since the shaft has to transmit torque equal to the braking torque

$$T_b = \frac{\pi}{16} d_S^3\ \tau$$

$$980 \times 10^3 = \frac{\pi}{16} d_S^3 \times 50$$

$$d_S = 46.388 \text{ mm}$$

Shaft diameter, $d_S \approx 50$ mm

Design of Lever

t_1 = thickness of the lever

B = width of the lever

Maximum bending moment at fulcrum O due to force P,

$$M = P \times l = 949.612 \times 600$$

$$= 569767.2 \text{ N.mm}$$

Section modulus,

$$Z = \frac{1}{6} t_1 \cdot B^2 = \frac{1}{6} t_1 \cdot (2t_1)^2 \qquad \text{(Assuming } B = 2t_1\text{)}$$

$$= 0.67\ t_1^3 \text{ mm}^3$$

$$\text{Bending stress} = \frac{M}{Z}$$

$$70 = \frac{569767.2}{0.67\ t_1^3}$$

$$t_1 = 22.988 \text{ mm} \approx 23 \text{ mm}$$

$B = 2t_1$

$= 46$ mm

Design of Pins

d_1 = diameter of pins

l_1 = length of pins = $1.25\ d_1$

The pins are designed for maximum tension in the band (i.e. T_1 = 12022.09 N)

Considering bearing of the pins, maximum tension (T_1),

$$12022.09 = d_1.l_1.p = d_1 \times 1.25\, d_1 \times 0.5$$

$$d_1 = 138.69 \text{ mm}$$

$$d_1 \approx 140 \text{ mm}$$

$l_1 = 1.25\, d_1 = 1.25 \times 140$

= 175 mm

Check the pin for induced shearing stress. Since the pin is in double shear, therefore maximum tension (T_1),

$$12022.09 = 2\times\frac{\pi}{4}(d_1^{\,2})\tau = 2\times\frac{\pi}{4}(140)^2\tau$$

$$= 30787.6\tau$$

$$\tau = \frac{12022.09}{30787.6}$$

= 0.39 MPa

This induced stress is quite within permissible limits.

The pin may be checked for induced bending stress.

$$\text{Maximum bending moment, } M = \frac{5}{24}\times W.l_1 = \frac{5}{24}\times 12022.09\times 175 \qquad (W = T_1)$$

= 438305.36 N-mm

$$\text{section modulus, } Z = \frac{\pi}{32}(d_1^{\,3}) = \frac{\pi}{32}(140)^3$$

$$= 269391.57 \text{ mm}^3$$

$$\text{Bending stress} = \frac{M}{Z} = \frac{438305.36}{269391.57}$$

=1.627 MPa

The induced bending stress is within safe limits of 70 MPa.

The lever has an eye hole for the pin and connectors at band have a forked end.

$$\text{Thickness of each eye, } t_2 = \frac{l_1}{2} = \frac{175}{2} = 87.5 \text{ mm}$$

Outer diameter of the eye, D= $2d_1$ = 2 x 140 = 280 mm

A clearance of 1.5 mm is provided on either side of the lever in the fork.

A brass bush of 3 mm thickness may be provided in the eye of the lever.

Diameter of hole in the lever = d_1 + 2 x 3 = 140 + 2 x 3 = 146 mm

The boss is made at pin joints whose outer diameter is taken equal to twice the diameter of the pin and length equal to the length of the pin.

The inner diameter of the boss is equal to the diameter of the hole in the lever.

Outer diameter of the boss = $2d_1 = 2(140) = 280$ mm

Length of the boss = $l_1 = 175$ mm

Check the bending stress induced in the lever at the fulcrum.

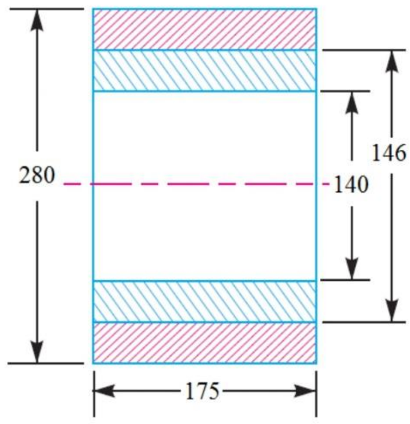
Maximum bending moment at the fulcrum,

M = P x l = 949.612 x 600

= 569767.2 N-mm

$$\text{Section modulus, } Z = \frac{\frac{1}{12}\times 175\left[280^3 - 146^3\right]}{280/2}$$

= 1962485.83 mm^3



All dimensions in mm.

$$\text{Bending stress} = \frac{M}{Z} = \frac{569767.2}{1962485.83} = 0.29 \text{ MPa}$$

The induced bending stress is within safe limits of 70 MPa.

Ex. 6.27 **A differential band brake has a drum with a diameter of 800 mm. The two ends of the band are fixed to the pins on the opposite sides of the fulcrum of the lever at distances of 40 mm and 200 mm from the fulcrum. The angle of contact is 270° and the coefficient of friction is 0.2. Determine the brake torque when a force of 600 N is applied to the lever at a distance of 800 mm from the fulcrum.**

Solution: Given data:

Differential Band Brake

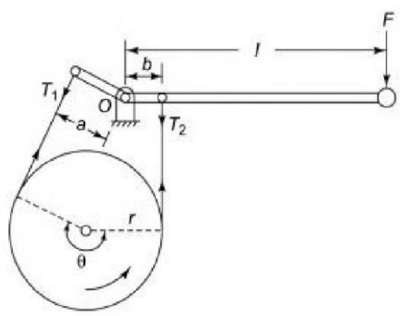
$d = 800 \text{ mm} \rightarrow r = 400 \text{ mm}$

$\theta = 270^\circ$

$\mu = 0.2$

Assuming a = 200 mm, b = 40 mm, i.e., a > b, F must act downward direction to apply the brake.

(a) Anticlockwise Rotation



Taking moment about fulcrum O,

$F \cdot l - T_1 a + T_2 b = 0$

$600 \times 800 - (2.57\,T_2)\,200 + T_2\,(40) = 0$

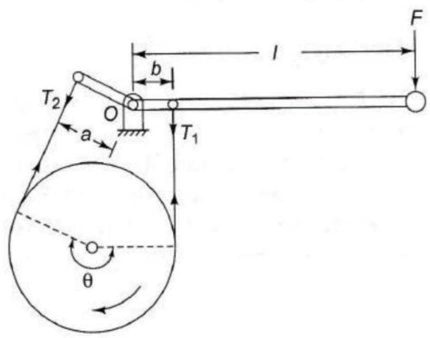
$T_2 = 1012.7\ \text{N}$ and

$T_1 = 2602.5\ \text{N}$

$$\left\{ \begin{aligned} &\frac{T_1}{T_2} = e^{\mu\theta} = e^{\mu \times 270 \times \frac{\pi}{180}} \\ &\therefore \frac{T_1}{T_2} = 2.57 \end{aligned} \right\}$$

Braking Torque, $T_B = (T_1 - T_2)\,r = (2602.5 - 1012.7)\,0.4 = 635.92\ \text{N} \cdot \text{m}$

(b) Clockwise Rotation



Taking moment about fulcrum O,

$F \cdot l + T_1 b - T_2 a = 0$

$600 \times 800 + 2.57\,T_2\,(40) - T_2\,(200) = 0$

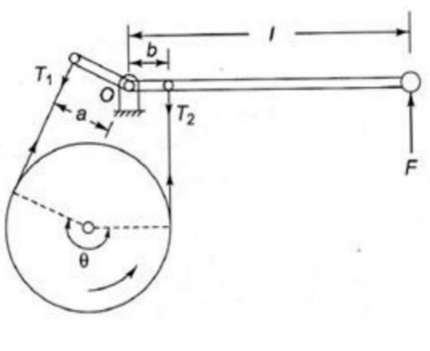
$T_2 = 4938\ \text{N}$ and

$T_1 = 2.57\,T_2 = 12691\ \text{N}$

Braking Torque, $T_B = (T_1 - T_2)\,r = (12691 - 4938)\,0.4 = 3101\ \text{N} \cdot \text{m}$

<u>Assuming a = 40 mm, b = 200 mm, i.e., a < b, F must act upward to apply the brake.</u>

(a) Anticlockwise Rotation



Taking moment about fulcrum O,

$$F \cdot l + T_1 a - T_2 b = 0$$

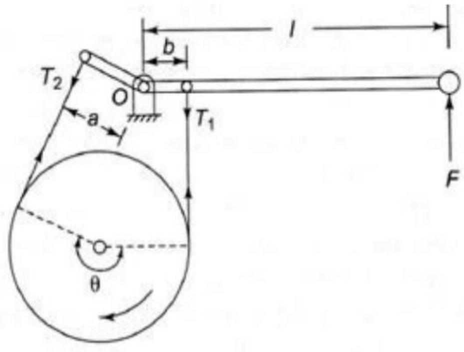
$$600 \times 800 + 2.57\, T_2 (40) - T_2\,(200) = 0$$

$T_2 = 4938\,\text{N}$ and

$T_1 = 12691\,\text{N}$

Braking Torque, $T_B = (T_1 - T_2) r = (12691 - 4938)\, 0.4 = 3101\,\text{N}\cdot\text{m}$

(b) Clockwise Rotation



Taking moment about fulcrum O,

$$F \cdot l + T_2 a - T_1 b = 0$$

$$600 \times 800 + T_2\,(40) - 2.57\, T_2\,(200) = 0$$

$T_2 = 1012.7\,\text{N}$ and

$T_1 = 2602.5\,\text{N}$

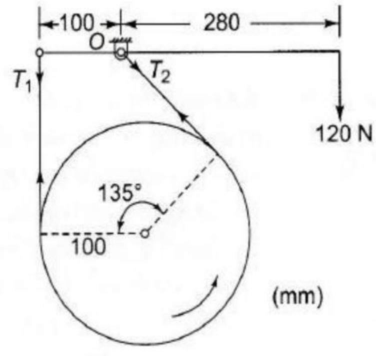
Braking Torque, $T_B = (T_1 - T_2) r = (1012.7 - 2602.5)\, 0.4 = 636\,\text{N}\cdot\text{m}$

Note: The above results show that the effectiveness of the brake in one direction of rotation is equal to the effectiveness in the other direction if the distance of the pins on the opposite sides of the fulcrum is changed and the force is applied in the proper direction so that the band is tightened.

Ex. 6.28 **A simple band brake is applied to a shaft carrying a flywheel of 250 kg mass and of the radius of gyration of 300 mm. the shaft speed is 200 rpm. The drum diameter is 200 mm and the coefficient of friction is 0.25. Determine the**

1. **brake torque when a force of 120 N is applied at the lever end**
2. **number of turns of the flywheel before it comes to rest**
3. **time is taken by the flywheel to come to rest.**

Solution:



Given data:

$$m = 250\,kg$$
$$k = 300\,mm$$
$$N = 200\,rpm$$
$$d = 200\,mm$$
$$\mu = 0.25$$
$$\theta = 360^\circ - 135^\circ = 225^\circ \times \frac{\pi}{180} = 3.93\,rad$$

1. Taking moment about O,

$$F \cdot l = T_1 \cdot a$$
$$120 \times 280 = T_1 \cdot 100$$
$$T_1 = 336\,N$$
$$T_2 = \frac{T_1}{2.67} = 125.8\,N$$

$$\left\{ \frac{T_1}{T_2} = e^{\mu\theta} = e^{0.25 \times 3.93} \quad \therefore \frac{T_1}{T_2} = 2.67 \right\}$$

Braking Torque, $T_B = (T_1 - T_2)\,r = (336 - 125.8)\,0.1 = 21\ N\cdot m$

2. K.E of the flywheel

$$= \frac{1}{2} I\omega^2 = \frac{1}{2} m k^2 \cdot \left(\frac{2\pi N}{60}\right)^2$$
$$= \frac{1}{2} \times 250 \times 0.3^2 \left(\frac{2\pi \times 200}{60}\right)^2$$
$$= 4935\ N\cdot m$$

This K.E is used to overcome the work done by the braking torque in n revolutions.

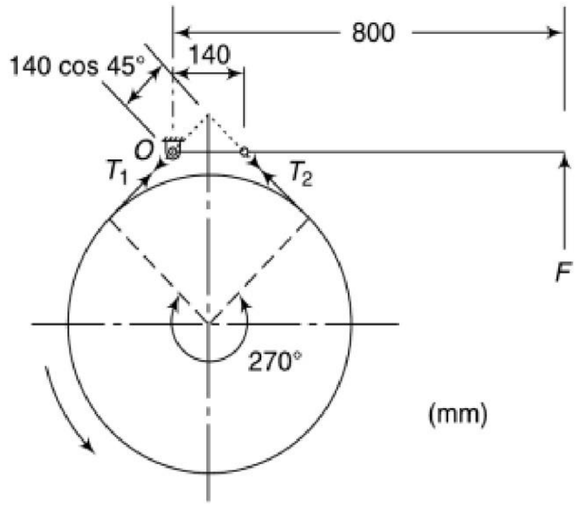
$$K.E \text{ of flywheel} = T_B \times \text{Angular displacement}$$
$$4935 = 21 \times 2\pi \times n$$
$$\therefore n = 37.4\ \text{Revolution}$$

3. For uniform retardation,

$$\text{Average speed} = \frac{200}{2} = 100\ rpm$$
$$\therefore \text{Time taken} = \frac{n}{N} = \frac{37.4}{100}\ \text{min.} = \frac{37.4}{100/60} = 22.44\ sec$$

Ex. 6.29 **A simple band brake is applied to a drum of 560 mm diameter which rotates at 240 RPM. The angle of contact of the band is 270°. One end of the band is fastened to a fixed pin and the other end to the brake lever, 140 mm from the fixed pin. The brake lever is 800 mm long and is placed perpendicular to the diameter that bisects the angle of contact. Assuming the coefficient of friction as 0.3, determine the necessary pull at the end of the lever to stop the drum if 40 kW of power is being absorbed. Also, find the width of the band if its thickness is 3 mm and the maximum tensile stress is limited to 40 N/mm^2.**

Solution:



Given data:

$d = 560\,\text{mm}$ $\quad N = 240\,\text{rpm}$

$\theta = 270^\circ$ $\quad a = 140\,\text{mm}$,

$l = 800\,\text{mm}$ $\quad \mu = 0.3$

$P = 40\,\text{kW}$

thickness $t = 3\,\text{mm}$

$\sigma = 40\,\text{N/mm}^2$

Note: It can be observed from the figure that to tighten the band, the force is to be applied upwards. If the drum rotates counterclockwise, the tight and slack sides will be as shown.

$$P = T_B \cdot \omega$$

$$= (T_1 - T_2)\, r \cdot \frac{2\pi N}{60}$$

$$40 \times 10^3 = (T_1 - T_2)\, 0.28 \times \frac{2\pi \times 240}{60}$$

$$\therefore T_1 - T_2 = 5684 \qquad \_\_\_\_\_\_\_\_(1)$$

and

$$\frac{T_1}{T_2} = e^{\mu\theta} = e^{0.3 \times 270 \times \frac{\pi}{180}} = 4.11 \qquad \_\_\_\_\_\_\_\_(2)$$

From equation (1) & (2)

$T_2 = 1828\,\text{N}$

$T_1 = 7514\,\text{N}$

Taking the moment about O,

$$F \cdot l = T_2 \cdot 140 \cos 45^\circ$$

$$F \times 800 = 1828 \cdot 140 \cos 45^\circ$$

$$\therefore F = 226.2\,\text{N}$$

Let maximum tension,

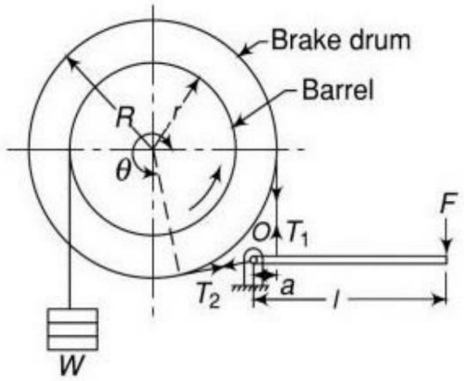
$$T_1 = \sigma \cdot b \cdot t$$
$$7514 = 40 \times b \times 3$$
$$\therefore b = 62.6\,\text{mm}$$

Note: If the drum rotates clockwise, the brake is less effective as in that case tight and slack sides are interchanged and the force required to apply the same braking torque is more.

$$F \cdot l = T_1 \cdot 140 \cos 40^\circ$$
$$F \times 800 = 7514 \cdot 140 \cos 40^\circ$$
$$\therefore F = 930\,\text{N}$$

Ex. 6.30 **A crane is required to support a load of 1.2 tonnes on the rope around its barrel of 400 mm diameter. The brake drum which is keyed to the same shaft as the barrel has a diameter of 600 mm. The angle of contact of the band brake is 275° and the coefficient of friction is 0.22. Determine the force required at the end of the lever to support the load. Take a = 150 mm and l = 750 mm.**

Solution:



Given data:

$W = 1.2 \times 1000 \times 9.81\,\text{N}$

$R = 300\,\text{mm}$ $\quad\quad r = 200\,\text{mm}$

$\mu = 0.22$ $\quad\quad \theta = 275^\circ$

For equilibrium position,

$$(T_1 - T_2)R = W \times r$$
$$(2.87\,T_2 - T_2)\,300 = (1.2 \times 1000 \times 9.81)\,200$$
$$\therefore T_2 = 4197\,\text{N} \cdot \text{m}$$
$$\text{and } T_1 = 12045\,\text{N} \cdot \text{m}$$

$$\left\{ \frac{T_1}{T_2} = e^{\mu\theta} = e^{0.22 \times 275 \times \frac{\pi}{180^\circ}} = 2.87 \right\}$$

Taking moment about O,

$$F \cdot l = T_1 \cdot a$$
$$F \times 750 = 12045 \times 150$$
$$\therefore F = 2409\,\text{N}$$

Ex. 6.31 **A band and block brake has 14 blocks. Each block subtends an angle of 14° at the center of the rotating drum. The diameter of the drum is 750 mm and the thickness of the blocks is 65 mm. The two ends of the band are fixed to the pins on the lever at distances of 50 mm and 210 mm from the fulcrum on the opposite sides. Determine the least force required to be applied at the lever at a distance of 600 mm from the fulcrum if the power absorbed by the blocks is 180 kW at 175 RPM. The coefficient of friction between the blocks and the drum is 0.35.**

Solution: Given data:

$n = 14 \text{ blocks}$ $\quad 2\theta = 14^\circ$

$d = 750 \text{ mm}$ $\quad t = 65 \text{ mm}$

$a = 210 \text{ mm},$ $\quad b = 50 \text{ mm}$

$l = 600 \text{ mm}$ $\quad P = 180 \text{ kW}$

$N = 175 \text{ rpm}$ $\quad \mu = 0.35$

$F = ?$

$$P = (T_{14} - T_0)v$$

$$= (T_{14} - T_0)\frac{\pi DN}{60}$$

$$180 \times 10^3 = (T_{14} - T_0)\frac{(0.75 + 2 \times 0.065) \times 175}{60}$$

$$\therefore (T_{14} - T_0) = 22323 \text{ N} \quad \text{\_\_\_\_\_(1)}$$

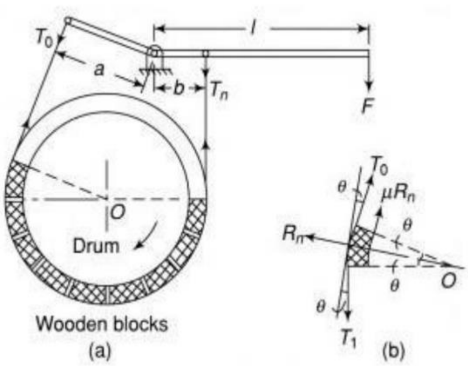
$$\frac{T_{14}}{T_0} = \left[\frac{1 + \mu \tan\theta}{1 - \mu \tan\theta}\right]^n$$

$$\frac{T_{14}}{T_0} = \left[\frac{1 + 0.35 \tan 7^\circ}{1 - 0.35 \tan 7^\circ}\right]^{14} = 3.334 \quad \text{\_\_\_\_\_\_\_(2)}$$

From equation (1) and (2)

$T_0 = 9564 \text{ N}$

$T_{14} = 31887 \text{ N}$



Assume a > b, F must be a downward and clockwise rotation for maximum braking torque.

The moment about O,

$$F \cdot l - T_0 a + T_{14} b = 0$$

$$F \times 600 - 9564 \times 210 + 31887 \times 50 = 0$$

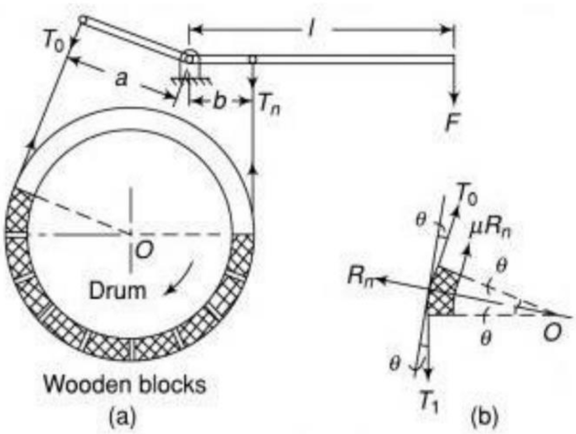
$$F = 690 \text{ N}$$

Ex. 6.32 **A band and block brake having 12 blocks, each of which subtends an angle of 16° at the center, is applied to a rotating drum with a diameter of 600 mm. The blocks are 75 mm thick. The drum and the flywheel mounted on the same shaft have a mass of 1800 kg and have a combined radius of gyration of 600 mm. The two ends of the band are attached to pins on the opposite sides of the brake fulcrum at distances of 40 mm and 150 mm from it. If a force of 250 N is applied on the lever at a distance of 900 mm from the fulcrum, find the**

1. **Maximum braking torque**
2. **Angular retardation of the drum**
3. **Time is taken by the system to be stationary from the rated speed of 300 rpm.**

Solution: Given data:

$\theta = 16^\circ$	$n = 12$ blocks
$d = 600$ mm	$t = 75$ mm
$m = 1800$ kg	$k = 600$ mm
$a = 150$ mm,	$b = 40$ mm
$F = 250$ N	$l = 900$ mm
$N = 300$ rpm	$\mu = 0.3$



$$\frac{T_{12}}{T_0} = \left[\frac{1+\mu \tan\theta}{1-\mu \tan\theta}\right]^n$$

$$\frac{T_{14}}{T_0} = \left[\frac{1+0.3\tan 8^\circ}{1-0.3\tan 8^\circ}\right]^{12} = 2.752$$

Assume a = 150 mm, b = 40 mm as a > b, F must be downwardsand rotation is clockwise.

Taking moment about fulcrum O,

$$F \cdot l - T_0 a + T_{12} b = 0$$

$$250 \times 900 - T_0(150) + T_{12}(40) = 0$$

$$T_0(150 - 2.752 \times 40) = 250 \times 900$$

$$\therefore T_0 = 5636 \text{ N}$$

$$T_{12} = 15511 \text{ N}$$

Maximum Braking torque,

$$T_B = \left(T_{12} - T_0\right) r$$

$$= \left(15511 - 5636\right) \times \frac{0.6 + 0.075 \times 2}{2} \qquad \left\{ \because r = \frac{d}{2} = \frac{d + 2(t)}{2} \right\}$$

$$= 3703\ \text{N} \cdot \text{m}$$

2. Let braking torque

$$T_B = I \cdot \alpha = m k^2 \cdot \alpha$$

$$3703 = 1800\,(0.6)^2 \cdot \alpha$$

$$\therefore \alpha = 5.71\ \text{rad/sec}^2$$

3. Initial Angular Speed

$$\omega = \omega_0 - \alpha t$$

$$0 = 31.4 - 5.71\,t \qquad \left\{ \begin{array}{l} \omega = \text{Final Angular speed } = 0 \\ \omega_0 = \dfrac{2\pi N}{60} = \dfrac{2\pi \times 300}{60} = 31.4\ \text{rad/sec} \end{array} \right\}$$

$$t = 5.5\ \text{sec}$$

Ex. 6.33 **A car moving on a level road at a speed 50 km/h has a wheelbase 2.8 meters, the distance of C.G. from ground level 600 mm, and the distance of C.G. from rear wheels 1.2metres. Find the distance traveled by car before coming to rest when brakes are applied,**
1. to the rear wheels,
2. to the front wheels, and
3. to all the four wheels.
The coefficient of friction between the tyres and the road may be taken as 0.6.

Solution: Given data:

$$u = 50\ \text{km/hr} = 13.89\ \text{m/sec}$$

$$L = 2.8\ \text{m} \qquad h = 600\ \text{mm}$$

$$x = 1.2\ \text{m} \qquad \mu = 0.6$$

1. Rear Wheels

Here vehicle moves on a level road, so retardation of the car is

$$a = \frac{\mu g\,(L - x)}{L + \mu h} = \frac{0.6 \times 9.81\,(2.8 - 1.2)}{2.8 + 0.6 \times 0.6} = 2.98\ \text{m/sec}^2$$

If retardation is uniform,

$$v^2 - u^2 = -2\,s\,a$$

$$0 - u^2 = -2\,s\,a$$

$$\therefore s = \frac{u^2}{2a} = \frac{13.89^2}{2 \times 2.98} = 32.4\ \text{m}$$

2. Front Wheels

Here vehicle moves on a level road, so retardation of the car is

$$a = \frac{\mu g x}{L - \mu h} = \frac{0.6 \times 9.81 \times 1.2}{2.8 - 0.6 \times 0.6} = 2.9\ \text{m/sec}^2$$

For uniform retardation,

$$s = \frac{u^2}{2\,a}$$

$$s = \frac{13.89^2}{2 \times 2.9} = 33.26\,\text{m}$$

3. All the four wheels

Here vehicle moves on a level road, so retardation of the car is

$$a\ =\ g\,\mu = 9.81 \times 0.6 = 5.886\ \ \text{m/sec}^2$$

For uniform retardation,

$$s = \frac{u^2}{2\,a} = \frac{13.89^2}{2 \times 5.886} = 16.4\,\text{m}$$

Ex. 6.34	**A vehicle moving on a rough plane inclined at 10° with the horizontal at a speed of 36 km/h has a wheelbase 1.8 meters. The center of gravity of the vehicle is 0.8 meters from the rear wheels and 0.9 meters above the inclined plane. Find the distance traveled by the vehicle before coming to rest and the time is taken to do so when 1. The vehicle moves up the plane, and 2. The vehicle moves down the plane. The brakes are applied to all the four wheels and the coefficient of friction is 0.5.**

Solution: Given data:

$$\alpha = 10^\circ$$
$$u\ = 36\,\text{km/h} = 10\,\text{m/sec}$$
$$L = 1.8\,\text{m}$$
$$x = 0.8\,\text{m}$$
$$h = 0.9\,\text{m}$$
$$\mu\ = 0.5$$

1. The vehicle moves up (All Four wheels)

$$\begin{aligned} a\ &=\ g\left(\mu\cos\alpha + \sin\alpha\right) \\ &= 9.81\left(0.5\cos 10^\circ + \sin 10^\circ\ \right) \\ &= 6.53\ \ \text{m/sec}^2 \end{aligned}$$

For uniform retardation,

$$s = \frac{u^2}{2\,a} = \frac{10^2}{2 \times 6.53} = 7.657\,\text{m}$$

The final velocity of a vehicle

$$v = u - a\,t \qquad \{-\text{ve sign due to retardation}\}$$
$$0 = 10 - 6.53\,t$$
$$\therefore t = 1.53\,\text{sec}$$

2. The vehicle moves down the plane (All Four wheels)

$$\begin{aligned} a &= g(\mu \cos\alpha - \sin\alpha) \\ &= 9.81(0.5 \cos 10^\circ - \sin 10^\circ) \\ &= 3.13 \ \text{m/sec}^2 \end{aligned}$$

For uniform retardation

$$s = \frac{u^2}{2a} = \frac{10^2}{2 \times 3.13} = 16 \text{ m}$$

The final velocity of a vehicle

$$v = u - a t$$

$$0 = 10 - 3.13 t$$

$$\therefore t = 3.2 \text{ sec}$$

7.1 Introduction

If power transmitted between two shafts is small, motion between them may be obtained by using two plain cylinders or discs 1 and 2.

If there is no slip of one surface relative to the other, a definite motion of 1 can be transmitted to 2 and vice-versa. Such wheels are termed as "***friction wheels***". However, as the power transmitted increases, slip occurs between the discs and the motion no longer remains definite.

Assuming no slipping of the two surfaces, the following kinematic relationship exists for their linear velocity:

- To transmit a definite motion of one disc to the other or to prevent slip between the surfaces, projection and recesses on the two discs can be made which can mesh with each other. This leads to the formation of teeth on the discs and the motion between the surfaces changes from rolling to sliding. The discs with the teeth are known as gears or gear wheels.
- It is to be noted that if the disc 1 rotates in the clockwise direction, 2 rotates in the counter-clockwise direction and vice-versa.

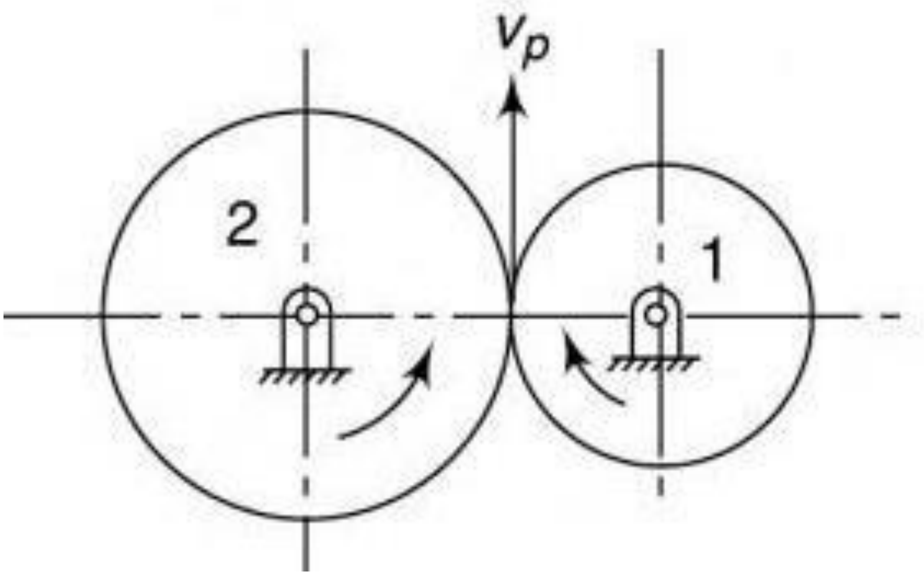


Fig.7.1 – Friction Wheels

7.1.1 Advantages of gear Drive

1. It transmits an exact velocity ratio.
2. It may be used to transmit large power.
3. It has high efficiency.
4. It has a reliable service.
5. It has a compact layout.

7.1.2 Disadvantages of gear Drive

1. The manufacture of gears required special tools and equipment.
2. The error in cutting teeth may cause vibrations and noise during operation.
3. They are costly.

7.1.3 Classification of Gears

7.1.3.1 According to the position of axes of the shafts

a) Parallel shaft

- **Spur gear**

The two parallel and co-planar shafts connected by the gears are called spur gears. These gears have teeth parallel to the axis of the wheel.

They have straight teeth parallel to the axes and thus are not subjected to axial thrust due to tooth load.

At the time of engagement of the two gears, the contact extends across the entire width on a line parallel to the axis of rotation. This results in the sudden application of the load, high impact stresses and excessive noise at high speeds.

If the gears have external teeth on the outer surface of the cylinders, the shaft rotates in the opposite direction.

In internal spur gear, teeth are formed on the inner surface of an annulus ring. An internal gear can mesh with an external pinion (smaller gear) only and the two shafts rotate in the same direction.

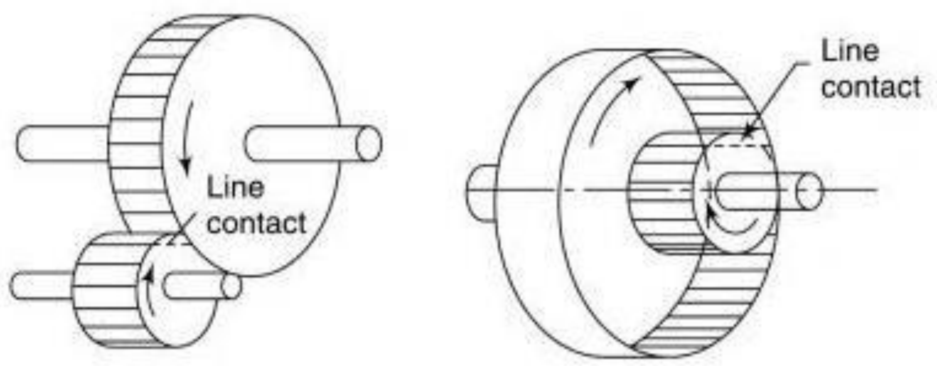


Fig.7.2 – Spur Gear

- **Spur rack and pinion**

Spur rack is a special case of a spur gear where it is made of infinite diameter so that the pitch surface is plane.

The spur rack and pinion combination converts rotary motion into translator motion, or vice-versa. It is used in a lathe in which the rack transmits motion to the saddle.

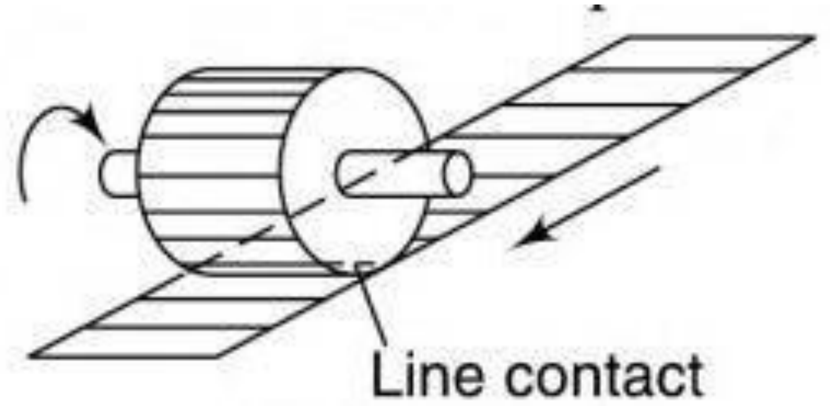


Fig.7.3 – Rack and Pinion

- **Helical Spur Gears**

In helical gears, the teeth are curved, each being helical in shape. Two mating gears have the same helix angle but have teeth of opposite hands.

At the beginning of the engagement, contact occurs only at the point of the leading edge of the curved teeth. As the gears rotate, the contact extends along a diagonal line across the teeth. Thus, the load application is gradual which results in low impact stresses and reduction in noise. Therefore, the helical gear can be used at higher velocities than the spur gears and have a greater load-carrying capacity.

Helical gears have the disadvantage of having end thrust as there is a force component along the gear axis. The bearing and assemblies mounting the helical gears must be able to withstand thrust loads.

Double helical: A double-helical gear is equivalent to a pair of helical gears secured together, one having a right-hand helix and another left-hand helix.

The teeth of two rows are separated by groove used for tool run out.

Axial thrust which occurs in the case of single-helical gears is eliminated in double-helical gears.

This is because the axial thrusts of the two rows of teeth cancel each other out. These can be run at highs speeds with less noise and vibrations.

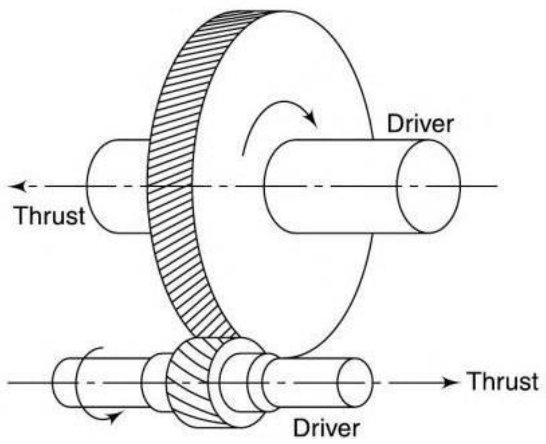


Fig.7.4 – Helical gear

- **Herringbone gear**

If the left and the right inclinations of a double-helical gear meet at a common apex and there is no groove in between, the gear is known as Herringbone gear.

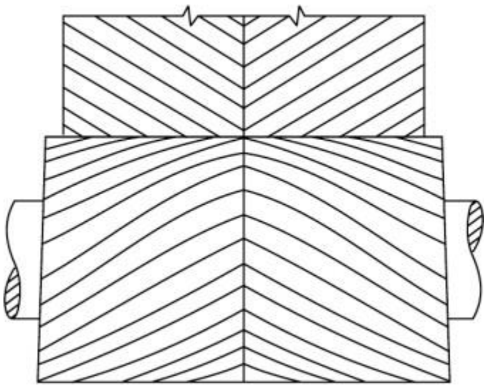


Fig.7.5 – Herringbone gear

b) Intersecting Shafts

The two non-parallel or intersecting, but coplanar shafts connected by gears are called bevel gears

When teeth formed on the cones are straight, the gears are known as bevel gears when inclined, they are known as spiral or helical bevel.

- **Straight Bevel Gears**

The teeth are straight, radial to the point of intersection of the shaft axes and vary in cross-section throughout their length.

Usually, they are used to connect shafts at right angles which run at low speeds

Gears of the same size and connecting two shafts at right angles to each other are known as “Mitre” gears.

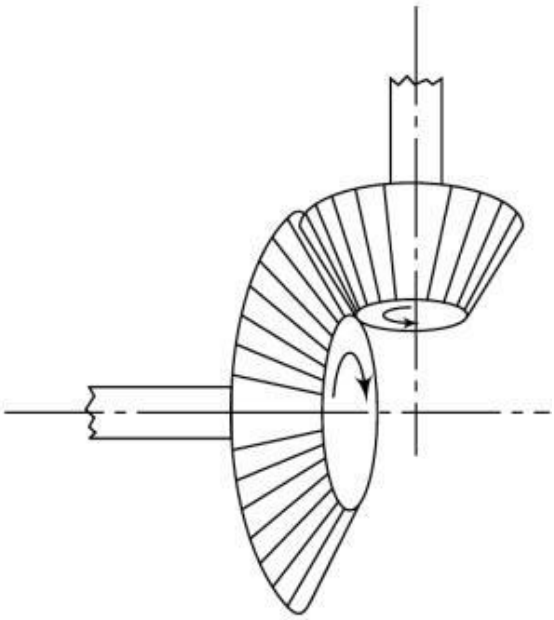


Fig.7.6 - Straight Bevel Gears

- **Spiral Bevel Gears**

When the teeth of a bevel gear are inclined at an angle to the face of the bevel, they are known as spiral bevels or helical bevels.

They are smoother in action and quieter than straight tooth bevels as there are gradual load applications and low impact stresses. Of course, there exists an axial thrust calling for stronger bearings and supporting assemblies.

These are used for the drive to the differential of an automobile.

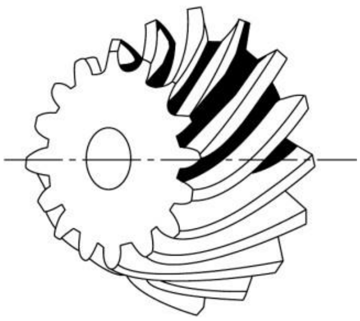


Fig.7.7 – Spiral Bevel Gears

- **Zero Bevel Gears**

Spiral bevel gears with curved teeth but with a zero degree spiral angle are known as zero bevel gears.

Their tooth action and the end thrust are the same as that of straight bevel gears and, therefore, can be used in the same mountings.

However, they are quieter in action than the straight bevel type as the teeth are curved.

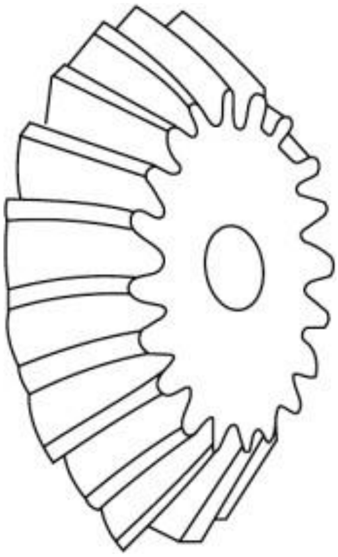


Fig.7.8 – Zero Bevel Gear

c) Non-intersecting and non-parallel shaft (Skew shaft)

The two non-intersecting and non-parallel i.e. non-coplanar shaft connected by gears are called skew bevel gears or spiral gears and the arrangement is known as skew bevel gearing or spiral gearing.

In these gears teeth have point contact.

These gears are suitable for transmitting small power.

A worm gear is a special case of a spiral gear in which the larger wheel, usually, has a hollow shape such that a portion of the pitch diameter of the other gear is enveloped on it.

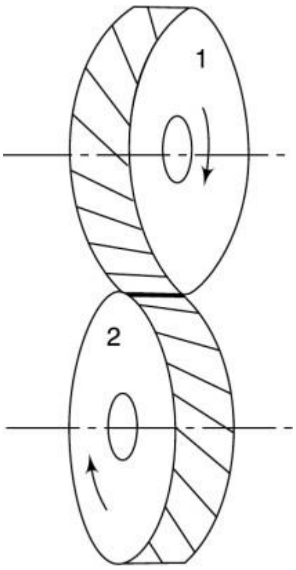


Fig.7.9 - Non-intersecting and non-parallel shaft

7.1.3.2 According to the peripheral velocity of the gears

a) Low velocity V < 3 m/sec

(b) Medium velocity 3 < V < 15 m/sec

(c) High velocity V > 15 m/sec

7.1.3.3 According to the position of teeth on the gear surface

(a) Straight,

(b) Inclined, and

(c) Curved

7.2 Terms Used in Gears

1. Pitch circle. It is an imaginary circle which by the pure rolling action, would give the same motion as the actual gear.

2. Pitch circle diameter. It is the diameter of the pitch circle. The size of the gear is usually specified by the pitch circle diameter. It is also known as **pitch diameter**.

3. Pitch point. It is a common point of contact between two pitch circles.

4. Pitch surface. It is the surface of the rolling discs which the meshing gears have replaced at the pitch circle.

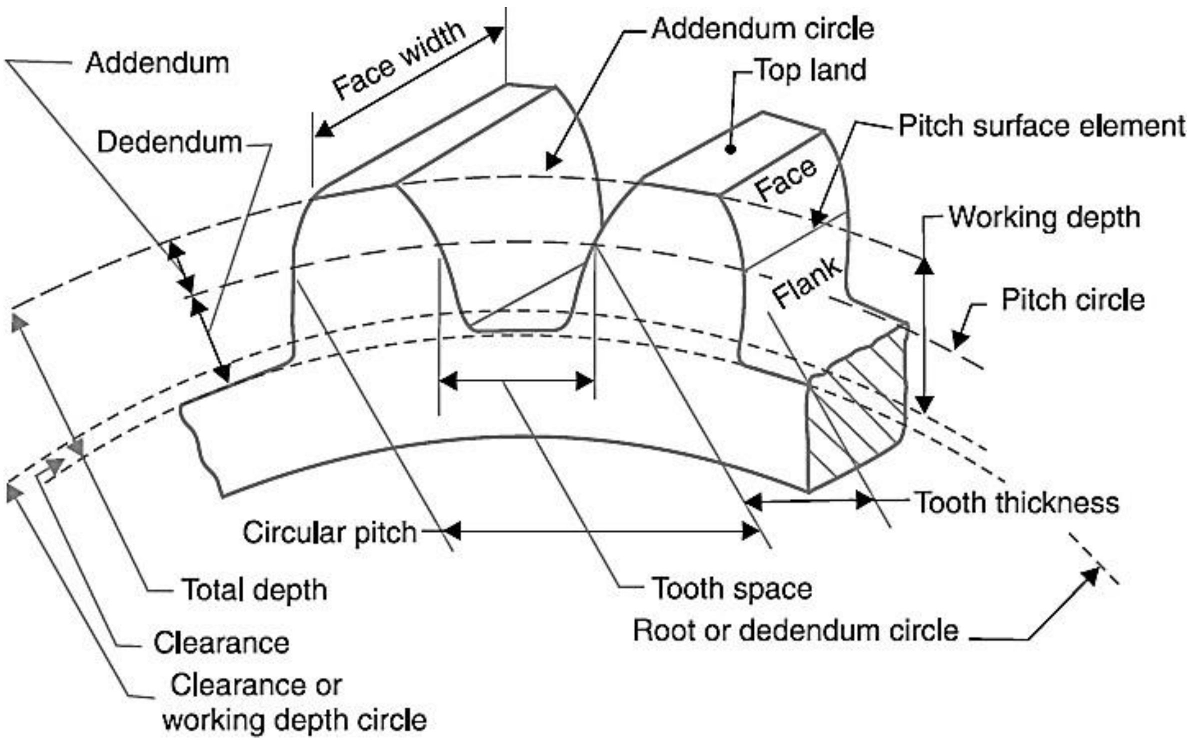


Fig.7.10 - Terms used in gears

5. Pressure angle or angle of obliquity. It is the angle between the common normal to two gear teeth at the point of contact and the common tangent at the pitch point.
For more power transmission lesser pressure on the bearing and pressure angle must be kept small.
It is usually denoted by ø.
The standard pressure angles are 20° and 25°.Gears with $14\frac{1}{2}°$ pressure angle has become obsolete.

6. Addendum. It is the radial distance of a tooth from the pitch circle to the top of the tooth.

- Standard value = 1 module

7. Dedendum. It is the radial distance of a tooth from the pitch circle to the bottom of the tooth.

- Standard value = 1.157 module

8. Addendum circle. It is the circle drawn through the top of the teeth and is concentric with the pitch circle.

9. Dedendum circle. It is the circle drawn through the bottom of the teeth. It is also called the root circle.

10. Clearance. It is the radial difference between the addendum and the Dedendum of a tooth.

Addendum circle diameter $= d + 2m$

Dedendum circle diameter $= d - 2 \times 1.157m$

Clearance $= 1.157m - m = 0.157m$

11. Full-depth of Teeth It is the total radial depth of the tooth space.

Full depth= Addendum + Dedendum

12. Working Depth of Teeth The maximum depth to which a tooth penetrates into the tooth space of the mating gear is the working depth of teeth.

- Working depth = Sum of addendums of the two gears.

13. Working depth. It is the radial distance from the addendum circle to the clearance circle.

It is equal to the sum of the addendum of the two meshing gears.

14. Tooth thickness. It is the width of the tooth measured along the pitch circle.

15. Tooth space. It is the width of space between the two adjacent teeth measured along the pitch circle.

16. Backlash. It is the difference between the tooth space and the tooth thickness, as measured along the pitch circle. Theoretically, the backlash should be zero, but in actual practice, some backlash must be allowed to prevent jamming of the teeth due to tooth errors and thermal expansion.

17. The face of a tooth. It is the surface of the gear tooth above the pitch surface.

18. The flank of the tooth. It is the surface of the gear tooth below the pitch surface.

19. Top land. It is the surface of the top of the tooth.

20. Face width. It is the width of the gear tooth measured parallel to its axis.

21. Fillet It is the curved portion of the tooth flank at the root circle.

22. *Circular pitch*. It is the distance measured on the circumference of the pitch circle from point of one tooth to the corresponding point on the next tooth.
It is usually denoted by p_c.

Mathematically,

$$\text{Circular pitch, } p_c = \frac{\pi d}{T}$$

Where *d*= Diameter of the pitch circle, and

T = Number of teeth on the wheel.

The angle subtended by the circular pitch at the center of the pitch circle is known as the **pitch angle**.

23. Module (m). It is the ratio of the pitch diameter in mm to the number of teeth.

$$m = \frac{d}{T}$$

$$\text{Also} \quad p_c = \frac{\pi d}{T} = \pi m$$

- Pitch of two mating gear must be the same.

24. Diametral Pitch (P) It is the number of teeth per unit length of the pitch circle diameter in inch.

OR

It is the ratio of no. of teeth to pitch circle diameter in inch.

$$P_d = \frac{T}{d}$$

- The recommended series of modules in Indian Standard are 1, 1.25, 1.5, 2, 2.5, 3, 4, 5, 6, 8, 10, 12, 16, and 20. The modules 1.125, 1.375, 1.75, 2.25, 2.75, 3.5, 4.5, 5.5, 7, 9, 11, 14 and 18 are of second choice.

25. Gear Ratio (G). It is the ratio of the number of teeth on the gear to that on the pinion.

$$G = \frac{T} {t} \quad \text{Where} \quad T = \text{No of teeth on a gear}$$

t = No. of teeth on the pinion

26. Velocity Ratio (VR) The velocity ratio is defined as the ratio of the angular velocity of the follower to the angular velocity of the driving gear.

$$VR = \frac{\omega_2}{\omega_1} = \frac{N_2}{N_1} = \frac{d_1}{d_2} = \frac{T_1}{T_2}$$

27. Length of the path of contact. It is the length of the common normal cut-off by the

Addendum circles of the wheel and pinion.

OR

The locus of the point of contact of two mating teeth from the beginning of the engagement to the end of engagement is known as the contact.

a. **Path of Approach** Portion of the path of contact from the beginning of the engagement to the pitch point.
b. **Path of Recess** Portion of the path of contact from the pitch point to the end of the engagement.

28. Arc of Contact The locus of a point on the pitch circle from the beginning to the end of the engagement of two mating gears is known as the arc of contact.

a. **Arc of Approach** It is the portion of the arc of contact from the beginning of the engagement to the pitch point.
b. **Arc of Recess** The portion of the arc of contact from the pitch point to the end of engagements the arc of recess.

29. The angle of Action (δ) It is the angle turned by gear from the beginning of the engagement to the end of engagement of a pair of teeth, i.e., the angle turned by arcs of contact of respective gear wheels.

$$\delta = \alpha + \beta \text{Where } \alpha = \text{Angle of approach}$$

β =Angle of recess

30. Contact ratio. It is the angle of action divided by the pitch angle

$$\text{Contact ratio} = \frac{\delta}{\gamma} = \frac{\alpha+\beta}{\gamma}$$

OR

$$\text{Contact ratio} = \frac{\text{Arc of contact}}{\text{Circular pitch}}$$

7.2.1 Condition for Constant Velocity Ratio of Toothed Wheels – Law of Gearing

To understand the theory consider the portions of two gear teeth gear 1 and gear 2. The two teeth come in contact at point C and the direction of rotation of gear 1 is anticlockwise & gear 2 is clockwise.

Let TT be the common tangent & NN be the common normal to the curve at the point of contact C. From points O, & O_2, draw O_1A & O_2B perpendicular to common normal NN. When the point D is considered on gear 1, the point C moves in the direction of "CD" & when it is considered on gear 2. The point C moves in the direction of "CE".

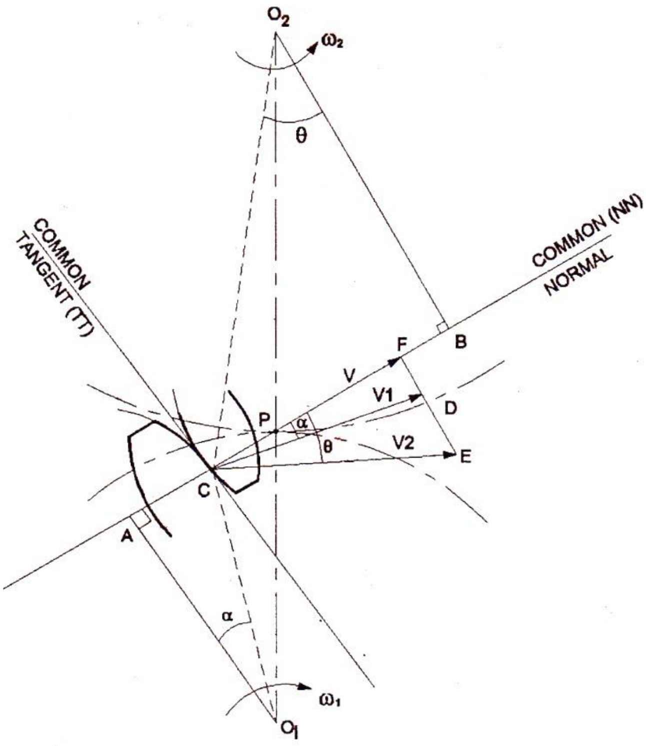


Fig.7.11 – Law of gearing

The relative motion between tooth surfaces along the common normal NN must be equal to zero in order to avoid separation. So, relative velocity

$$V_1 \cos\alpha = V_2 \cos\theta$$

$$(\omega_1 \times O_1 C)\cos\alpha = (\omega_2 \times O_2 C)\cos\theta \qquad (\because V = r\,\omega) \qquad \textit{Eq. (7.1)}$$

But from $\Delta O_1 AC$, $\cos\alpha = \dfrac{O_1 A}{O_1 C}$

and from $\Delta O_2 BC$, $\cos\theta = \dfrac{O_2 B}{O_2 C}$

Putting the above value in *Eq. (7.1)* it becomes

$$(\omega_1 \times O_1 C)\frac{O_1A}{O_1C} = (\omega_2 \times O_2 C)\frac{O_2B}{O_2C}$$

$$\therefore (\omega_1 \times O_1 A) = (\omega_2 \times O_2 B)$$

$$\therefore \frac{\omega_1}{\omega_2} = \frac{O_2B}{O_2A} \qquad \textit{Eq. (7.2)}$$

From a similar triangle $\Delta O_1 AP$ & $\Delta O_2 BP$

$$\frac{O_2B}{O_2A} = \frac{O_2P}{O_2P} \qquad \textit{Eq. (7.3)}$$

Equating ***Eq. (7.2)*** and ***Eq. (7.3)***

$$\frac{\omega_1}{\omega_2} = \frac{O_2\,B}{O_1\,A} = \frac{O_2\,P}{O_1\,P} = \frac{PB}{AP}$$

From the above, we can conclude that the angular velocity ratio is inversely proportional to the ratio of the distances of the point P from the central O_1& O_2.

If it is desired that the angular velocities of two gear remain constant, the common normal at the point of contact of two teeth always passes through a fixed point P. This fundamental condition is called as law of gearing which must be satisfied while designing the profiles of teeth for gears.

7.2.2 Standard Tooth Profiles or Systems

Following four types of tooth profiles or systems are commonly used in practice for interchangeability:

a) $14\frac{1}{2}^{\circ}$ composite system

b) $14\frac{1}{2}^{\circ}$ full depth involute system

c) 20° full depth involute system

d) 20° stub involute system

a) $14\frac{1}{2}^{\circ}$ composite system:

This type of profile is made with circular arcs at the top and bottom portion and the middle portion is a straight line. The straight portion corresponds to the involute profile and the circular arc portion corresponds to the cycloidal profile. Such profiles are used for general purpose gears.

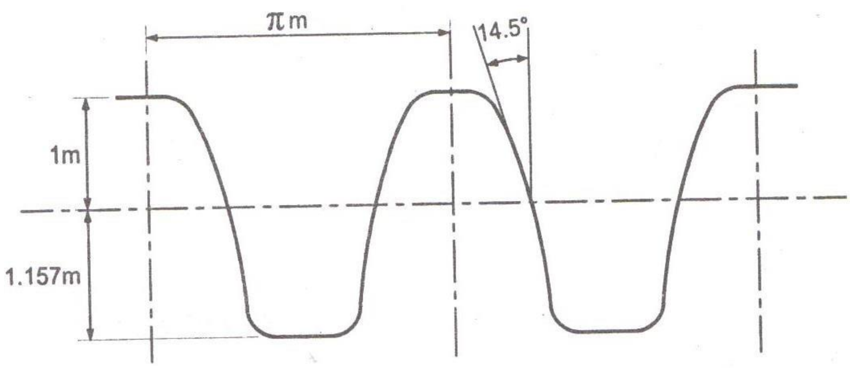


Fig.7.12 - $14\frac{1}{2}^{\circ}$ composite system

b) $14\frac{1}{2}^{\circ}$ **full depth involute system:**

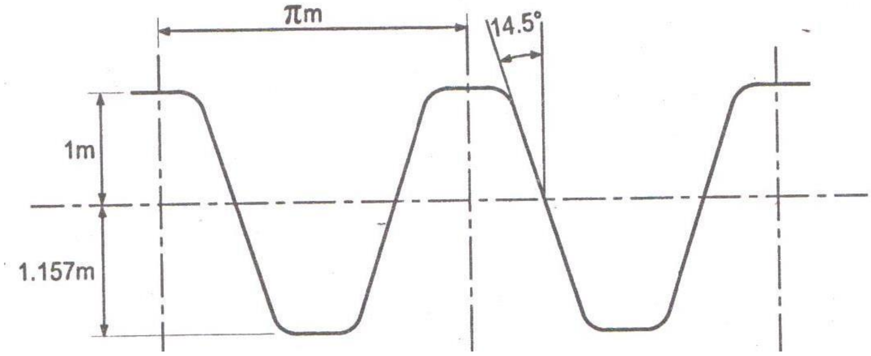


Fig.7.13 - $14\frac{1}{2}^{\circ}$ *full depth involute system*

This type of profile is made straight line except for the fillet arcs. The whole profile corresponds to the involute profile. Therefore manufacturing of such profiles is easy but they have interface problems.

c) 20° **full depth involute system:**

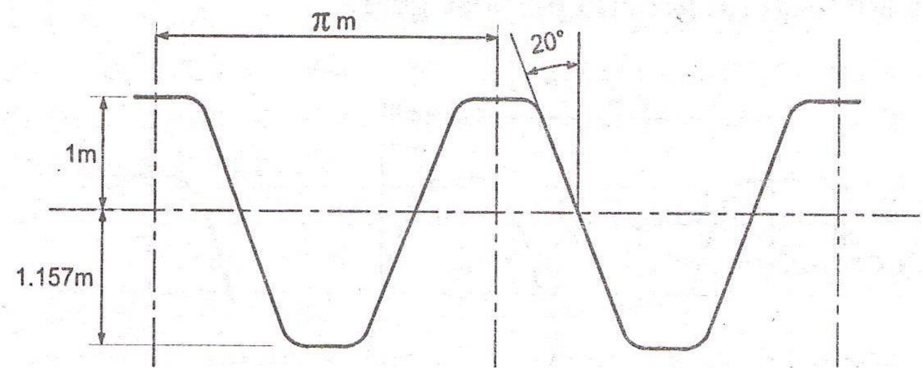


Fig.7.14 - 20° *full depth involute system*

This type of profile is the same as $14\frac{1}{2}^{\circ}$ the full depth involute system except for the pressure angle. The increase of pressure angle from $14\frac{1}{2}^{\circ}$ to 20° results in a stronger tooth, since the tooth acting like a beam, is wider at the base. This type of gears also has an interference problem if the number of teeth is less.

d) 20° **stub involute system:**

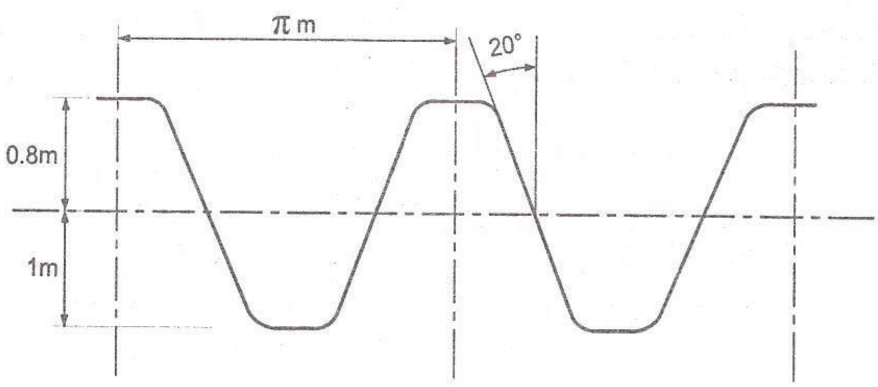


Fig.7.15 - 20° *stub involute system*

The problem of interference in 20° full depth involute system is minimized by removing extra addendum of gear tooth which causes interference. Such a modified tooth profile is called "Stub tooth profile". This type of gears is used for heavy load.

7.3 Length of Path of Contact And Length of Arc of Contact

7.3.1 Length of Path of Contact

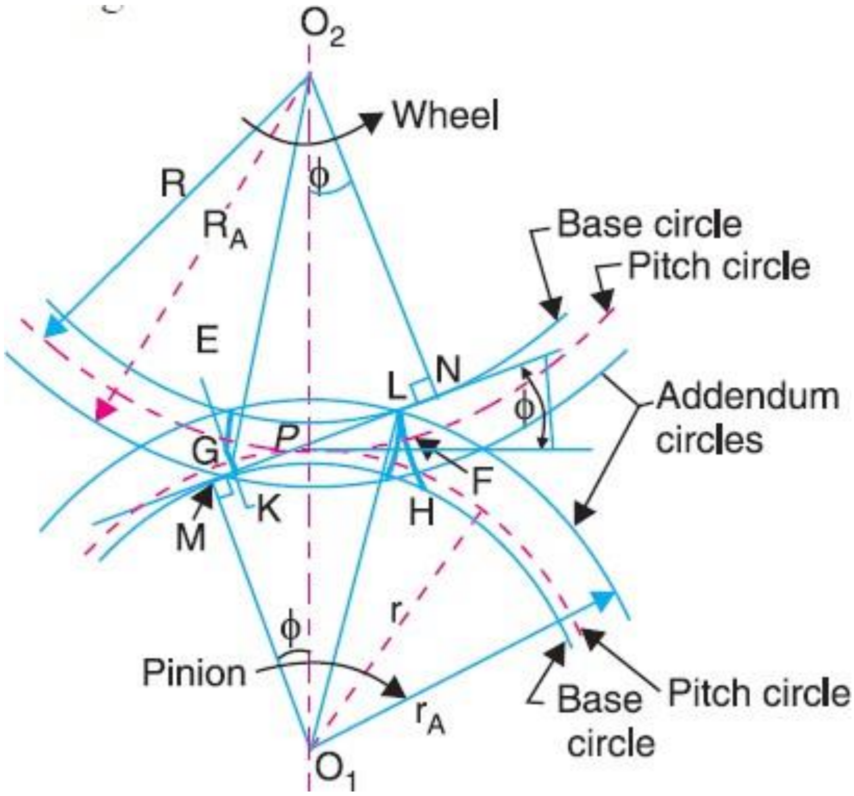


Fig.7.16 - Length of Path of Contact

- When the pinion rotates in the clockwise direction, the contact between a pair of involute teeth begins at *K* (on the flank near the base circle of pinion or the outer end of the tooth face on the wheel) and ends at L (on the flank near the base circle of the wheel).
- MN is the common normal at the point of contact and the common tangent to the base circles.
- The point K is the intersection of the addendum circle of the wheel and the common tangent.
- The point L is the intersection of the addendum circle of pinion and common tangent.
- **The length of the path of contact** is the length of the common normal cutoff by the addendum circles of the wheel and the pinion.
- Thus the length of the path of contact is KL which is the sum of the parts of the path of contacts KP and PL. The part of the path of contact KP is known as the **path of approach** and the part of the path of contact PL is known as the **path of recess.**

$$\text{L.P.C} = \text{KL}$$

$$= \text{KP} + \text{PL}$$

where, KP = path of approach

PL = path of recess

Let,

R = O_2P = pitch circle radius of wheel

R_A = O_2K = addendum circleradius of wheel

r = O_1P = pitch circle radius of pinion

r_A = O_1L = addendumcircleradius of pinion

Length of the path of contact = Path of approach + path of recess

$$= KP + PL$$

$$= (KN - PN) + (ML - MP)$$

$$= \left(\sqrt{O_2K^2 - O_2N^2} - PN\right) + \left(\sqrt{O_1L^2 - O_1M^2} - MP\right)$$

$$= \left(\sqrt{R_A{}^2 - (R\cos\phi)^2} - R\sin\phi\right) + \left(\sqrt{r_A{}^2 - (r\cos\phi)^2} - r\sin\phi\right)$$

7.3.2 Length of Arc of Contact

- The arc of contact is the path traced by a point on the pitch circle from the beginning to the end of the engagement of a given pair of teeth.
- The arc of contact is EPF or GPH.
- Considering the arc of contact GPH, it is divided into two parts i.e. arc GP and arc PH. The arc GP is known as **arc of approach** and the arc PH is called the **arc of recess**.
- The angles subtended by these arcs at O1 are called **angle of approach** and **angle of recess** respectively.

Length of the arc of contact GPH $= (GP + PH)$

$$= \text{Arc of approach} + \text{Arc of recess}$$

$$= \frac{KP}{\cos\phi} + \frac{PL}{\cos\phi}$$

$$= \frac{KP + PL}{\text{cos}\cos\phi} = \frac{KL}{\cos\phi}$$

$$= \frac{\text{Length of path of contact}}{\cos\phi}$$

7.3.3 Contact Ratio (or Number of Pairs of Teeth in Contact)

The contact ratio or the number of pairs of teeth in contact is defined as the ratio of the length of the arc of contact to the circular pitch.

Mathematically, Contact ratio or number of pairs of teeth in contact

$$= \frac{\text{Length of arc of contact}}{\text{Circular pitch}}$$

$$= \frac{\text{Length of arc of contact}}{\pi\, m}$$

Notes:

- For continuous transmission of the motion, at least one tooth of anyone wheel must be in contact with another tooth of the second wheel so ‘n’ must be greater than unity.
- If ‘n’ lies between 1& 2, no. of teeth in contact at any time will not be less than one and will never mate two.
- If ‘n’ lies between 2 & 3, it is never less than two pairs of teeth and not more than three pairs and so on.
- If ‘n’ is 1.6, one pair of teeth are always in contact whereas two pair of teeth are in contact for 60% of the time.

7.4 Interference in Involute Gears

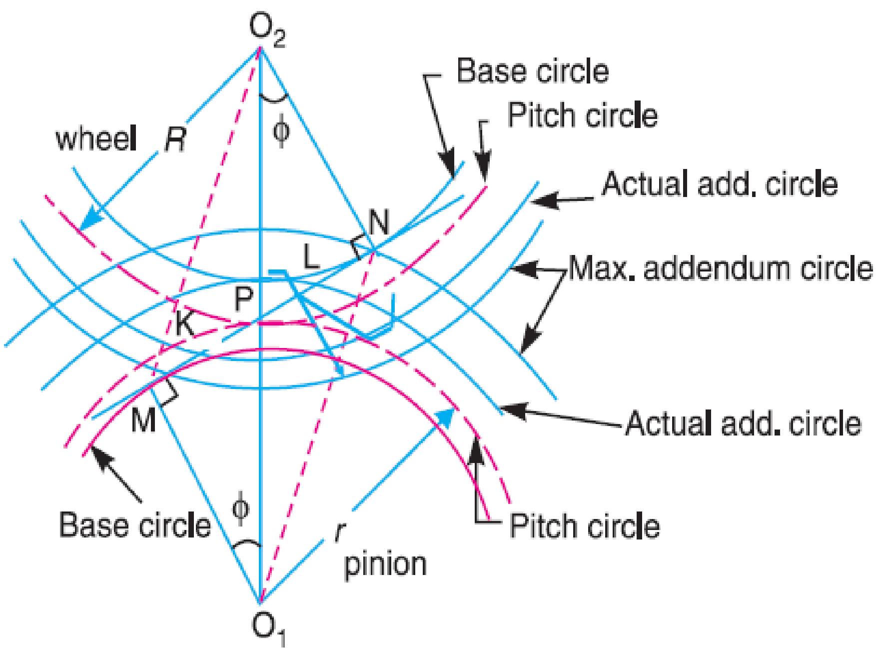


Fig.7.17 - Interference in involute gears

- A pinion with centreO_1, in mesh with wheel or gear with centreO_2. MN is the common tangent to the base circles and KL is the path of contact between the two mating teeth.
- A little consideration will show that if the radius of the addendum circle of pinion is increased to O_1N, the point of contact L will move from L to N. When this radius is further increased, the point of contact L will be on the inside of base circle of wheel and not on the involute profile of tooth on wheel. The tip of the tooth on the pinion will then undercut the tooth on the wheel at the root and remove part of the involute profile of the tooth on the wheel. This effect is known as **interference** and occurs when the teeth are being cut. In brief, the phenomenon when the tip of the tooth undercuts the root on its mating gear is known as interference.
- Similarly, if the radius of the addendum circles of the wheel increases beyond O_2M, then the tip of the tooth on the wheel will cause interference with the tooth on pinion.
- The points M and N are called **interference points.** Interference may be avoided if the path of contact does not extend beyond interference points. The limiting value of the radius of the addendum circle of the pinion is O_1N and the wheel is O_2M.

7.4.1 How to avoid interference?

- The interference may only be avoided if the point of contact between the two teeth is always on the involute profiles of both the teeth.

OR

- Interference may only be prevented if the addendum circles of the two mating gears cut the common tangent to the base circles between the points of tangency.
  When interference is just avoided, the maximum length of the path of contact is *MN.*

The maximum length of the path of contact $= MN$

$$= MP + PN$$

$$= r\sin\phi + R\sin\phi$$

$$= (r+R)\sin\phi$$

The maximum length of the arc of contact $= \dfrac{(r+R)\sin\phi}{\cos\phi}$

Notes :

In case the addenda on pinion and wheel are such that the path of approach and path of recess are half of their maximum possible values, then

Path of approach, $KP = \dfrac{1}{2} MP$

$$\left(\sqrt{R_A^{\,2} - (R\cos\phi)^2} - R\sin\phi\right) = \frac{1}{2} r\sin\phi$$

Path of recess, $PL = \dfrac{1}{2} PN$

$$\left(\sqrt{r_A^{\,2} - (r\cos\phi)^2} - r\sin\phi\right) = \frac{1}{2} R\sin\phi$$

Length of the path of contact $= KP + PL$

$$= \frac{1}{2} MP + \frac{1}{2} PN$$

$$= \frac{(r+R)\sin\phi}{2}$$

7.4.2 Minimum Number of Teeth on the Pinion in Order to Avoid Interference

- In order to avoid interference, the addendum circles for the two mating gears must cut the common tangent to the base circles between the points of tangency.
- The limiting condition reaches when the addendum circles of pinion and wheel pass-through point *N* and *M* respectively.

Let t = Number of teeth on the pinion,

T = Number of teeth on the wheel,

m = Module of the teeth,

r = Pitch circle radius of pinion = $mt/2$

G = Gear ratio = $T/t = R/r$

ϕ = Pressure angle or angle of obliquity.

From Δ O_1NP,

$$O_1N^2 = O_1P^2 + PN^2 - 2O_1P \times PN\cos(O_1PN)$$

$$\therefore O_1N^2 = r^2 + (R\sin\phi)^2 - 2r(R\sin\phi)\times\cos(90+\phi)$$

$$\therefore O_1N^2 = r^2 + (R\sin\phi)^2 - 2r(R\sin\phi)\times\cos(90+\phi)$$

$$\therefore O_1N^2 = r^2 + R^2\sin^2\phi + 2rR\sin^2\phi$$

$$\therefore O_1N^2 = r^2\left[1 + \frac{R^2\sin^2\phi}{r^2} + \frac{2R\sin^2\phi}{r}\right]$$

$$\therefore O_1N^2 = r^2\left[1 + \frac{R^2\sin^2\phi}{r^2} + \frac{2R\sin^2\phi}{r}\right]$$

$$\therefore O_1N^2 = r^2\left[1 + \frac{R}{r}\left(\frac{R}{r} + 2\right)\sin^2\phi\right]$$

$$\therefore O_1N = r\sqrt{1 + \frac{R}{r}\left(\frac{R}{r} + 2\right)\sin^2\phi}$$

$$\therefore O_1N = \frac{mt}{2}\sqrt{1 + \frac{R}{r}\left(\frac{R}{r} + 2\right)\sin^2\phi}$$

Let $A_p \cdot m$ = Addendum of the pinion, where A_P is a fraction by which the standard addendum of one module for the pinion should be multiplied in order to avoid interference.

Addendum of the pinion $= O_1N - O_1P$

$$A_P.m = \frac{mt}{2}\sqrt{1 + \frac{T}{t}\left(\frac{T}{t} + 2\right)\sin^2\phi} - \frac{mt}{2}$$

$$\therefore A_P.m = \frac{mt}{2}\sqrt{1 + \frac{T}{t}\left(\frac{T}{t} + 2\right)\sin^2\phi} - \frac{mt}{2}$$

$$\therefore A_P.m = \frac{mt}{2}\left[\sqrt{1 + \frac{T}{t}\left(\frac{T}{t} + 2\right)\sin^2\phi} - 1\right]$$

$$\therefore A_P.m = \frac{mt}{2}\left[\sqrt{1 + \frac{T}{t}\left(\frac{T}{t} + 2\right)\sin^2\phi} - 1\right]$$

$$\therefore A_P. = \frac{t}{2}\left[\sqrt{1 + \frac{T}{t}\left(\frac{T}{t} + 2\right)\sin^2\phi} - 1\right]$$

$$\therefore A_P = \frac{t}{2}\left[\sqrt{1+\frac{T}{t}\left(\frac{T}{t}+2\right)\sin^2\phi}-1\right]$$

$$\therefore A_P = \frac{t}{2}\left[\sqrt{1+\frac{T}{t}\left(\frac{T}{t}+2\right)\sin^2\phi}-1\right]$$

$$\therefore t = \frac{2A_P}{\left[\sqrt{1+\frac{T}{t}\left(\frac{T}{t}+2\right)\sin^2\phi}-1\right]}$$

$$\therefore t = \frac{2A_P}{\left[\sqrt{1+G(G+2)\sin^2\phi}-1\right]}$$

Note:
If the pinion and wheel have equal teeth, then $G = 1$.

$$\therefore t = \frac{2A_P}{\left[\sqrt{1+3\sin^2\phi}-1\right]}$$

Table 7.1 - Min. no of teeth on pinion

Sr. No.	System of Gear Tooth	Min. no. of teeth on the pinion
1	$14\frac{1}{2}°$ Composite	12
2	$14\frac{1}{2}°$Full depth involute	32
3	20° Full-depth involute	18
4	20°Stub involute	14

7.4.3 Minimum Number of Teeth on the Wheel in Order to Avoid Interference

Let T = Minimum number of teeth required on the wheel in order to avoid interference,

$A_w \cdot m$= Addendum of the wheel, where A_W is a fraction by which the standard

The addendum for the wheel should be multiplied.

From ΔO_2MP

$$O_2M^2 = O_2P^2 + PM^2 - 2O_2P \times PM\cos(O_2PM)$$

$$\therefore O_2M^2 = R^2 + (r\sin\phi)^2 - 2r(R\sin\phi)\times\cos(90+\phi)$$

$$\therefore O_2M^2 = R^2 + r^2\sin^2\phi + 2rR\sin^2\phi$$

$$\therefore O_2M^2 = R^2\left[1+\frac{r^2\sin^2\phi}{R^2}+\frac{2r\sin^2\phi}{R}\right]$$

$$\therefore O_2M^2 = R^2\left[1+\frac{r}{R}\left(\frac{r}{R}+2\right)\sin^2\phi\right]$$

$$\therefore O_2M = R\sqrt{1+\frac{r}{R}\left(\frac{r}{R}+2\right)\sin^2\phi}$$

$$\therefore O_2M = \frac{mT}{2}\sqrt{1+\frac{r}{R}\left(\frac{r}{R}+2\right)\sin^2\phi}$$

Addendum of the wheel $= O_2M - O_2P$

$$A_w m = \frac{mT}{2}\sqrt{1+\frac{t}{T}\left(\frac{t}{T}+2\right)\sin^2\phi} - \frac{mT}{2}$$

$$\therefore A_w m = \frac{mT}{2}\left[\sqrt{1+\frac{t}{T}\left(\frac{t}{T}+2\right)\sin^2\phi} - 1\right]$$

$$\therefore A_w m = \frac{mT}{2}\left[\sqrt{1+\frac{t}{T}\left(\frac{t}{T}+2\right)\sin^2\phi} - 1\right]$$

$$\therefore A_w = \frac{T}{2}\left[\sqrt{1+\frac{t}{T}\left(\frac{t}{T}+2\right)\sin^2\phi} - 1\right]$$

$$\therefore T = \frac{2A_w}{\left[\sqrt{1+\frac{t}{T}\left(\frac{t}{T}+2\right)\sin^2\phi} - 1\right]}$$

$$\therefore T = \frac{2A_w}{\left[\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\phi} - 1\right]}$$

Note:

From the above equation, we may also obtain the minimum number of teeth on the pinion. Multiplying both sides by t/T,

$$T \times \frac{t}{T} = \frac{2A_w \times \frac{t}{T}}{\left[\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\phi} - 1\right]}$$

$$\therefore t = \frac{2A_w}{G\left[\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\phi} - 1\right]}$$

If wheel and pinion have equal teeth, then $G = 1$,

$$\therefore T = \frac{2A_w}{\left[\sqrt{1+3\sin^2\phi} - 1\right]}$$

7.4.4 Minimum Number of Teeth on a Pinion for Involute Rack in Order to Avoid Interference

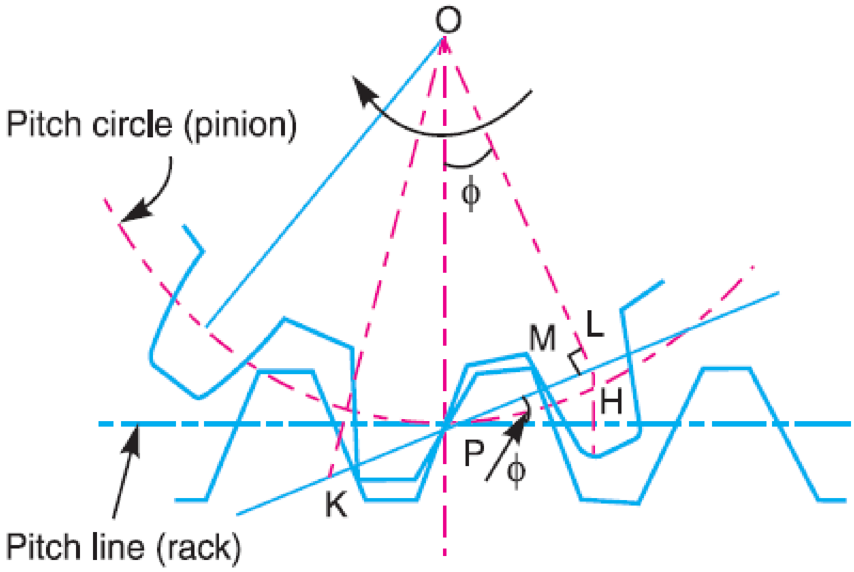


Fig.7.18 - Rack, and pinion in mesh

Let t = Minimum number of teeth on the pinion,

r = Pitch circle radius of the pinion = $\frac{m \cdot t}{2}$ and

ø= Pressure angle or angle of obliquity, and

$A_R \cdot m$= Addendum for the rack, where A_R is the fraction by which the standard addendum of one module for the rack is to be multiplied.

Addendum for the rack, $A_R \cdot m = LH$

$$\therefore A_R \cdot m = PL \sin\phi$$

$$\therefore A_R \cdot m = r \sin\phi \times \sin\phi$$

$$\therefore A_R \cdot m = r \sin^2\phi$$

$$\therefore A_R \cdot m = \frac{mt \sin^2\phi}{2}$$

$$\therefore t = \frac{2A_R}{\sin^2\phi}$$

Note:

In the case of pinion, max. value of addendum radius to avoid interference if AF

$$= O_2M^2 + AF^2$$

$$= (r\cos ø)^2 + (R\sin ø + r\sin ø)^2$$

Max value of addendum of the pinion is

$$(A_p)_{max} = r\sqrt{1 + \frac{R}{r}\left(\frac{R}{r} + 2\right)\sin^2 ø} - 1$$

$$= \frac{mt}{2}\left[\sqrt{1 + G(G+2)\sin^2 ø} - 1\right]$$

7.5 Comparison of Cycloidal and Involute tooth forms

Table 7.2 - Comparison of Cycloidal and Involute tooth forms

Cycloidal teeth	Involute teeth
Pressure angle varies from a maximum at the beginning of the engagement, reduces to zero at the pitch point and again increases to a maximum at the end of the engagement resulting in the smooth running of gears.	The pressure angle is constant throughout the engagement of teeth. This results in the smooth running of the gears.
It involves double curves for the teeth, epicycloid, and hypocycloid. This complicates the manufacturer.	It involves the single curves for the teeth resulting in simplicity of manufacturing and of tool
Owing to the difficulty of the manufacturer, these are costlier.	These are simple to manufacture and thus are cheaper.
Exact center distance is required to transmit a constant velocity ratio.	A little variation in a center distance does not affect the velocity ratio.
The phenomenon of interference does not occur at all.	Interference can occur if the condition of minimum no. of teeth on a gear is not followed.
The teeth have spreading flanks and thus are stronger.	The teeth have radial flanks and thus are weaker as compared to the Cycloidal form for the same pitch.
In this, a convex flank always has contact with a concave face resulting in less wear.	Two convex surfaces are in contact and thus there is more wear.

7.6 Helical and Spiral Gears

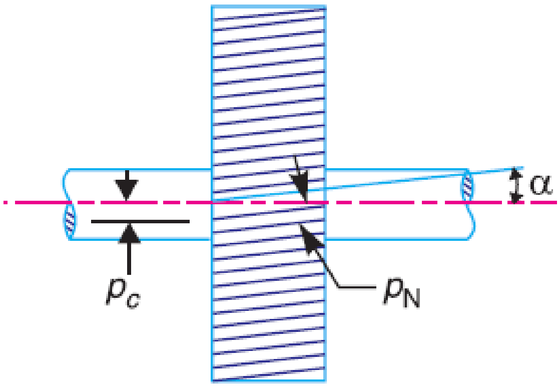


Fig.7.19 – Helical Gear

- A helical gear has teeth in the form of the helix around the gear. Two such gears may be used to connect two parallel shafts in place of the spur gear. The helixes may be right-handed on one wheel and left-handed on the other.

- The pitch surfaces are cylindrical as in spur gearing, but the teeth instead of being parallel to the axis, wind around the cylinders helically like screw threads.
- The teeth of helical gears with parallel axis have line contact, as in spur gearing. This provides gradual engagement and continuous contact of the engaging teeth. Hence helical gears give smooth drive with high efficiency of transmission.
- The helical gears may be of the single helical type or double helical type. In the case of single helical gears, there is some axial thrust between the teeth, which is a disadvantage.
- In order to eliminate this axial thrust, double helical gears are used. It is equivalent to two single helical gears, in which equal and opposite thrusts are produced on each gear and the resulting axial thrust is zero.
- The following definitions may be clearly understood in connection with a helical gear:

  1. Normal pitch. It is the distance between similar faces of adjacent teeth, along a helix on the pitch cylinder normal to the teeth. It is denoted by p_N.

  2. Axial pitch. It is the distance measured parallel to the axis, between similar faces of adjacent teeth. It is the same as circular pitch and is therefore denoted by pc. If α is the helix angle, then

$$\text{Circular pitch, } p_c = \frac{p_N}{\cos\alpha}$$

Note: The **helix angle** is also known as the **spiral angle** of the teeth.

7.6.1 The efficiency of Spiral Gears

A pair of spiral gears 1 and 2 are in mesh. Let the gear 1 be the driver and the gear 2 the driven. The forces acting on each of a pair of teeth in contact. The forces are assumed to act at the center of the width of each teeth and in the plane tangential to the pitch cylinders

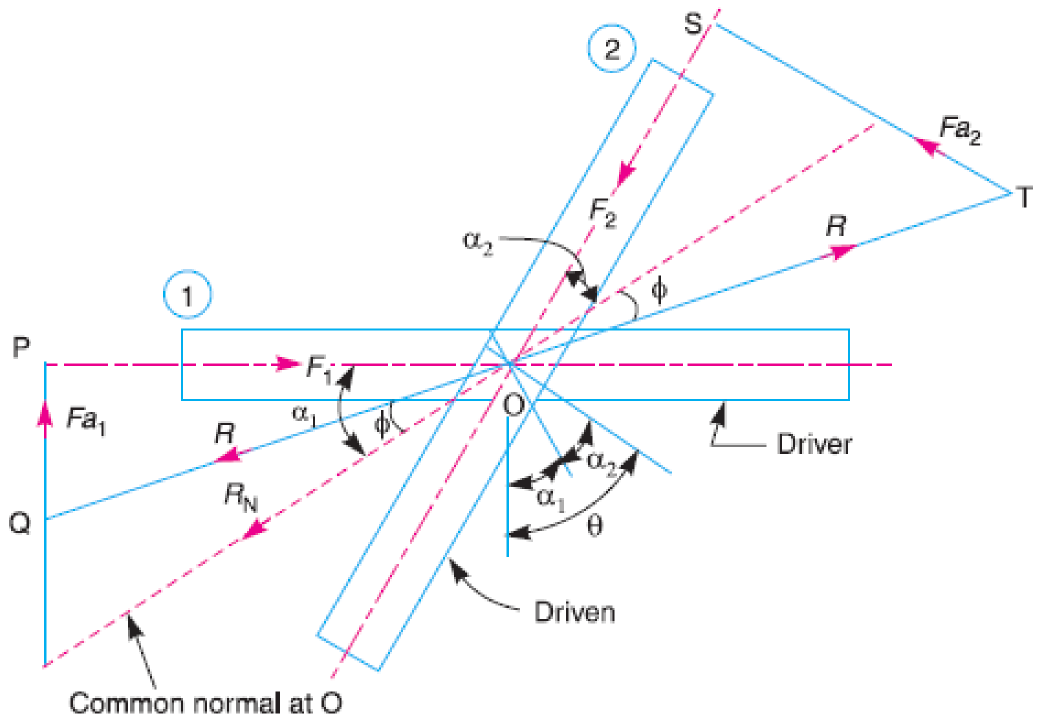


Fig.7.20 - Efficiency of Spiral Gears

Let F_1 = Force applied tangentially on the driver,

F_2 = Resisting force acting tangentially on the driven,

F_{a1} = Axial or end thrust on the driver,

F_{a2} = Axial or end thrust on the driven,

R_N = Normal reaction at the point of contact

ϕ = Angle of friction,

R = Resultant reaction at the point of contact, and

θ = Shaft angle $=\alpha_1+\alpha_2$

...($\because$ Both gears are of the same hand)

From triangle OPQ, $F1 = R\cos(\alpha_1-\phi)$

$\therefore$ Work input to the driver = $F_1 \times \pi d_1 \cdot N_1 = R\cos(\alpha_1-\phi)\times \pi d_1 \cdot N_1$

From triangle OST, $F_2 = R\cos(\alpha_2+\phi)$

$\therefore$ Work output of the driven = $F_2 \times \pi d_2 \cdot N_2 = R\cos(\alpha_2+\phi)\times \pi d_2 \cdot N_2$

$\therefore$ Efficiency of spiral gears,

$$\eta = \frac{\text{Work output}}{\text{Work input}} = \frac{R\cos(\alpha_2+\phi)\times \pi d_2 \cdot N_2}{R\cos(\alpha_1-\phi)\times \pi d_1 \cdot N_1}$$

$$= \frac{\cos(\alpha_2+\phi)\times d_2 \cdot N_2}{\cos(\alpha_1-\phi)\times d_1 \cdot N_1}$$

Pitch circle diameter of gear 1,

$$d_1 = \frac{p_{c1}\times T_1}{\pi} = \frac{p_N}{\text{Cos}\alpha_1}\times\frac{T_1}{\pi}$$

Pitch circle diameter of gear 2,

$$d_2 = \frac{p_{c2}\times T_2}{\pi} = \frac{p_N}{\text{Cos}\alpha_2}\times\frac{T_2}{\pi}$$

$$\therefore \frac{d_2}{d_1} = \frac{T_2\text{Cos}\alpha_1}{T_1\text{Cos}\alpha_2} \qquad \cdots\cdots\cdots(2)$$

$$\frac{N_2}{N_1} = \frac{T_1}{T_2} \qquad \cdots\cdots\cdots(3)$$

Multiplying equation (2) and (3) we get

$$\frac{d_2N_2}{d_1N_1} = \frac{\cos\alpha_1}{\cos\alpha_2}$$

Substituting this value in equation (1)

$$\eta = \frac{\cos(\alpha_2+\phi)\times\cos\alpha_1}{\cos(\alpha_1-\phi)\times\cos\alpha_2} \qquad \cdots\cdots(4)$$

$$= \frac{\cos(\alpha_1 + \alpha_2 + \phi) + \cos(\alpha_1 - \alpha_2 - \phi)}{\cos(\alpha_1 + \alpha_2 - \phi) + \cos(\alpha_1 - \alpha_2 + \phi)}$$

$$\left(\because \cos A \cdot \cos B = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]\right)$$

$$= \frac{\cos(\theta + \phi) + \cos(\alpha_1 - \alpha_2 - \phi)}{\cos(\theta - \phi) + \cos(\alpha_1 - \alpha_2 + \phi)} \quad \cdots\cdots\cdots(5)$$

$$(\because \theta = \alpha_1 + \alpha_2)$$

Since the angle θ and ϕ are constants, therefore the efficiency will be maximum, when $\cos(\alpha_1 - \alpha_2 + \phi)$ is maximum, *i.e.* $\cos(\alpha_1 - \alpha_2 + \phi) = 1$

$$\therefore \alpha_1 - \alpha_2 + \phi = 0$$

$$\therefore \alpha_1 = \alpha_2 + \phi \quad \text{and} \quad \alpha_2 = \alpha_1 - \phi$$

Since $\alpha_1 + \alpha_2 = \theta$ therefore

$$\alpha_1 = \theta - \alpha_2 = \theta - \alpha_1 + \phi \quad \text{OR} \quad \alpha_1 = \frac{\theta + \phi}{2}$$

Similarly $\alpha_2 = \frac{\theta - \phi}{2}$

Substituting $\alpha_1 = \alpha_2 + \phi$ and $\alpha_2 = \alpha_1 - \phi$ in equation (5) we get

$$\eta_{max} = \frac{\cos(\theta + \phi) + 1}{\cos(\theta - \phi) + 1}$$

7.7 Introduction to Gear Trains and Its Classification

When two or more gears are made to mesh with each other to transmit power from one shaft to another. Such a combination is called gear train or train of toothed wheels.

The nature of the train used depends upon the velocity ratio required and the relative position of the axes of shafts. A gear train may consist of a spur, bevel or spiral gears.

Types of Gear Trains

1. Simple gear train
2. Compound gear train
3. Reverted gear train
4. Epicyclic gear train
5. Compound epicyclic gear train

7.7.1 Simple gear train

When there is only one gear on each shaft, it is known as a **simple gear train.** The gears are represented by their pitch circles. When the distance between the two shafts is small, the two gears 1 and 2 are made to mesh with each other to transmit motion from one shaft to the other. Since the gear 1 drives the gear 2, therefore gear 1 is called the **driver** and the gear 2 is called the **driven** or **follower.** It may be noted that the motion of the driven gear is opposite to the motion of driving gear.

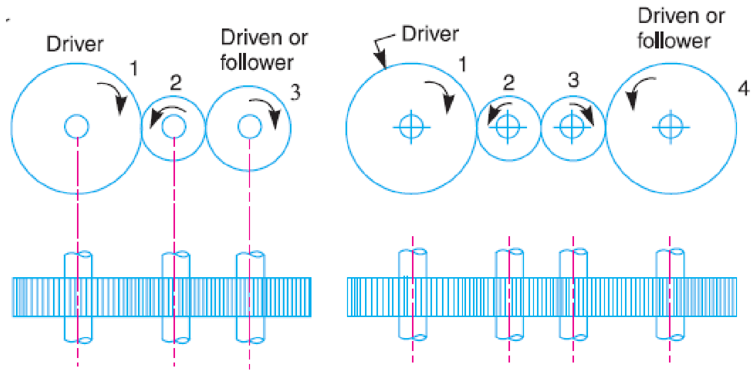


Fig.7.21 - Simple gear train

Let,

N_1 = Speed of driver rpm

N_2 = Speed of intermediate wheel rpm

N_3 = Speed of follower rpm

T_1 = Number of teeth on driver

T_2 = Number of teeth on intermediate wheel

T_3 = Number of teeth on follower

Since the driving gear, 1 is in mesh with the intermediate gear 2, so speed ratio for these two gears is

$$\frac{N_1}{N_2} = \frac{T_2}{T_1} \qquad \cdots\cdots\cdots(1)$$

As the intermediate gear, 2 is in mesh with the driven gear 3, so speed ratio for these two gears is

$$\frac{N_2}{N_3} = \frac{T_3}{T_2} \qquad \cdots\cdots\cdots(2)$$

The speed ratio of the gear train is obtained by multiplying the equations (1) and (2).

$$\frac{N_1}{N_2} \times \frac{N_2}{N_3} = \frac{T_2}{T_1} \times \frac{T_3}{T_2}$$

$$\therefore \frac{N_1}{N_3} = \frac{T_3}{T_1}$$

Sometimes, the distance between the two gears is large. The motion from one gear to another, in such a case, maybe transmitted by either of the following two methods:

1. By providing the large-sized gear, or
   - A little consideration will show that this method (i.e. providing large-sized gears) is a very inconvenient and uneconomical method.
2. By providing one or more intermediate gears.
   - This method (i.e. providing one or more intermediate gear) is very convenient and economical.

It may be noted that when the number of intermediate gears is **odd**, the motion of both the gears (i.e. driver and driven or follower) will rotate in the same direction.

If the numbers of intermediate gears are **even**, the motion of the driven or follower will be in the **opposite direction** of the driver.

- **Speed ratio** (or velocity ratio) of the gear train is the ratio of the speed of the driver to the speed of the driven or follower and the ratio of speeds of any pair of gears in the mesh is the inverse of their number of teeth.

$$\text{Speed ratio} = \frac{N_1}{N_2} = \frac{T_2}{T_1}$$

- **The train value** of the gear train is the ratio of the speed of the driven or follower to the speed of the driver.

$$\text{Trainvalue} = \frac{N_2}{N_1} = \frac{T_1}{T_2}$$

7.7.2 Compound Gear Train

When there is more than one gear on a shaft, it is called a **compound train of gear.**

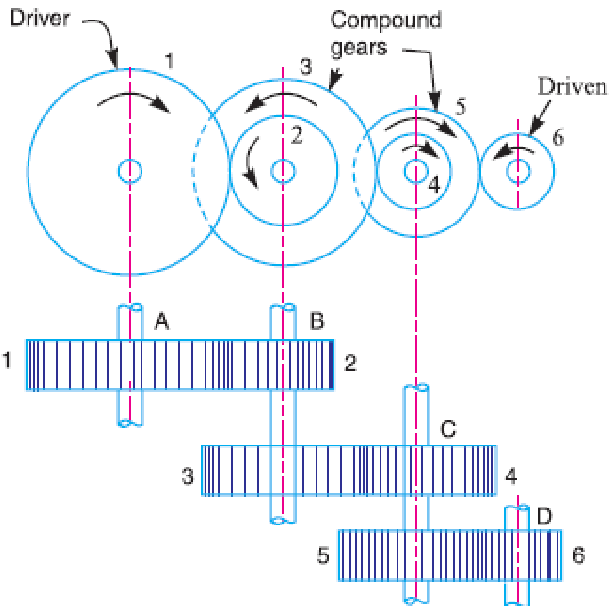


Fig.7.22 - Compound Gear Train

The idle gears, in a simple train of gears, do not affect the speed ratio of the system. But these gears are useful in bridging over the space between the driver and the driven.

But whenever the distance between the driver and the driven or follower has to be bridged over by intermediate gears and at the same time a great (or much less) speed ratio is required, then the advantage of intermediate gears is intensified by providing compound gears on intermediate shafts.

In this case, each intermediate shaft has two gears rigidly fixed to it so that they may have the same speed. One of these two gears meshes with the driver and the other with the driven or follower attached to the next shaft.

In a compound train of gears, the gear 1 is the driving gear mounted on shaft A; gears 2 and 3 are compound gears which are mounted on shaft B. The gears 4 and 5 are also compound gears which are mounted on shaft C and the gear 6 is the driven gear mounted on shaft D.

Let,

N1 = Speed of driving gear 1,

T1 = Number of teeth on driving gear 1,

N2 ,N3 ..., N6 = Speed of respective gears in r.p.m., and

T2 ,T3..., T6 = Number of teeth on respective gears.

Since gear 1 is in mesh with gear 2, therefore its speed ratio is

$$\frac{N_1}{N_2} = \frac{T_2}{T_1} \qquad \cdots\cdots\cdots(1)$$

Similarly, for gears 3 and 4, the speed ratio is

$$\frac{N_3}{N_4} = \frac{T_4}{T_3} \qquad \cdots\cdots\cdots(2)$$

And for gears 5 and 6, speed ratio is

$$\frac{N_5}{N_6} = \frac{T_6}{T_5} \qquad \cdots\cdots\cdots(3)$$

The speed ratio of the compound gear train is obtained by multiplying the equations (1), (2) and (3),

$$\frac{N_1}{N_2} \times \frac{N_3}{N_4} \times \frac{N_5}{N_6} = \frac{T_2}{T_1} \times \frac{T_4}{T_3} \times \frac{T_6}{T_5}$$

The **advantage** of a compound train over a simple gear train is that a much larger speed reduction from the first shaft to the last shaft can be obtained with small gears. If a simple gear train is used to give a large speed reduction, the last gear has to be very large. Usually, for a speed reduction in excess of 7 to 1, a simple train is not used and a compound train or worm gearing is employed.

7.7.3 Reverted Gear Train

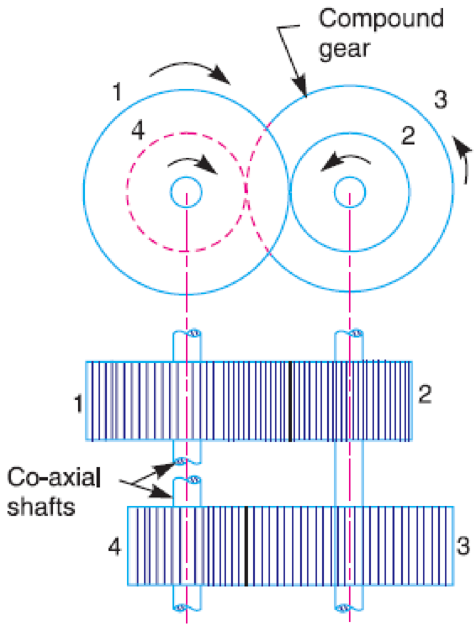


Fig.7.23 - Reverted Gear Train

When the axes of the first gear (i.e. first driver) and the last gear (i.e. last driven or follower) are co-axial, then the gear train is known as a **reverted gear train.**

Gear 1 (i.e. first driver) drives the gear 2 (i.e. first driven or follower) in the opposite direction. Since the gears 2 and 3 are mounted on the same shaft, therefore they form a compound gear and the gear 3 will rotate in the same direction as that of gear 2.

The gear 3 (which is now the second driver) drives the gear 4 (i.e. the last driven or follower) in the same direction as that of gear 1. Thus we see that in a reverted gear train, the motion of the first gear and the last gear is **like.**

Let,

T1 = Number of teeth on gear 1,

r1 = Pitch circle radius of gear 1, and

N1 = Speed of gear 1 in r.p.m.

Similarly,

T2, T3, T4 = Number of teeth on respective gears,

r2, r3, r4 = Pitch circle radii of respective gears, and

N2, N3, N4 = Speed of respective gears in r.p.m.

Since the distance between the centers of the shafts of gears 1 and 2 as well as gears 3 and 4 is the same, therefore

$$r_1 + r_2 = r_3 + r_4$$

Also, the circular pitch or module of all the gears is assumed to be the same; therefore the number of teeth on each gear is directly proportional to its circumference or radius.

$$T_1 + T_2 = T_3 + T_4$$

$$\text{Speedratio} = \frac{\text{Product of number of teeth on drivens}}{\text{Product of number of teeth on drivers}}$$

$$\frac{N_1}{N_4} = \frac{T_2 \times T_4}{T_1 \times T_3}$$

Applications:

The reverted gear trains are used in automotive transmissions**,** lathe back gears, industrial speed reducers, and in clocks (where the minute and hour hand shafts are co-axial).

7.7.4 Epicyclic Gear Train

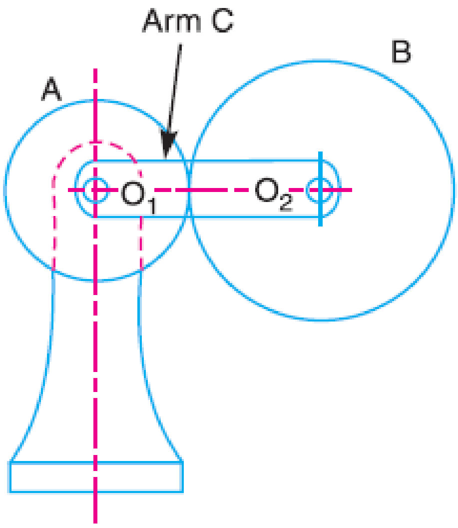


Fig.7.24 - Epicyclic Gear Train

In an epicyclic gear train, the axes of the shafts, over which the gears are mounted, may move relative to a fixed axis. A gear A and the arm C have a common axis at O1 about which they can rotate. The gear B meshes with gear A and has its axis on the arm at O2, about which the gear B can rotate.

If the arm is fixed, the gear train is simple and gear A can drive gear B or **vice- versa**, but if gear A is fixed and the arm is rotated about the axis of gear A (i.e. O1), then the gear B is forced to rotate **upon** and **around** gear A.

Such a motion is called **epicyclic** and the gear trains arranged in such a manner that one or more of their members move upon and around another member is known as **epicyclic gear trains** (**epi.** means upon and **cyclic** means around). The epicyclic gear trains may be **simple** or **compound.**

Applications:

The epicyclic gear trains are useful for transmitting high-velocity ratios with gears of moderate size in a comparatively lesser space. The epicyclic gear trains are used in the back gear of lathe, differential gears of the automobiles, hoists, pulley blocks, wristwatches, etc.

Table 7.3 - Table of Motions

Step No.	Condition of Motion	Revolution of Elements		
		Arm C	Gear A	Gear B
1	Arm fixed - gear A rotates +1 revolution (anticlockwise)	0	+1	$-\frac{T_A}{T_B}$
2	Arm fixed - gear A rotates through + *x* revolutions	0	$+x$	$-x\frac{T_A}{T_B}$
3	Add + *y* revolutions to all elements	$+y$	$+y$	$+y$
4	Total motion	$+y$	$x+y$	$y-x\frac{T_A}{T_B}$

Applications:

The epicyclic gear trains are useful for transmitting high-velocity ratios with gears of moderate size in a comparatively lesser space. The epicyclic gear trains are used in the back gear of lathe, differential gears of the automobiles, hoists, pulley blocks, wristwatches, etc.

7.7.5 Compound Epicyclic Gear Train—Sun and Planet Gear

A compound epicyclic gear train consists of two co-axial shafts S1 and S2, an annulus gear A which is fixed, the compound gear (or planet gear) B-C, the sun gear D, and the arm H. The annulus gear has internal teeth and the compound gear is carried by the arm and revolves freely on a pin of the arm H. The sun gear is co-axial with the annulus gear and the arm but independent of them.

The annulus gear A meshes with the gear B and the sun gear D meshes with the gear C. It may be noted that when the annulus gear is fixed, the sun gear provides the drive and when the sun gear is fixed, the annulus gear provides the drive. In both cases, the arm acts as a follower.

Note: The gear at the center is called the **sun gear** and the gears whose axes move are called **planet gears.**

Let T_A, T_B, T_C, and T_D be the teeth and N_A, N_B, N_C, and N_D be the speeds for the gears *A*, *B*, *C,* and *D* respectively. A little consideration will show that when the arm is fixed and the sun gear *D* is turned anticlockwise, then the compound gear *B-C* and the annulus gear A will rotate in the clockwise direction.

Table 7.4 - Table of motions

Step No.	Conditions of motion	Revolution of Elements			
		Arm	Gear D	Compound Gear (B-C)	Gear A
1	Arm fixe, gear D rotates +1 revolution (anticlockwise)	0	+1	$-\frac{T_D}{T_C}$	$-\frac{T_D}{T_C}\times\frac{T_B}{T_A}$
2	Arm fixed gear D rotates through + *x* revolutions	0	+x	$x\frac{T_D}{T_C}$	$-x\frac{T_D}{T_C}\times\frac{T_B}{T_A}$
3	Add + *y* revolutions to all elements	+y	+y	+y	+y
4	Total motion	+y	x + y	$y-x\frac{T_D}{T_C}$	$y-x\frac{T_D}{T_C}\times\frac{T_B}{T_A}$

7.8 Problems

Ex. 7.1 **Two spur gears have a velocity ratio of 1/3 the driven gear has 72 teeth of 8 mm module and rotates at 300 RPM. Calculate the number of teeth and Speed of driver. What will be**

Solution:

Given data

$VR = 1/3$

$T_2 = 72$ teeth

$m = 8$ mm

$N_2 = 300$

Find

$V_p = ?$

$T_1 = ?$

$$VR = \frac{N_2}{N_1} = \frac{T_1}{T_2}$$

$$\therefore \frac{1}{3} = \frac{300}{N_1} = \frac{T_1}{72}$$

$$\therefore T_1 = 24 \;\&\; N_1 = 900 \text{ rpm}$$

Pitch line velocity,

$$V_P = r_1\omega_1 = r_2\omega_2$$

$$= \frac{2\pi N_1}{60} \times \frac{d_1}{2}$$

$$= \frac{2\pi N_1}{60} \times \frac{mT_1}{2}$$

$$= \frac{2\pi \times 900}{60} \times \frac{8 \times 24}{2}$$

$$= 9047.78 \text{ mm / sec}$$

Ex. 7.2 **The number of teeth of spur gear is 30 and it rotates at 200 RPM. What will be its circular**

Solution :

Given data

$T = 30$

$N = 200$ rpm

$m = 2$ mm

Find:

$P_c = ?$

$V_p = ?$

Circular pitch, $P_c = \pi \cdot m$

$$= \pi \cdot 2$$

$$= 6.28 \text{mm}$$

Pitch line velocity, $V_p = \omega \cdot r$

$$= \frac{2\pi N}{60} \times \frac{d}{2}$$

$$= \frac{2\pi \times 200}{60} \times \frac{2 \times 30}{2}$$

$$= 628.3 mm/s$$

Ex. 7.3 **The following data relate to two meshing gears velocity ratio = 1/3, module = 1mm, Pressure angle 20°, center distance= 200 mm. Determine the number of teeth and the**

Solution :

Given data | Find:

$VR = 1/3$ | $T_1 = ?$

$\emptyset = 20^\circ$ | $T_2 = ?$

$C = 200$ mm | Base circle radius of gear wheel = ?

$m = 4$ mm

$$VR = \frac{N_2}{N_1} = \frac{1}{3} = \frac{T_1}{T_2}$$

$$\therefore T_2 = 3T_1 \qquad \cdots\cdots\cdots(1)$$

Center distance $C = \dfrac{d_1 + d_2}{2}$

$$\therefore 200 = \frac{m(T_1 + T_2)}{2} \qquad \left(\because m = \frac{d}{2}\right)$$

$$\therefore 200 = \frac{4(T_1 + T_2)}{2}$$

$$\therefore T_1 + T_2 = 100 \qquad \cdots\cdots\cdots(2)$$

By solving equation (1) & (2)

$$T_1 = 25$$

$$T_2 = 75$$

No of teeth of the gear wheel $T_2 = 75$

But $m = \dfrac{d_2}{T_2}$

$$\therefore d_2 = mT_2$$

$$\therefore d_2 = 300mm$$

Base circle radius $d_{b2} = \dfrac{d_2}{2}\cos\varphi$

$$= \frac{300}{2} \times \cos 20^\circ$$

$$= 141mm$$

Ex. 7.4	**Each of the gears in a mesh has 48 teeth and a module of 8 mm. The teeth are of a 20° involute profile. The arc of' contact is 2.25 times the circular pitch. Determine**

Solution :

Given data

$T = t = 48$

$m = 8$ mm

$\phi = 20°$

Arc of contact $= 2.25\, P_c$

Find:

Addendum $= ?$

Arc of contact $= 2.25 P_c$

$$= 2.25 \times \pi m$$

$$= 2.25 \times \pi \times 8$$

$$= 56.55 mm$$

Let, $m = \frac{d}{t} = \frac{2r}{T}$

$$\therefore R = r = \frac{mT}{2} = \frac{8 \times 48}{2}$$

$$\therefore R = r = 192 mm$$

$$(\because \text{tooth size same})$$

Also, $R_a = r_a$

$$L.A.C = \frac{L.P.C}{\cos\varphi}$$

$$\therefore 56.55 = \frac{L.P.C}{\cos 20^\circ}$$

$$\therefore L.P.C = 53.14 mm$$

$$L.P.C = \left(\sqrt{R_A^2 - (R\cos\phi)^2} - R\sin\phi\right) + \left(\sqrt{r_A^2 - (r\cos\phi)^2} - r\sin\phi\right)$$

$$\therefore 53.14 = 2\left[\sqrt{R_A^2 - (R\cos\phi)^2}\right] - (R+r)\sin\phi \qquad (\because R_A = r_A)$$

$$\therefore 53.14 = 2\left[\sqrt{R_A^2 - (192\cos 20)^2}\right] - (192+192)\sin 20^\circ$$

$$\therefore 53.14 = 2\left[\sqrt{R_A^2 - 32551.73}\right] - 131.33$$

$$\therefore \sqrt{R_A^2 - 32551.73} = 92.23 mm$$

$$\therefore R_A = 202.63 mm$$

Now $R_A = R +$ Addendum

$$\therefore \text{Addendum} = R_A - R$$

$$\therefore \text{Addendum} = 10.63 mm$$

Ex. 7.5 **Two involute gears in mesh have a 20° pressure angle. The gear ratio is 3 and the number of teeth on the pinion is 24. The teeth have a module of 6 mm. The pitch line velocity is 1.5 m/s and the addendum equal to one module. Determine the angle of action of a pinion**

Solution: Given data

$$\varnothing = 20^\circ$$
$$G = T/t = 3$$
$$t = 24$$
$$m = 6\text{ mm}$$
$$V_p = 1.5\text{ m/s}$$
$$\text{Addendum} = 1\text{ module}$$

Find:

Angle of action of the pinion $=?$

Max. velocity of sliding $=?$

$$r = \frac{mt}{2} = \frac{6\times 24}{2} = 72\text{mm}$$
$$(\because T = 24\times 3 = 72)$$
$$R = \frac{mT}{2} = \frac{6\times 72}{2} = 216\text{mm}$$

$$r_a = r + \text{Add.} = 72 + (1\times 6) = 78\text{mm}$$
$$R_A = R + \text{Add.} = 216 + (1\times 6) = 222\text{mm}$$

Let the length of the path of contact $KL = KP + PL$

$$KP = \left(\sqrt{R_A^{\,2} - (R\cos\varnothing)^2} - R\sin\varnothing\right)$$
$$= \left(\sqrt{222^2 - (216\cos 20^\circ)^2} - 216\sin 20^\circ\right)$$
$$= 16.04\text{mm}$$

$$PL = \left(\sqrt{r_A^{\,2} - (r\cos\varnothing)^2} - r\sin\varnothing\right)$$
$$= \left(\sqrt{78^2 - (72\cos 20^\circ)^2} - 72\sin 20^\circ\right)$$
$$= 14.18\text{mm}$$

$$\text{Arcofcontact} = \frac{\text{Pathofcontact}}{\cos\varphi}$$
$$= \frac{16.04 + 14.18}{\cos 20^\circ}$$
$$= 32.16\text{mm}$$

$$\text{Angleturnedthroughbypinion}(\theta) = \frac{\text{Lengthof arcof contact}\times 360^\circ}{\text{circumferenceofpinion}}$$
$$= \frac{32.16\times 360^\circ}{2\pi\times 72}$$
$$= 25.59^\circ$$

$$\text{Max.velocity of sliding} = (\omega_p + \omega_g) \times KP$$

$$= \left(\frac{V}{r} + \frac{V}{R}\right) \times KP \qquad (\because V = r\omega)$$

$$= \left(\frac{1500}{72} + \frac{1500}{216}\right) \times 16.04$$

$$= 445.6 \text{ mm/sec}$$

Ex. 7.6 **Two involute gears in a mesh have a module of 8 mm and a pressure angle of 20°. The larger gear has 57 while the pinion has 23 teeth. If the addendum on pinion and gear wheels are equal to one module, Determine**

i. **Contact ratio (No. of pairs of teeth in contact)**
ii. **The angle of action of pinion and gear wheel**
iii. **The ratio of sliding to the rolling velocity at the**
   a. **Beginning of the contact.**
   b. **Pitch point.**

Solution: Given data

$\emptyset = 20°$

$m = 8$ mm

$T = 57$

$t = 23$

Addendum $= 1$ module $= 8$ mm

Find:

1. Contact ratio = ?
2. Angle of action of pinion and gear = ?
3. Ratio of sliding to rolling velocity at the
   a. Beginning of contact
   b. Pitch point
   c. End of contact

i. Let the length of the path of contact $KL = KP + PL$

$$KP = \left(\sqrt{R_A^{\,2} - (R\cos\emptyset)^2} - R\sin\emptyset\right)$$

$$= \left(\sqrt{236^2 - (228\cos20°)^2} - 228\sin20°\right)$$

$$= 20.97 \text{ mm}$$

$$PL = \left(\sqrt{r_A^{\,2} - (r\cos\emptyset)^2} - r\sin\emptyset\right)$$

$$= \left(\sqrt{100^2 - (92\cos20°)^2} - 92\sin20°\right)$$

$$= 18.79 \text{mm}$$

$$\text{Arc of contact} = \frac{\text{Path of contact}}{\cos\varphi}$$

$$= \frac{KP + KP}{\cos\varphi}$$

$$= \frac{20.97 + 18.79}{\cos20°}$$

$$= 42.29 \text{mm}$$

$$\text{Contact ratio} = \frac{\text{Length of arc of contact}}{P_c}$$

$$= \frac{42.21}{\pi m} = 1.68 \quad \text{say} \quad 2$$

ii.

$$\text{Angle of action of pinion}(\delta_p) = \frac{\text{Length of arc of contact} \times 360^\circ}{\text{circumference of pinion}}$$

$$= \frac{42.31 \times 360^\circ}{2\pi \times 92}$$

$$= 26.34^\circ$$

$$\text{Angle of action of pinion}(\delta_g) = \frac{\text{Length of arc of contact} \times 360^\circ}{\text{circumference of gear}}$$

$$= \frac{42.31 \times 360^\circ}{2\pi \times 228}$$

$$= 10.63^\circ$$

iii. The ratio of sliding to rolling velocity:

a. Beginning of contact

$$\frac{\text{Sliding velocity}}{\text{Rolling velocity}} = \frac{(\omega_p + \omega_g)KP}{\omega_p r}$$

$$= \frac{\left(\omega_p + \frac{92}{228}\omega_p\right) \times 20.97}{\omega_p \times 92}$$

$$= 0.32$$

b. Pitch point

$$\frac{\text{Sliding velocity}}{\text{Rolling velocity}} = \frac{(\omega_p + \omega_g)KP}{\omega_p r}$$

$$= \frac{(\omega_p + \omega_g) \times 0}{\omega_p r}$$

$$= 0$$

c. End of the contact

$$\frac{\text{Sliding velocity}}{\text{Rolling velocity}} = \frac{(\omega_p + \omega_g)PL}{\omega_p r}$$

$$= \frac{\left(\omega_p + \frac{92}{228}\omega_p\right) \times 18.79}{\omega_p \times 92}$$

$$= 0.287$$

Ex. 7.7 Two 20° gears have a module pitch of 4 mm. The number of teeth on gears 1 and 2 is 40 and 24 respectively. If the gear 2 rotates at 600 rpm, determine the velocity of sliding when the contact is at the tip of the tooth of gear 2. Take addendum equal to one module.

Solution: Given data Find:

$\phi = 20^\circ$ Velocity of sliding = ?

$m = 4$ mm Max. velocity of sliding = ?

$N_p = 600$ rpm

$T = 40$

$t = 24$

Addendum = 1 module

= 4 mm

$$r = \frac{mt}{2} = \frac{4\times24}{2} = 48mm \qquad R = \frac{mT}{2} = \frac{4\times40}{2} = 80mm$$

$$r_a = r + Add. = 48 + (1\times4) = 52mm \qquad R_A = R + Add. = 80 + (1\times4) = 84mm$$

(Note: The tip of the driving wheel is in contact with a tooth of driving wheel at the end of the engagement. So it is required to find the path of recess.)

Path of recess,

$$PL = \left(\sqrt{{r_A}^2 - (r\cos\phi)^2} - r\sin\phi\right)$$

$$= \left(\sqrt{52^2 - (48\cos20^\circ)^2} - 48\sin20^\circ\right)$$

$$= 9.458mm$$

The velocity of sliding,

$$= (\omega_p + \omega_g)\times PL$$

$$= \frac{2\pi}{60}(600 + 360)\times 9.458$$

$$\left(\because \frac{N_g}{N_p} = \frac{t}{T} \Rightarrow N_g = 600\times\frac{24}{40} = 360rpm\right)$$

$$= 956.82mm/sec$$

Path of recess,

$$\therefore KP = \left(\sqrt{{R_A}^2 - (R\cos\phi)^2} - R\sin\phi\right)$$

$$= \left(\sqrt{84^2 - (80\cos20^\circ)^2} - 80\sin20^\circ\right)$$

$$= 10.108mm$$

Max. Velocity of sliding,

$$= (\omega_p + \omega_g)\times KP$$

$$= \frac{2\pi}{60}(600 + 360)\times 10.108$$

$$= 1016.16mm/sec$$

Ex. 7.8 **Two 20° involute spur gears mesh externally and give a velocity ratio of 3. The module is**

3 mm and the addendum is equal to 1.1 module. If the pinion rotates at the 120

Solution : Given data Find:

$\varnothing = 20°$ t_{min} & T_{min} = ?

$VR = 3$ Contact ratio = ?

$m = 3$

$N_P = 120$

Addendum = 1.1 module

I.

$$T = \frac{2A_w}{\left[\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\varnothing}-1\right]}$$

$$\therefore T = \frac{2\times1.1}{\left[\sqrt{1+\frac{1}{3}\left(\frac{1}{3}+2\right)\sin^2 20^\circ}-1\right]}$$

$$\therefore T = 49.44 \text{ teeth}$$

$$\therefore T = 51 \text{teeth}$$ And $$t = \frac{T}{3} = \frac{51}{3} = 17 \text{teeth}$$

II.

$$r = \frac{mt}{2} = \frac{3\times17}{2} = 25.5mm \quad R = \frac{mT}{2} = \frac{3\times51}{2} = 76.5mm$$

$$r_a = r + Add. = 25.5+(1.1\times3) = 28.8mm \quad R_A = R + Add. = 76.5+(1.1\times3) = 28.8mm$$

$$\text{Contactratio} = \frac{\text{Lengthofpathofcontact}}{\cos\varphi \times P_c}$$

$$= \frac{\left(\sqrt{R_A{}^2-(R\cos\varnothing)^2}-R\sin\varnothing\right)+\left(\sqrt{r_A{}^2-(r\cos\varnothing)^2}-r\sin\varnothing\right)}{\cos20^\circ\times\pi\times3}$$

$$= \frac{\left(\sqrt{79.8^2-(76.5\cos20^\circ)^2}-76.5\sin20^\circ\right)+\left(\sqrt{28.8^2-(25.5\cos0^\circ)^2}-25.5\sin20^\circ\right)}{\cos20^\circ\times\pi\times3}$$

$$= 1.78$$

Thus 1 pair of teeth will always remain in contact whereas, for 78 % of the time, 2 pairs of teeth will be in contact.

Ex. 7.9 **Two involute gears in a mesh have a velocity ratio of 3. The arc of approach is not to be less than the circular pitch when the pinion is the driver The pressure angle of the involute teeth is 20°. Determine the least no of teeth on each gear. Also, find the addendum of the**

Solution: Given data Find:

$\varnothing = 20°$ leastno of teeth on the each gear = ?

$VR = 3$ Addendum = ?

Arc of approach = Circular pitch $= \pi \cdot m$

$$\therefore \text{Path of approach} = \text{Arc of approach} \times \cos 20^\circ$$

$$= \pi \cdot m \cdot \cos 20^\circ$$

$$= 2.952m \qquad \cdots\cdots\cdots(1)$$

Let the max length of path of approach $= r \sin\varphi$

$$= \frac{mt}{2} \sin 20^\circ$$

$$= 0.171mt \qquad \cdots\cdots(2)$$

From eq. 1. and 2,

$$\therefore 0.171mt = 0.2952m$$

$$\therefore t = 17.26 \cong 18 \text{ teeth}$$

$$T = 18 \times 3 = 54 \text{ teeth}$$

Max. Addendum of the wheel,

$$A_{wmax} = \frac{mt}{2}\left[\sqrt{1 + \frac{1}{G}\left(\frac{1}{G} + 2\right)\sin^2\varphi} - 1\right]$$

$$= \frac{m \times 54}{2}\left[\sqrt{1 + \frac{1}{3}\left(\frac{1}{3} + 2\right)\sin^2 20^\circ} - 1\right]$$

$$= 1.2m$$

Ex. 7.10 **Two 20° involute spur gears have a module of 10 mm. The addendum is equal to one module. The larger gear has 40 teeth while the pinion has 20 teeth will the gear interfere**

Solution: Given Data:

Ø = 20°

m = 10 mm

Addendum = 1 module

$= 1 \times 10$

$= 10mm$

To be Calculated: Interference or not?

Let the pinion is the driver,

t = 20 teeth

T = 40 teeth

$$r = \frac{mt}{2} = \frac{10 \times 20}{2} = 100mm \qquad R = \frac{mT}{2} = \frac{10 \times 40}{2} = 200mm$$

$$r_a = r + Add. = 100 + 10 = 110mm \qquad R_A = R + Add. = 200 + 10 = 210mm$$

$$\text{Pathofapproach} = \left(\sqrt{R_A{}^2 - (R\cos\varnothing)^2} - R\sin\varnothing\right)$$

$$= \left(\sqrt{210^2 - (200\cos 20^\circ)^2} - 200\sin 20^\circ\right)$$

$$= 25.29mm$$

To avoid interference,

$$\text{Maxlengthofpathofapproach} = r\text{Sin}\varphi$$

$$= 100 \times \text{Sin}20^\circ$$

$$= 34.20mm > 25.29mm$$

Therefore, interference will **not occur.**

Ex. 7.11 **Two 20° involute spur gears have a module of 10 mm. The addendum is one module. The**
larger gear has 50 teeth and the pinion has l3 teeth. Does interference occur? If it

Solution: Given Data:

$\varnothing = 20°$

M = 10 mm

Addendum = 1 module = 10 mm

T = 50 and t = 13

To be Calculated: Interference or not?

$$r = \frac{mt}{2} = \frac{10 \times 13}{2} = 65mm \qquad R = \frac{mT}{2} = \frac{10 \times 50}{2} = 250mm$$

$$r_a = r + Add. = 65 + 10 = 75mm \; R_A = R + Add. = 250 + 10 = 260mm$$

$$R_{amax} = \sqrt{(R\text{Cos}\varphi)^2 + (R\text{Sin}\varphi + r\text{Sin}\varphi)}$$

$$= \sqrt{(250\text{Cos}20^\circ)^2 + (250\text{Sin}20^\circ + 65\text{Sin}20^\circ)}$$

$$= 258.45mm$$

Here actual addendum radius R_a (260 mm) > $R_{a\,max}$ value

So interference will occur.

The new value of ø can be found by comparing

$$R_{a\,max} = R_a$$

$$\therefore R_a = R_{a\,max}$$

$$\therefore R_a = \sqrt{\left(RCos\varphi\right)^2 + \left(RSin\varphi + rSin\varphi\right)}$$

$$\therefore 260 = \sqrt{\left(250Cos\varphi\right)^2 + \left(250Sin\varphi + 65Sin\varphi\right)^2}$$

$$\therefore 260^2 = \left(250Cos\varphi\right)^2 + \left(250Sin\varphi + 65Sin\varphi\right)^2$$

$$\therefore Cos^2\varphi = 0.861$$

$$\therefore \varphi = 21.88^{\circ}$$

Note: If pressure angle is increased to 21.88° interference can be avoided.

Ex. 7.12 **The following data related to meshing involute**

gears: No. of teeth on gear wheel = 60

Pressure angle = 20°

Gear ratio = 1.5

Speed of gear wheel = 100 rpm

Module = 8 mm

The addendum on each wheel is such that the path of approach and path of recess on

each side are 40 % of the maximum possible length each. Determine the addendum

Solution: Given Data:

T = 60

Ø = 20°

G = 1.5

N_g = 100 rpm

m = 8 mm

To be Calculated:

Addendum for gear and pinion

Length of arc of contact

Let pinion is the driver.

Max. Possible length of the path of approach $= r\sin\phi$

$\therefore$ The actual length of the path of approach $= 0.4\, r\sin\phi$ (Given in data)

Same way, the actual length of the path of the recess $= 0.4\, R\sin\phi$ (Given in data)

$$\therefore 0.4\ r\sin\phi = \left(\sqrt{R_A{}^2 - \left(R\cos\text{ø}\right)^2} - R\sin\text{ø}\right)$$

$$\therefore 0.4 \times 160\sin 20 = \left(\sqrt{R_A{}^2 - \left(240\cos 20\right)^2} - 240\sin 20\right)$$

$$\therefore R_a = 248.33\ \text{mm}$$

$$\therefore \text{Addendumof wheel} = 248.3 - 240 = 8.3\text{mm}$$

Also,

$$0.4\,R\sin\phi = \sqrt{r_A^{\,2} - (r\cos\phi)^2} - r\sin\phi$$

$$\therefore 0.4 \times 240 \times \sin 20 = \sqrt{r_A^{\,2} - (160\cos 20)^2} - 160\sin 20$$

$$\therefore r_a = 173.98 = 174\text{ mm}$$

$$\therefore \text{Addendum of pinion} = 174 - 160 = 14\text{ mm}$$

$$\text{Length of Arc of contact} = \frac{\text{Path of contact}}{\cos\varphi}$$

$$= \frac{(r\sin\varphi + R\sin\phi) \times 0.4}{\cos\phi}$$

$$= \frac{(160 + 240) \times \sin 20 \times 0.4}{\cos 20}$$

$$= 58.2\text{ mm}$$

Ex. 7.13 **A pinion of 20 ° involute teeth rotating at 274 rpm meshes with gear and provides a gear ratio of 1.8. The no. of teeth on the pinion is 20 and the module is 8 mm. If interference is just avoided, determine:**

1. Addendum on wheel and pinion

2. Path of contact

3. Maximum velocity of sliding on both side of pitch point

Solution : Given data

$\phi = 20°$

$m = 8\text{ mm}$

$N_p = 275\text{ rpm}$

$T = 36$

$t = 20$

Find:

1. Addndum on wheel and pinion = ?
2. Path of contact = ?
3. Max. velocity of sliding on both side of pitch point = ?

Max. Addendum on wheel

$$\therefore A_{w\,max} = R\left[\sqrt{1 + \frac{1}{G}\left(\frac{1}{G} + 2\right)\sin^2\phi} - 1\right]$$

$$\therefore A_{w\,max} = 144\left[\sqrt{1 + \frac{1}{1.8}\left(\frac{1}{1.8} + 2\right)\sin^2 20} - 1\right]$$

$$= 11.5\text{ mm}$$

Max. Addendum on pinion

$$\therefore A_{p\,max} = r\left[\sqrt{1 + G(G+2)\sin^2\phi} - 1\right]$$

$$\therefore A_{p\,max} = 80\left[\sqrt{1 + 1.8(1.8 + 2)\sin^2 20} - 1\right]$$

$$= 27.34\text{ mm}$$

Path of contact when interference is just avoided

$$= \text{Max. path of approach} + \text{Max. path of recess}$$

$$= r\sin\phi + R\sin\phi$$

$$= 80\sin 20 + 144\sin 20$$

$$= 27.36 + 49.25$$

$$= 76.6\,\text{mm}$$

The velocity of sliding on one side of the approach

$$= (\omega_p + \omega_g)\,\text{Path of approach} \qquad \left(\begin{aligned} \omega_p &= \frac{2\pi\times 275}{60} = 28.8\,\text{rad/sec} \\ \omega_g &= \frac{28.8}{1.8 - G} = 16\,\text{rad/sec} \end{aligned} \right)$$

$$= (28.8 + 16)\times 27.36$$

$$= 1225.72\,\text{mm/sec}$$

The velocity of sliding on side of the path of recess

$$= (\omega_p + \omega_g)\,\text{Path of recess}$$

$$= (28.8 + 16)\times 49.25$$

$$= 2206\,\text{mm/sec}$$

Ex. 7.14 **A pinion of 20 involute teeth and 125 mm pitch circle diameter drives a rack. The addendum of both pinion and rack is 6.25 mm. What is the least pressure angle which can be used to avoid interference? With this pressure angle, find the length of the arc of**

Solution: Given data

$T = 20$

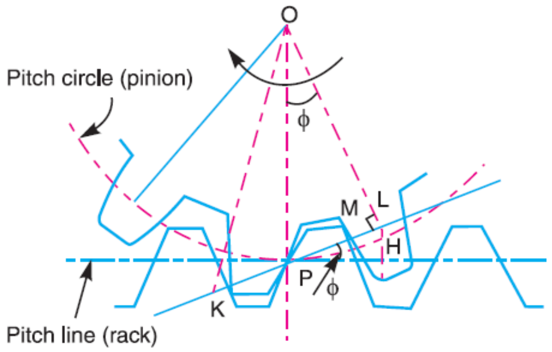
$d = 125\text{mm}$

$r = OP = 62.5\,\text{mm}$

Addendum for rack / pinion, $LH = 6.25\,\text{mm}$

Find:

1. Least pressure angle to avoid interference = ?
2. Lengthof arc of contact = ?
3. Min. no. of teeth in contact = ?



Least pressure angle to avoid interference

Let, ϕ = Least pressure angle to avoid interference

We know that for no interference, rack addendum,

From fig..... $LH = PL\sin\phi$

$$= r\sin\phi \times \sin\phi$$

$$= r\sin^2\phi$$

$$\therefore \sin^2\phi = \frac{LH}{r} = \frac{6.25}{62.5}$$

$$\therefore \phi = (18.4349)^\circ$$

Length of the arc of contact

Now, $KL = \sqrt{OK^2 - OL^2}$

$$= \sqrt{(OP+6.25)^2 - (r\cos\phi)^2}$$

$$= \sqrt{(62.5+6.25)^2 - (62.5\times\cos 18.439^\circ)^2}$$

$$= 34.8\ \text{mm}$$

$$\text{Length of Arc of Contact} = \frac{KL}{\cos\phi} = \frac{34.8}{\cos 18.439^\circ} = 36.68\ \text{mm}$$

$$\text{Min. no. of teeth in contact} = \frac{\text{Length of arc of contact}}{p_c}$$

$$= \frac{\text{Length of arc of contact}}{\pi \cdot m}$$

$$= \frac{36.68}{19.64}$$

$$= 1.87$$

$$\cong 2$$

Ex. 7.15 **In a spiral gear drive connecting two shafts, the approximate center distance is 400 mm and the speed ratio = 3. The angle between the two shafts is 50° and the normal pitch is 18 mm. The spiral angles for the driving and driven wheels are equal. Find:**

1. Number of teeth on each wheel,

2. Exact center distance,

3. The efficiency of the drive, if friction angle = 6°,

Solution: Given data:

$L = 400\ \text{mm}$ $\quad \theta = 50^\circ$ $\quad G = \frac{T_2}{T_1} = 3$

$\phi = 6$ $\quad P_N = 18\ \text{mm}$

1. No. of teeth on a wheel:

$$L = \frac{P_N \cdot T_1}{2\pi}\left[\frac{1}{\cos\alpha_1} + \frac{G}{\cos\alpha_2}\right]$$

$$\therefore 400 = \frac{P_N \cdot T_1}{2\pi} \times \frac{1+G}{\cos\alpha_1}$$

$$\therefore 400 = \frac{18 \cdot T_1}{2\pi} \times \frac{1+3}{\cos 25^\circ} \qquad \left(\begin{array}{l} \because \alpha_1 = \alpha_2 \\ \theta = \alpha_1 + \alpha_2 \\ \therefore 50 = 2\alpha_1 \\ \therefore \alpha_1 = 25^\circ \end{array}\right)$$

$$\therefore T_1 = 31.64 \simeq 32$$

$$\therefore T_2 = 3T_1 = 96$$

2. Exact center distance (L):

$$L = \frac{P_N \cdot T_1}{2\pi}\left[\frac{1}{\cos\alpha_1} + \frac{G}{\cos\alpha_2}\right]$$

$$= \frac{P_N \cdot T_1}{2\pi}\left[\frac{1+G}{\cos\alpha_1}\right] \qquad (\because \alpha_1 = \alpha_2)$$

$$= \frac{18 \cdot 32}{2\pi}\left[\frac{1+3}{\cos 25^\circ}\right]$$

$$= 404.600\ \text{mm}$$

3. The efficiency of drive:

$$\eta = \frac{\cos(\alpha_2 + \phi) \times \cos\alpha_1}{\cos(\alpha_1 - \phi) \times \cos\alpha_2}$$

$$= \frac{\cos(\alpha_1 + \phi)}{\cos(\alpha_1 - \phi)} \qquad (\because \alpha_1 = \alpha_2)$$

$$= \frac{\cos(25+6)}{\cos(25-6)}$$

$$= 90.655\ \%$$

4. Maximum efficiency:

$$\eta_{max} = \frac{\cos(\theta + \phi) + 1}{\cos(\theta - \phi) + 1}$$

$$= \frac{\cos(50+6) + 1}{\cos(50-6) + 1}$$

$$= 90.685\ \%$$

Ex. 7.16 **A drive on a machine tool is to be made by two spiral gear wheels, the spirals of which are of the same hand and has a normal pitch of 12.5 mm. The wheels are of equal diameter and the center distance between the axes of the shafts is approximately 134**
mm. The angle between the shafts is 80° and the speed ratio 1.25. Determine:
1. the spiral angle of each wheel,
2. The number of teeth on each wheel,
3. The efficiency of the drive, if the friction angle is 6°, and

Solution : Given data:

$$P_N = 12.5\,mm$$
$$L = 134\,mm$$
$$G = 1.25$$
$$\theta = 80^\circ$$

1. The spiral angle of each wheel

Weknow that........

$$\therefore \frac{d_2}{d_1} = \frac{T_2 Cos\alpha_1}{T_1 Cos\alpha_2}$$

$$\therefore T_1 Cos\alpha_2 = T_2 Cos\alpha_1 \qquad (\because d_1 = d_2)$$

$$\therefore Cos\alpha_1 = 1.25 Cos\alpha_2 \qquad (\because \frac{T_1}{T_2} = G = 1.25)$$

$$\therefore Cos\alpha_1 = 1.25\, Cos(\theta - \alpha_1) \qquad (\because \alpha_1 + \alpha_2 = \theta)$$

$$\therefore Cos\alpha_1 = 1.25\, Cos(80 - \alpha_1)$$

$$\therefore Cos\alpha_1 = 1.25\left(Cos80 \cdot Cos\alpha_1 + Sin80 \cdot Sin\alpha_1\right)$$

$$\left(\because Cos(A - B) = CosA \cdot CosB + SinA \cdot SinB\right)$$

$$By \quad solving,$$
$$\tan\alpha_1 = 0.636$$
$$\therefore \alpha_1 = 32.46^\circ$$
$$and \qquad \alpha_2 = 80^\circ - 32.46^\circ = 47.54^\circ$$

2. The efficiency of drive:

$$\eta = \frac{\cos(\alpha_2 + \phi) \times \cos\alpha_1}{\cos(\alpha_1 - \phi) \times \cos\alpha_2}$$

$$= \frac{\cos(47.24 + 6) \times \cos 32.46}{\cos(32.46 - 6) \times \cos 47.24}$$

$$= 83\,\%$$

3. No. of teeth on the wheel:

$$L=\frac{d_1+d_2}{2}$$

$$\therefore 134=\frac{2d_1}{2} \qquad (\because d_1=d_2)$$

$$\therefore d_1=134\,mm$$

$$\text{Let}\quad p_{c1}=\frac{\pi d_1}{T_1} \Rightarrow d_1=\frac{p_{c1}\cdot T_1}{\pi}$$

$$\therefore d_1=\frac{P_N}{\cos\alpha_1}\times\frac{T_1}{\pi}$$

$$\therefore T_1=\frac{d_1\cdot\cos\alpha_1\cdot\pi}{P_N}$$

$$\therefore T_1=\frac{134\times\cos32.24\times\pi}{12.5}$$

$$\therefore T_1=28.4\simeq 30\,nos.$$

$$\text{Now, } G=\frac{T_1}{T_2}=1.25 \Rightarrow T_2=\frac{T_1}{G}=\frac{30}{1.25}$$

$$T_2=24\,nos.$$

4. Maximum efficiency:

$$\eta_{max}=\frac{\cos(\theta+\phi)+1}{\cos(\theta-\phi)+1}$$

$$=\frac{\cos(80+6)+1}{\cos(80-6)+1}$$

$$=83.8\,\%$$

Ex. 7.17 **The addendum of the teeth is 0.84 module and the power component is 0.95 times the normal thrust. Find the minimum no. of teeth on the gear wheel and the arc of contact (in**

I. The gear ratio is unity

II. The gear ratio is 3

Solution : Here $A_w=0.84$

$$\text{Cos}\phi=0.95 \Rightarrow \phi=18.19^\circ$$

$$\therefore \sin\phi=0.3122$$

I. Gear ratio is unity

Let, min. no of teeth on gear wheel T

$$\therefore T = \frac{2A_w}{\left[\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\phi}-1\right]}$$

$$= \frac{2A_w \cdot G}{\left[\sqrt{G^2+(1+2G)\sin^2\phi}-G\right]}$$

$$= \frac{2\times 0.84\times 1}{\left[\sqrt{1^2+(1+2)(0.3123)^2}-1\right]}$$

$$= 12.73$$

$$\therefore T \cong 13 \text{ teeth}$$

$$\therefore t \cong 13 \text{ teeth}$$

Length of the arc of contact:

$$L.P.C = \left(\sqrt{R_A{}^2-(R\cos\phi)^2}-R\sin\phi\right)+\left(\sqrt{r_A{}^2-(r\cos\phi)^2}-r\sin\phi\right)$$

$$= 2\left(\sqrt{r_A{}^2-(r\cos\phi)^2}-r\sin\phi\right) \qquad \left(\begin{aligned} \because r &= \frac{m\cdot t}{2} = \frac{m\cdot 13}{2} = 6.5m \\ r_A &= r + \text{addendum} \\ &= 6.5m + 0.84m \\ &= 7.34m \end{aligned}\right)$$

$$= 2m\left(\sqrt{(7.34)^2-(6.5\times 0.95)^2}-(6.5\times 0.3123)\right)$$

$$= 3.876m$$

$$L.A.C = \frac{3.876m}{\cos\phi} = \frac{3.876m}{0.95}$$

$$\therefore L.A.C = 4.08m$$

II. Gear ratio G = 3

Let, min. no of teeth on gear wheel T

$$\therefore T = \frac{2A_w}{\left[\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\phi}-1\right]}$$

$$= \frac{2A_w \cdot G}{\left[\sqrt{G^2+(1+2G)\sin^2\phi}-G\right]}$$

$$= \frac{2\times 0.84\times 3}{\left[\sqrt{3^2+(1+2\times 3)(0.3123)^2}-3\right]}$$

$$= 45.11$$

$$\therefore T \cong 45 \text{ teeth}$$

$$\therefore t \cong 15 \text{ teeth}$$

Length of the arc of contact:

$$\text{L.P.C} = \left(\sqrt{R_A{}^2-(R\cos\phi)^2}-R\sin\phi\right)+\left(\sqrt{r_A{}^2-(r\cos\phi)^2}-r\sin\phi\right)\cdots\cdots\cdots\cdots\cdots(1)$$

$$\left(\begin{array}{l} \because r = \frac{m\cdot t}{2} = \frac{m\cdot 15}{2} = 7.5m \\ r_A = r + \text{addendum} \\ \quad = 7.5m + 0.84m \\ \quad = 8.34m \\ R = \frac{mT}{2} = \frac{m\times 45}{2} = 22.5m \\ R_A = R + \text{addendum} \\ \quad = 22.5m + 0.84m \\ \quad = 23.34m \end{array}\right)$$

putting all values in equation (1)

$$= \left(\sqrt{(23.34m)^2-(22.5m\times 0.95)^2}-22.5m\times 0.3122\right)+\left(\sqrt{(8.34m)^2-(7.5m\times 0.95)^2}-7.5m\times 0.3122\right)$$

$$= 4.343m$$

$$\text{L.A.C} = \frac{4.343m}{\cos\phi} = \frac{3.876m}{0.95}$$

$$\therefore \text{L.A.C} = 4.57m$$

III. Pinion gear with a rack

Min. no. of teeth on pinion t

$$t = \frac{2A_R}{\sin^2\phi} = \frac{2\times 0.84}{(0.3123)^2}$$

$$\therefore t = 17.23$$

$$\therefore t \cong 18$$

Length of the arc of contact:

$$L.P.C = \left(\sqrt{R_A^{\ 2} - (R\cos\phi)^2} - R\sin\phi\right) + \left(\sqrt{r_A^{\ 2} - (r\cos\phi)^2} - r\sin\phi\right)$$

$$= 2\left(\sqrt{r_A^{\ 2} - (r\cos\phi)^2} - r\sin\phi\right) \qquad (\because \text{assume rack and pinion same dimension})$$

$$\left(\begin{aligned} &\because r = \frac{mt}{2} = \frac{18m}{2} = 9m \\ &r_A = r + \text{addendum} \\ &\quad = 9m + 0.84m \\ &\quad = 9.84m \end{aligned}\right)$$

$$= 2\left(\sqrt{(9.84)^2 - (9m \times 0.95)^2} - 9m \times 0.3123\right)$$

$$= 4.12m$$

$$L.A.C = \frac{4.12m}{\cos\phi} = \frac{4.12m}{0.95}$$

$$\therefore L.A.C = 4.337m$$

Ex. 7.18 **The gearing of a machine tool is shown in the figure. The motor shaft is connected to gear A and rotates at 975 RPM. The gear wheels B, C, D, and E are fixed to parallel shafts rotating together. The final gear F is fixed on the output shaft. What is the speed of gear F? The number of teeth on each gear is as given below:**

Gear	**A**	**B**	**C**	**D**	**E**	**F**
No. of teeth	**20**	**50**	**25**	**75**	**26**	**65**

Solution: Given Data:

$T_A = 20$ $\qquad N_F = ?$

$T_B = 50$

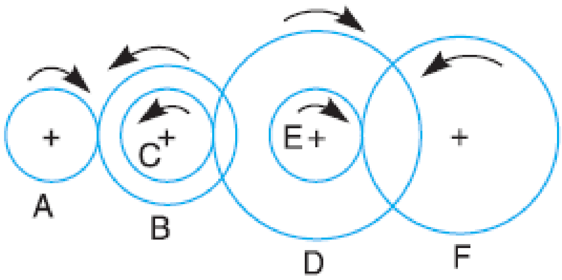
$T_C = 25$

$T_D = 75$

$T_E = 26$

$T_F = 65$

$N_A = 975 \ \text{rpm}$

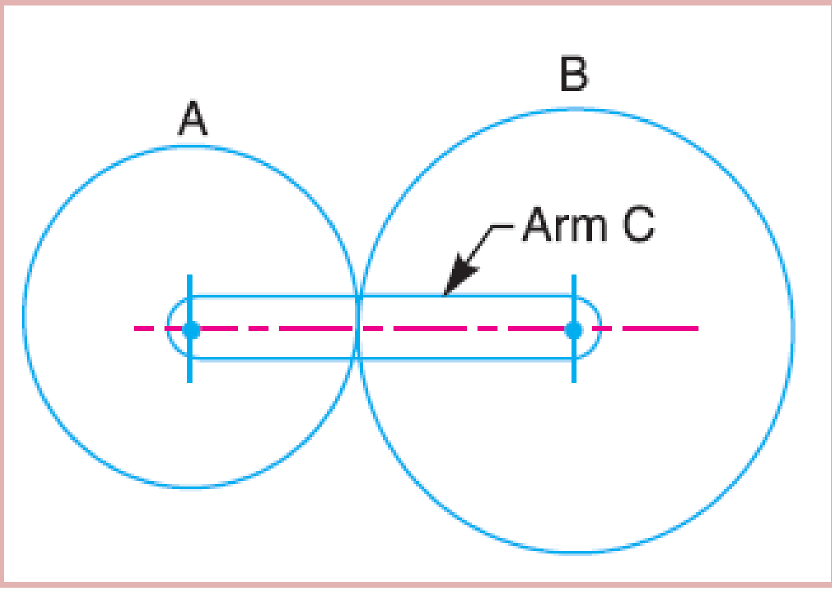


$$\frac{N_F}{N_A} = \frac{T_A}{T_B} \times \frac{T_C}{T_D} \times \frac{T_E}{T_F}$$

$$\therefore \frac{N_F}{975} = \frac{20}{50} \times \frac{20}{75} \times \frac{26}{65}$$

$$\boxed{\therefore N_F = 52 \ \text{rpm}}$$

Ex. 7.19 **In an epicyclic gear train, an arm carries two gears A and B having 36 and 45 teeth respectively. If the arm rotates at 150 rpm in the anticlockwise direction about the center of the gear A which is fixed, determine the speed of gear B. If the gear A instead of being**



Solution: Given Data:

$T_A = 36$ Gear Afixed $\Rightarrow N_B = ?$

$T_B = 45$ $N_A = -300(\text{Clockwise}) \Rightarrow N_B = ?$

$N_C = 150(\text{Anticlockwise})$

Sr. No.	Condition of motion	Revolution of element		
		Arm C	Gear A	Gear B
1	Arm fixe, gear A rotates +1 revolution(anticlockwise)	0	+1	$-\frac{T_A}{T_B}$
2	Arm fixed gear A rotates through $+x$ revolutions	0	$+x$	$-x\frac{T_A}{T_B}$
3	Add $+y$ revolutions to all elements	$+y$	$+y$	$+y$
4	Total motion	$+y$	$x+y$	$y - x\frac{T_A}{T_B}$

1. Speed of gear B (N_B) when gear A is

fixed Here, gear A fixed

$$\Rightarrow x + y = 0$$
$$\Rightarrow x + 150 = 0$$
$$\Rightarrow x = -150$$

Speed of gear B (N_B)

$$= y - x\frac{T_A}{T_B}$$

$$= y - (-150)\frac{36}{45}$$

$$\boxed{= +270\text{rpm (Anticlockwise)}}$$

2. Speed of gear B (N_B) when gear N_A = -300 (Clockwise) Here given

$$x + y = -300$$
$$\therefore x + 150 = -300$$
$$\therefore x = -450 \quad \text{rpm}$$

Speed of gear B (N_B)

$$= y - x\frac{T_A}{T_B}$$

$$= 150 - (-450)\frac{36}{45}$$

$$\boxed{= +510 \quad \text{rpm(Anti clockwise)}}$$

Ex. 7.20 **In a reverted epicyclic gear train, the arm A carries two gears B and C and a compound gear D - E. The gear B meshes with gear E and the gear C meshes with gear D. The number of teeth on gears B, C and D are 75, 30 and 90 respectively. Find the speed and direction**

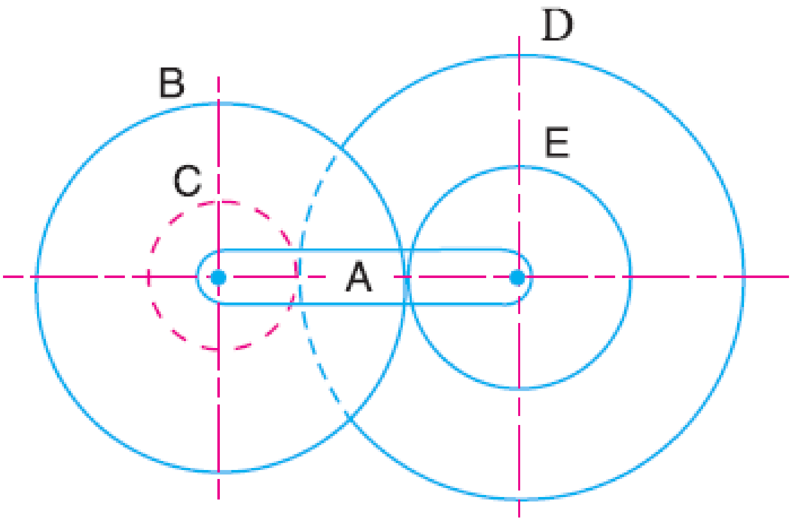
Solution: Given Data:

$T_B = 75$ Gear B fixed $\Rightarrow N_C = ?$

$T_C = 30$ $N_A = -100 \Rightarrow N_C = ?$

$T_D = 90$

$N_A = -100$(Clockwise)



Let $\quad d_C + d_D = d_B + d_E \qquad (r_C + r_D = r_B + r_E)$

$\therefore T_C + T_D = T_B + T_E$

$\therefore 30 + 90 = 75 + T_E$

$\therefore T_E = 45$

Sr. No.	Condition of motion	Revolution of element			
		Arm C	Gear A	Gear B	Gear C
1	Arm fixe, gear A rotates +1 revolution(anticlockwise)	0	+1	$-\frac{T_E}{T_B}$	$-\frac{T_D}{T_C}$
2	Arm fixed gear A rotates through + *x* revolutions	0	$+x$	$-x\frac{T_E}{T_B}$	$-x\frac{T_D}{T_C}$
3	Add + *y* revolutions to all elements	$+y$	$+y$	$+y$	$+y$
4	Total motion	$+y$	$x+y$	$y-x\frac{T_E}{T_B}$	$y-x\frac{T_D}{T_C}$

$$\text{Gear B is fixed} \Rightarrow y - x\frac{T_E}{T_B} = 0$$

$$\Rightarrow -100 - x\frac{45}{75} = 0$$

$$\Rightarrow x = -166.67$$

$$\text{Speed of gear C } (N_C) = y - x\frac{T_D}{T_C}$$

$$= -100 - (-166.67) \times \frac{90}{30}$$

$$\boxed{= +400 \text{ rpm(Anti clockwise)}}$$

Sr. No.	Condition of motion	Revolution of element			
		Arm C	Gear A	Gear B	Gear C
1	Arm fixe, gear A rotates +1 revolution(anticlockwise)	0	+1	$-\frac{T_B}{T_E}$	$+\frac{T_B}{T_E}\times\frac{T_D}{T_C}$
2	Arm fixed gear A rotates through + *x* revolutions	0	$+x$	$x\frac{T_B}{T_E}$	$+x\frac{T_B}{T_E}\times\frac{T_D}{T_C}$
3	Add + *y* revolutions to all elements	$+y$	$+y$	$+y$	$+y$
4	Total motion	$+y$	$x+y$	$y-x\frac{T_B}{T_E}$	$y+x\frac{T_B}{T_E}\times\frac{T_D}{T_C}$

From fig $\quad (r_C + r_D = r_B + r_E)$

$$\therefore T_C + T_D = T_B + T_E$$
$$\therefore T_E = 90 + 30 - 75$$
$$\therefore T_E = 45$$

When gear B is fixed

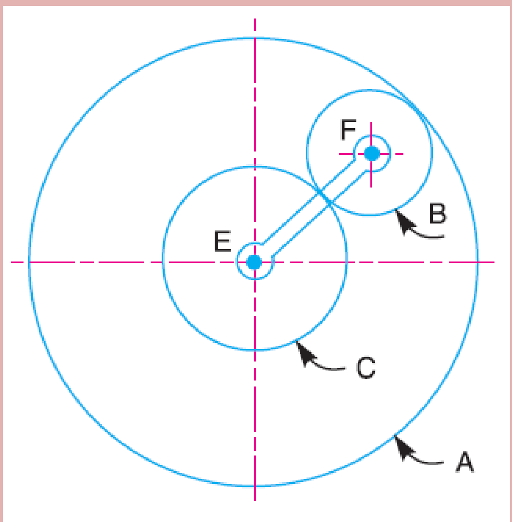
$$\therefore x + y = 0$$
$$\therefore x + (-100) = 0$$
$$\therefore x = 100$$

Now $\quad N_C = y + x \dfrac{T_B}{T_E} \times \dfrac{T_D}{T_C}$

$$= -100 + 100 \times \frac{75}{45} \times \frac{90}{30}$$

$$\boxed{N_C = 400 \text{ rpm(Anticlockwise)}}$$

Ex. 7.21 **An epicyclic gear consists of three gears A, B and C as shown in the figure. The gear A has 72 internal teeth and gear C has 32 external teeth. The gear B meshes with both A and C and is carried on an arm EF which rotates about the center of A at 18 RPM. If the**



Solution: Given Data:

$T_B = 72$ (Internal) $\qquad$ Gear A fixed $\Rightarrow N_B = ?$

$T_C = 32$ (External) $\qquad \Rightarrow N_C = ?$

Arm EF = 18rpm

From the geometry of the figure,

$$r_A = r_C + 2r_B$$
$$\therefore T_A = T_C + 2T_B$$
$$\therefore T_B = 20$$

Sr. No.	Condition of motion	Revolution of element			
		Arm C	Gear A	Gear B	Gear C
1	Arm fixe, gear A rotates +1 revolution (anticlockwise)	0	+1	$-\frac{T_C}{T_B}$	$-\frac{T_C}{T_B}\times\frac{T_B}{T_A}=-\frac{T_C}{T_A}$
2	Arm fixed gear A rotates through + *x* revolutions	0	+x	$-x\frac{T_C}{T_B}$	$-x\frac{T_C}{T_A}$
3	Add + *y* revolutions to all elements	+y	+y	+y	+y
4	Total motion	y	x+y	$y-x\frac{T_C}{T_B}$	$y-x\frac{T_C}{T_A}$

1. Speed of gear C (N_c)

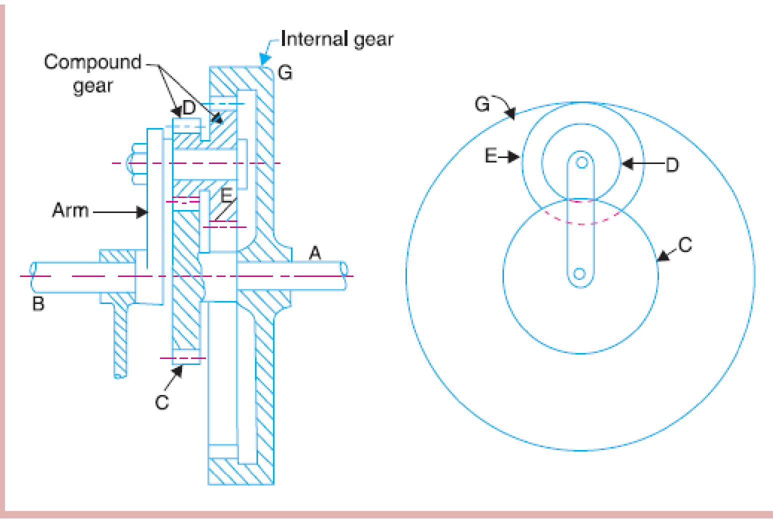
$$\text{Gear A is fixed} \Rightarrow y - x\frac{T_C}{T_A} = 0$$
$$\Rightarrow -18 - x\frac{32}{72} = 0$$
$$\Rightarrow x = -40.5$$

$$\text{Speed of gear C } (N_C) = x + y$$
$$= 40.5 + 18$$
$$\boxed{= 58.5 \text{ rpm (in the direction of arm)}}$$

2. Speed of gear B (N_B)

$$\text{Speed of gear B} = y - x\frac{T_C}{T_B}$$
$$= -18 - 40.5 \times \frac{32}{20}$$
$$= -46.8 \text{ rpm}$$
$$\boxed{= 46.8 \text{ rpm (in the opposite direction of arm)}}$$

Ex. 7.22 **Two shafts A and B are co-axial. A gear C (50 teeth) is rigidly mounted on shaft A. A compound gear D-E gears with C and an internal gear G. D has 20 teeth and gears with C and E has 35 teeth and gears with an internal gear G. The gear G is fixed and is concentric with the shaft axis. The compound gear D-E is mounted on a pin which projects from an arm keyed to the shaft B. Sketch the arrangement and find the number of teeth on internal gear G assuming that all gears have the same module. If the shaft A rotates at 110 rpm,**



Solution: Given Data:

$T_C = 50$ No. of teeth on internal gear = ?

$T_D = 20$ Speed of shaft B = ?

$T_E = 35$

$N_C = 110$ (Rotation of shaft)

Sr. No.	Condition of motion	Revolution of element			
		Arm C	Gear C (Shaft A)	Compound Gear (D-E)	Gear G
1	Arm fixe, gear A rotates +1 revolution(anticlockwise)	0	+1	$-\frac{T_C}{T_D}$	$-\frac{T_C}{T_D}\times\frac{T_E}{T_G}$
2	Arm fixed gear A rotates through $+x$ revolutions	0	+x	$-x\frac{T_C}{T_D}$	$-x\frac{T_C}{T_D}\times\frac{T_E}{T_G}$
3	Add $+y$ revolutions to all elements	+y	+y	+y	+y
4	Total motion	y	x+y	$y-x\frac{T_C}{T_D}$	$y-x\frac{T_C}{T_D}\times\frac{T_E}{T_G}$

From the geometry of the figure,

$$\frac{d_G}{2}=\frac{d_C}{2}+\frac{d_D}{2}+\frac{d_E}{2}$$

$$\therefore d_G = d_C + d_D + d_E$$

$$\therefore T_G = T_C + T_D + T_E$$

$$\therefore T_G = 50 + 20 + 35$$

$$\therefore T_G = 105$$

Speed of shaft B

Here given gear G is fixed

$$\therefore y - x\frac{T_C}{T_D}\times\frac{T_E}{T_G} = 0$$

$$\therefore y - x\frac{50}{20}\times\frac{35}{105} = 0$$

$$\therefore y - x\times\frac{5}{6} = 0 \qquad \cdots\cdots\cdots\cdots(1)$$

Also given gear C is rigidly mounted on shaft A

$$\therefore x + y = 110 \qquad \cdots\cdots\cdots\cdots(2)$$

Solving Eq. (1) & (2)

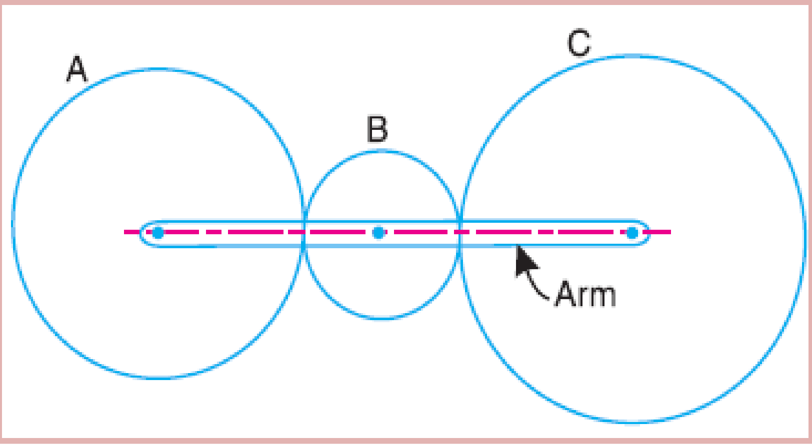
$$x = 60$$
$$y = 50$$

$$\boxed{\text{Speed of shaft B} = \text{Speed of arm} = +y = 50 \text{ rpm}}$$

Ex. 7.23 **In an epicyclic gear train, as shown in the figure, the number of teeth on wheels *A*, *B* and**
***C* are 48, 24 and 50 respectively. If the arm rotates at 400 rpm, clockwise,**

Find:

1. Speed of wheel *C* when *A* is fixed, and



Solution: Given Data:

$T_A = 48$ Gear A fixed $\Rightarrow N_C = ?$

$T_B = 24$ Gear C fixed $\Rightarrow N_A = ?$

$T_C = 50$ $y = -400$ rpm (Arm rotation clockwise)

Sr. No.	Condition of motion	Revolution of element			
		Arm C	Gear A	Gear B	Gear C
1	Arm fixe, gear A rotates +1 revolution(anticlockwise)	0	+1	$-\frac{T_A}{T_B}$	$\left(-\frac{T_A}{T_B}\right)\times\left(-\frac{T_B}{T_C}\right)=+\frac{T_A}{T_C}$
2	Arm fixed gear A rotates through + *x* revolutions	0	$+x$	$-x\frac{T_A}{T_B}$	$+x\frac{T_A}{T_C}$
3	Add + *y* revolutions to all elements	$+y$	$+y$	$+y$	$+y$
4	Total motion	y	$x+y$	$y-x\frac{T_A}{T_B}$	$y+x\frac{T_A}{T_C}$

1. Speed of wheel C when A is

fixed When A is fixed

$$\Rightarrow x+y=0$$
$$\Rightarrow x-400=0$$
$$\Rightarrow x=0$$

$$N_C=y+x\frac{T_A}{T_C}$$
$$=-400+400\times\frac{48}{50}$$
$$=-16\text{ rpm}$$

$$\boxed{N_C=16\text{ rpm(Clockwise direction)}}$$

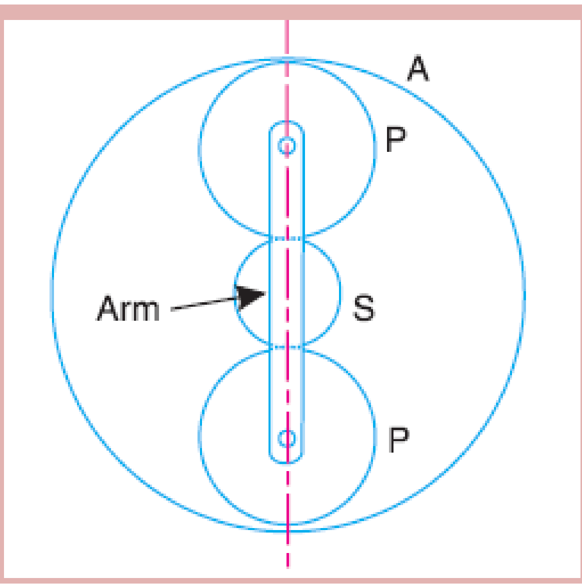
2. Speed wheel A when C is fixed When C is fixed

$$\therefore N_C=0$$
$$\therefore y+x\frac{T_A}{T_C}=0$$
$$\therefore -400+x\frac{48}{50}=0$$
$$\therefore x=416.67$$

$$N_A=x+y$$
$$=416.67-400$$

$$\boxed{N_A=16.67\text{ (Anticlockwise)}}$$

Ex. 7.24 **An epicyclic gear train, as shown in the figure, has a sun wheel *S* of 30 teeth and two planet wheels *P-P* of 50 teeth. The planet wheels mesh with the internal teeth of a fixed annulus *A*. The driving shaft carrying the sun wheel transmits 4 kW at 300 RPM. The driven shaft is connected to an arm which carries the planet wheels. Determine the speed**



Solution: Given Data:

$$T_S = 30 \quad T_P = 50 \quad T_A = 130$$
$$N_S = 300\,\text{rpm} \quad P = 4\,\text{KW}$$

From the geometry of the figure,

$$r_A = 2r_P + r_S$$
$$\therefore T_A = 2T_P + T_S$$
$$= 2 \times 50 + 30$$
$$= 130$$

Sr. No.	Condition of motion	Revolution of element			
		Arm C	Gear A	Gear B	Gear C
1	Arm fixe, gear A rotates +1 revolution(anticlockwise)	0	+1	$-\frac{T_S}{T_P}$	$\left(-\frac{T_S}{T_P}\right)\times\left(\frac{T_P}{T_A}\right) = +\frac{T_S}{T_A}$
2	Arm fixed gear A rotates through + x revolutions	0	$+x$	$-x\frac{T_S}{T_P}$	$-x\frac{T_S}{T_A}$
3	Add + y revolutions to all elements	$+y$	$+y$	$+y$	$+y$
4	Total motion	y	$x+y$	$y-x\frac{T_S}{T_P}$	$y-x\frac{T_S}{T_A}$

Here
,

$$N_S = 300\text{rpm}$$
$$\therefore x + y = 300 \qquad \cdots\cdots\cdots(1)$$

Also, Annular gear A is fixed

$$\therefore y - x\frac{T_S}{T_A} = 0$$

$$\therefore y - x \times \frac{30}{130} = 0$$

$$\therefore y = 0.23x \qquad \cdots\cdots\cdots(2)$$

Solving equation eq. (1) & (2)

$$x = 243.75$$

$$y = 56.25$$

Speed of Arm = Speed of driven shaft = y = 56.25 rpm

Here, P = 4 KW & η= 95%

$$\therefore \eta = \frac{P_{out}}{P_{in}}$$

$$\therefore P_{out} = \eta \times P_{in}$$

$$= \frac{95}{100} \times 4$$

$$= 3.8 \quad KW$$

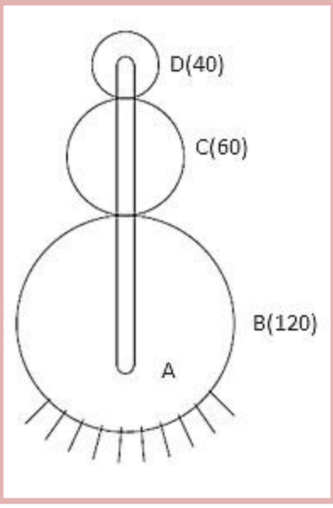
Also,

$$P_{out} = \frac{2\pi NT}{60}$$

$$\therefore 3.8 \times 10^3 = \frac{2\pi \times 56.30T}{60}$$

$$\therefore T = 644.5 \quad N \cdot m$$

Ex. 7.25 An epicyclic gear train is shown In figure. Find out the rpm of pinion D if arm A rotates at



Solution: Given Data:

$T_D = 40$ $\qquad N_D = ?$

$T_C = 60$

$T_B = 120$

$N_A = +60\,\text{rpm(Anticlockwise)}$

Sr. No.	Condition of motion	Revolution of element			
		Arm C	Gear B	Gear C	Gear D
1	Arm fixe, gear A rotates +1 revolution(anticlockwise)	0	+1	$-\frac{T_B}{T_C}$	$\frac{T_B}{T_C} \times \frac{T_C}{T_D} = +\frac{T_B}{T_D}$
2	Arm fixed gear A rotates through + *x* revolutions	0	$+x$	$-x\frac{T_B}{T_C}$	$+x\frac{T_B}{T_D}$
3	Add + *y* revolutions to all elements	$+y$	$+y$	$+y$	$+y$
4	Total motion	y	$x+y$	$y-x\frac{T_B}{T_C}$	$y+x\frac{T_B}{T_D}$

From fig. Gear B is fixed

$\therefore x + y = 0$

$\therefore x + 60 = 0 \qquad (\because \text{rpm of arm A} = 60 = y)$

$\therefore x = -60$

Now the motion of gear D

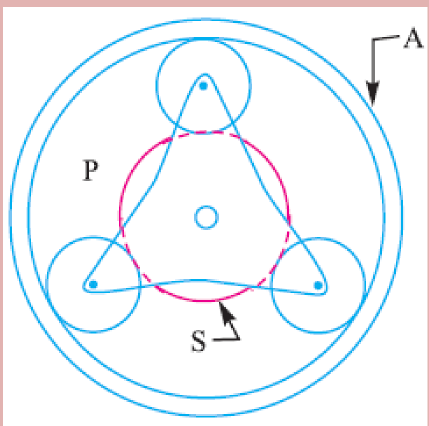
$= y + x\frac{T_B}{T_D}$

$= 60 - 60 \times \frac{120}{40}$

$= -120\,\text{rpm}$

D rotates 120 rpm in the clockwise direction.

Ex. 7.26 **An epicyclic gear train for an electric motor is shown in the figure. The wheel *S* has 15 teeth and is fixed to the motor shaft rotating at 1450 RPM. The planet *P* has 45 teeth, gears with fixed annulus *A* and rotates on a spindle carried by an arm that is fixed to the output shaft. The planet *P* also gears with the sun wheel *S*. Find the speed of the output**



Solution: Given Data:

$T_S = 15$ Speed of output shaft = ?

$T_P = 45$ Torque = ?

From figure,

$r_A = r_s + 2r_P$

$\therefore T_A = T_S + 2T_P$

$\therefore T_A = 105$

Sr. No.	Condition of motion	Revolution of element			
		Spindle	Gear S	Gear P	Gear A
1	Sector/Spindle fixed, gear S rotates +1 revolution (anticlockwise)	0	+1	$-\frac{T_S}{T_P}$	$-\frac{T_S}{T_P}\times\frac{T_P}{T_A}=-\frac{T_S}{T_A}$
2	Spindle fixed gear S rotates through + *x* revolutions	0	$+x$	$-x\frac{T_S}{T_P}$	$-x\frac{T_S}{T_A}$
3	Add + *y* revolutions to all elements	$+y$	$+y$	$+y$	$+y$
4	Total motion	y	$x+y$	$y-x\frac{T_S}{T_P}$	$y-x\frac{T_S}{T_A}$

Motor shaft is fixed with gear S

$\therefore x + y = 1450 \quad \cdots\cdots\cdots(1)$

And Annular A is fixed

$\therefore y - x\frac{T_S}{T_A} = 0$

$\therefore y - x\frac{15}{105} = 0$

$\therefore y = x\frac{15}{105} \quad \cdots\cdots\cdots(2)$

By solving equation (1) & (2)

$x = 1268.76$

$y = 181.25$

Speed of output shaft y = 181.25 rpm

- Torque on sun wheel (S) (input torque)

$$P = \frac{2\pi N T_i}{60}$$

$$\therefore T_i = \frac{P\times 60}{2\pi N}$$

$$= \left(\frac{2\times10^3}{1.35}\right)\times\frac{60}{2\pi\times1450} \qquad \left(\because 1.35\,\text{HP} = 1\,\text{KW} \Rightarrow 2\,\text{HP} = \frac{2}{1.35}\,\text{KW}\right)$$

$$= 9.75\,\text{N}\cdot\text{m}$$

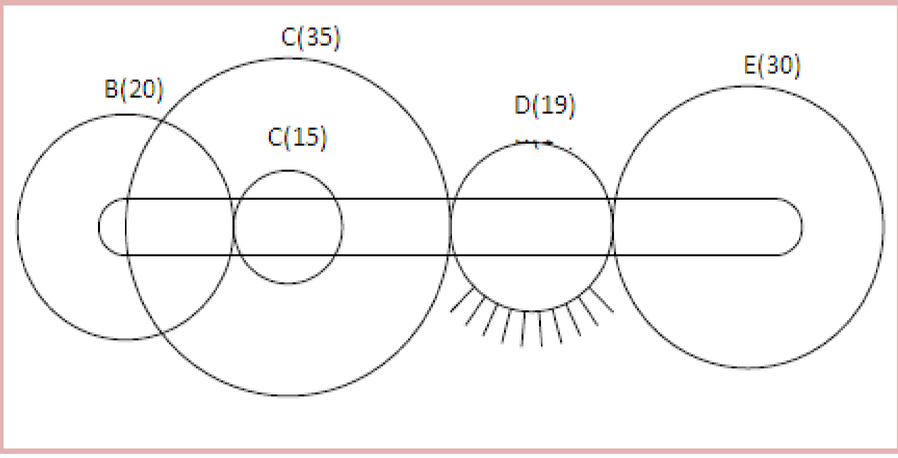
- Torque on output shaft (with 100% mechanical efficiency)

$$\therefore T_o = \frac{P \times 60}{2\pi N}$$
$$= \left(\frac{2\times10^3}{1.35}\right) \times \frac{60}{2\pi\times181.25}$$
$$= 78.05 \quad N\cdot m$$

- Fixing torque

$$= T_o - T_i$$
$$= 78.05 - 9.75$$
$$= 68.3\, N\cdot m$$

Ex. 7.27 **If wheel D of gear train as shown in the figure, is fixed and the arm A makes 140 revolutions in a clockwise direction. Find the speed and direction of rotation of B & E. C**



Solution: Given Data:

$T_B = 30 \;\; T_C = 35 \;\; T_D = 19 \;\; T_E = 30$

Sr. No.	Condition of motion	Revolution of element			
		Spindle	Gear S	Gear P	Gear A
1	Arm fixe, gear A rotates +1 revolution(anticlockwise)	0	+1	$-\frac{20}{15}$	$\left(-\frac{20}{15}\right)\times\left(-\frac{35}{19}\right)\times\left(-\frac{19}{30}\right)$
2	Arm fixed gear A rotates through + *x* revolutions	0	$+x$	$-1.33x$	$-1.555x$
3	Add + *y* revolutions to all elements	$+y$	$+y$	$+y$	$+y$
4	Total motion	y	$x+y$	$y-1.33x$	$y-1.555x$

- When gear D is fixed

$$y + 2.456x = 0$$
$$\therefore -140 + 2.456x = 0 \qquad (\because y = -140\text{rpm given})$$
$$\therefore x = 57$$

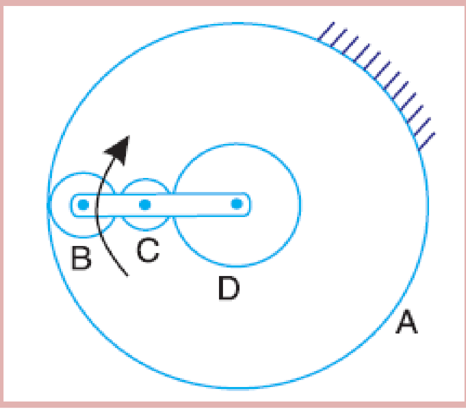
- Speed of gear B

$$N_B = x + y$$
$$= +57 - 140$$
$$= -83 \text{rpm(Clockwise)}$$

- Speed of gear E

$$N_E = y - 1.555x$$
$$= -140 - 1.555(57)$$
$$= -228.63 \text{rpm (Clockwise)}$$

Ex. 7.28 **The epicyclic train as shown in the figure is composed of a fixed annular wheel A having 150 teeth. Meshing with A is a wheel b which drives wheel D through an idle wheel C, D being concentric with A. Wheel B and C is carried on an arm which revolves clockwise at 100 rpm about the axis of A or D. If the wheels B and D are having 25 teeth and 40 teeth**



Solution:

From the geometry of figure,

$$r_A = 2r_B + 2r_C + r_D$$
$$\therefore T_A = 2T_B + 2T_C + T_D$$
$$\therefore 150 = 50 + 2T_C + 40$$
$$\therefore T_C = 30$$

Sr. No.	Condition of motion	Revolution of element				
		Arm	Gear D	Gear C	Gear B	Gear A
1	Arm fixe, gear D rotates +1 revolution (anticlockwise)	0	+1	$-\frac{T_D}{T_C}$	$+\frac{T_D}{T_B}$	$+\frac{T_D}{T_A}$
2	Arm fixed gear D rotates through $+x$ revolutions	0	$+x$	$-x\frac{T_D}{T_C}$	$+x\frac{T_D}{T_B}$	$+x\frac{T_D}{T_A}$
3	Add $+y$ revolutions to all elements	$+y$	$+y$	$+y$	$+y$	$+y$
4	Total motion	$+y$	$x+y$	$y-x\frac{T_D}{T_C}$	$y+x\frac{T_D}{T_B}$	$y+x\frac{T_D}{T_A}$

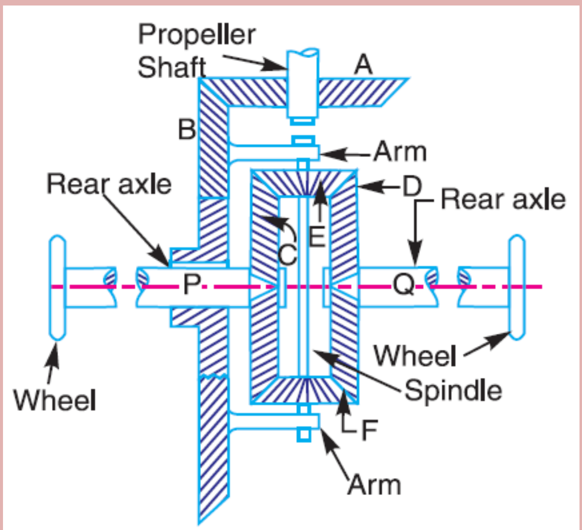
Now

$$N_A = 0$$
$$\therefore y + x\frac{T_D}{T_A} = 0$$
$$\therefore -100 + x \times \frac{40}{150} = 0$$
$$\therefore x = 375$$

Let

$$N_C = y - x\frac{T_D}{T_C}$$
$$= -100 - 375 \times \frac{40}{30}$$
$$= -600 \text{rpm}$$

Ex. 7.29 **The figure shows a differential gear used in a motor car. The pinion A on the propeller shaft has 12 teeth and gears with the crown gear B which has 60 teeth. The shafts P and Q from the rear axles to which the road wheels are attached. If the propeller shaft rotates at 1000 rpm and the road wheel attached to axle Q has a speed of 210 rpm. while taking**



Solution: Given Data:

$$T_A = 12$$
$$T_B = 60$$
$$N_Q = N_D = 210 \text{rpm}$$
$$N_A = 1000 \text{rpm}$$

Let,

$$N_A \times T_A = N_B T_B$$
$$\therefore N_B = N_A \times \frac{T_A}{T_B}$$
$$= 1000 \times \frac{12}{60}$$
$$= 200 \text{rpm}$$

Sr. No.	Condition of motion	Revolution of element			
		Gear B	Gear C	Gear E	Gear D
1	Gear B is fixed, gear C rotates +1 revolution(anticlockwise)	0	+1	$+\frac{T_C}{T_E}$	-1
2	Gear B is fixed gear C rotates through + x revolutions	0	$+x$	$+x\frac{T_C}{T_E}$	$-x$
3	Add + y revolutions to all elements	$+y$	$+y$	$+y$	$+y$
4	Total motion	$+y$	$x+y$	$y+x\frac{T_C}{T_E}$	$y-x$

Let here speed of gear B is 200 rpm

$$N_B = 200 = y$$

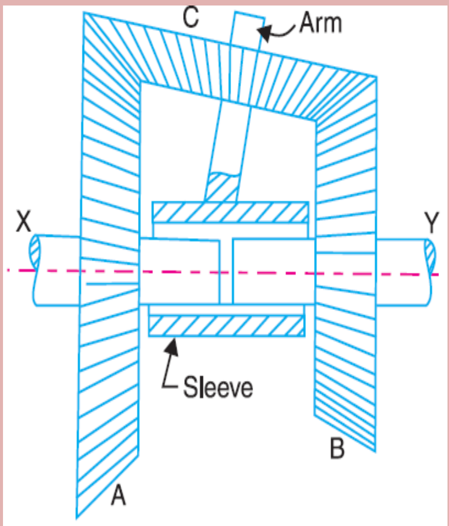
From table

$$N_D = y - x = 210$$
$$\therefore x = y - 210$$
$$\therefore x = 200 - 210$$
$$\therefore x = -10 rpm$$

Let the speed of road wheel attached to the axle P = Speed of gear C

$$= x + y$$
$$= -10 + 200$$
$$= 180 rpm$$

Ex. 7.30 **Two bevel gears A and B (having 40 teeth and 30 teeth) are rigidly mounted on two co- axial shafts X and Y. A bevel gear C (having 50 teeth) meshes with A and B and rotates freely on one end of an arm. At the other end of the arm is welded a sleeve and the sleeve is riding freely loose on the axes of the shafts X and Y. Sketch the arrangement. If the shaft X rotates at 100 RPM. clockwise and arm rotates at 100 RPM. anticlockwise, find**



Solution: Given Data:

$T_A = 40 \quad T_C = 50 \quad T_B = 30$

$N_X = N_A = -100\text{rpm(Clockwise)}$

$\text{Speedof arm} = 100\text{rpm}$

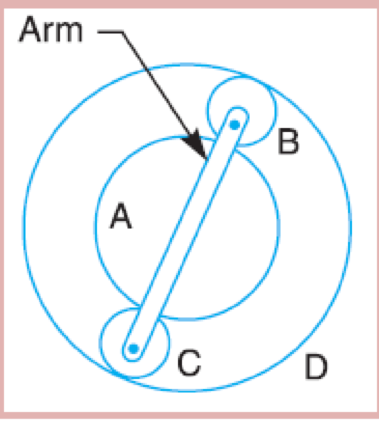
Sr. No.	Condition of motion	Revolution of element			
		Arm	Gear A	Gear C	Gear B
1	Arm fixe, gear A rotates +1 revolution(anticlockwise)	0	+1	$\pm\frac{T_A}{T_C}$	$-\frac{T_A}{T_B}$
2	Arm fixed gear A rotates through + x revolutions	0	$+x$	$\pm x\frac{T_A}{T_C}$	$-x\frac{T_A}{T_B}$
3	Add + y revolutions to all elements	$+y$	$+y$	$+y$	$+y$
4	Total motion	$+y$	$x+y$	$y \pm x\frac{T_A}{T_C}$	$y - x\frac{T_A}{T_B}$

Here speed of arm = y =+100 rpm (given)

$$\text{Also given} \quad N_A = N_X = -100\text{rpm}$$
$$\therefore N_A = x + y$$
$$\therefore -100 = x + 100$$
$$\therefore x = -200$$

$$\text{Speed of shaft } Y = N_B$$
$$= y - x\frac{T_A}{T_B}$$
$$= 100 + 200 \times \frac{40}{30}$$
$$= +366.7\text{rpm (Anticlockwise)}$$

Ex. 7.31 **An epicyclic train of gears is arranged as shown in the figure. How many revolutions does the arm, to which the pinions B and C are attached, make: 1. when A makes one revolution clockwise and D makes half a revolution anticlockwise, and 2. when A makes one revolution clockwise and D is stationary? The number of teeth on the gears A and D are**



Solution: Given Data:

$T_A = 40$

$T_D = 90$

First of all, let us find the number of teeth on gear B and C (i.e. T_B and T_C). Let d_A, d_B, d_C, d_D be the pitch circle diameter of gears A, B, C, and D respectively. Therefore from the geometry of fig, $d_A + d_B + d_C = d_D$ or $d_A + 2\, d_B = d_D$ ...($d_B = d_C$)

Since the number of teeth is proportional to their pitch circle diameters, therefore,

$$T_A + 2\,T_B = T_D \quad \text{or} \quad 40 + 2\,T_B = 90$$

$$\therefore \quad T_B = 25, \quad \text{and} \quad T_C = 25 \qquad ...(T_B = T_C)$$

Sr. No.	Conditions of motion	Revolutions of elements			
		Arm	Gear A	Compound Gear B-C	Gear D
1	Arm fixe, gear A rotates -1 revolution(clockwise)	0	-1	$+\frac{T_A}{T_B}$	$\left(+\frac{T_A}{T_B}\right)\times\left(+\frac{T_B}{T_D}\right)=+\frac{T_A}{T_D}$
2	Arm fixed gear A rotates through - *x* revolutions	0	$-x$	$+x\frac{T_A}{T_B}$	$+x\frac{T_A}{T_D}$
3	Add - *y* revolutions to all elements	$-y$	$-y$	$-y$	$-y$
4	Total motion	$-y$	$-x-y$	$x\frac{T_A}{T_B}-y$	$x\frac{T_A}{T_C}-y$

1. **Speed of arm when A makes 1 revolution clockwise and D makes half revolution anticlockwise**

   Since the gear A makes 1 revolution clockwise, therefore from the fourth row of the table,

   $$-x-y=-1 \quad \text{or} \quad x+y=1 \qquad ...(1)$$

   Also, the gear D makes half revolution anticlockwise, therefore

   $$x\times\frac{T_A}{T_D}-y=\frac{1}{2}$$

   $$\therefore x\times\frac{40}{90}-y=\frac{1}{2}$$

   $$\therefore 40x-90y=45$$

   $$\therefore x-2.25y=1.125 \qquad ...(2)$$

   From equations (1) and (2),

   $$x=1.04 \quad \text{and} \quad y=-0.04$$

   $$\text{Speedofarm}=-y=-(-0.04)=+0.04$$

   $$\boxed{=0.04\ \text{revolution(Anticlockwise)}}$$

2. **Speed of arm when A makes 1 revolution clockwise and D is stationary**

Since the gear A makes 1 revolution clockwise, therefore from the fourth row of the table,

$$-x - y = -1$$

$$\therefore x + y = 1 \qquad ...(3)$$

Also, the gear *D* is stationary, therefore

$$x \times \frac{T_A}{T_D} - y = 0$$

$$\therefore x \times \frac{40}{90} - y = 0$$

$$\therefore 40x - 90y = 0$$

$$\therefore x - 2.25y = 0 \qquad ...(4)$$

From equations (3) and (4),

$$\therefore \text{Speed of arm} = -y = -0.308$$

$$\boxed{\therefore \text{Speed of arm} = 0.308 \text{ revolution (Clockwise)}}$$

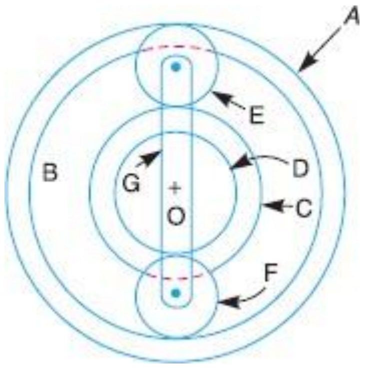
Ex. 7.32 **In an epicyclic gear train, the internal wheels A and B and compound wheels C and D rotate independently about axis O. The wheels E and F rotate on pins fixed to the arm G. E gears with A and C and F gears with B and D. All the wheels have the same module and the number of teeth are: TC = 28; TD = 26; TE = TF = 18. 1. Sketch the arrangement; 2. Find the number of teeth on A and B; 3. If the arm G makes 100 r.p.m. clockwise and A is fixed, find the speed of B; and 4. If the arm G makes 100 r.p.m. clockwise and wheel A**

Solution: Given Data:

$$T_C = 28\ ;\ T_D = 26\ ;\ T_E = T_F = 18$$

1. Sketch the arrangement

The arrangement is shown in the figure.



2. Number of teeth on wheels A and B

*T*A = Number of teeth on wheel *A*, and

*T*B = Number of teeth on wheel *B*.

If d_A, d_B, d_C, d_D, d_E and d_F are the pitch circle diameters of wheels *A*, *B*, *C*, *D*, *E,* and *F* respectively, then from the geometry of Fig.

$$d_A = d_C + 2\, d_E$$

$$\text{And} \quad d_B = d_D + 2\, d_F$$

Since the number of teeth is proportional to their pitch circle diameters, for the same module, therefore

$$T_A = T_C + 2\,T_E = 28 + 2 = 64$$

$$\text{And} \quad T_B = T_D + 2\,T_F = 26 + 2 = 62$$

3. Speed of wheel B when arm G makes 100 r.p.m. clockwise and wheel A is fixed

First of all, the table of motions is drawn as given below:

Sr. No	Conditions of motion	Revolutions of elements					
		Arm G	Wheel A	Wheel E	Compound wheel C-D	Wheel F	Wheel B
1	Arm fixe, A rotates +1 revolution (Anti clockwise)	0	$+1$	$+\frac{T_A}{T_E}$	$-\frac{T_A}{T_E}\times\frac{T_E}{T_C}$ $=-\frac{T_A}{T_C}$	$+\frac{T_A}{T_C}\times\frac{T_D}{T_F}$	$+\frac{T_A}{T_C}\times\frac{T_D}{T_F}\times\frac{T_F}{T_B}$ $=+\frac{T_A}{T_C}\times\frac{T_D}{T_B}$
2	Arm fixed A rotates through + *x* revolutions	0	$+x$	$+x\frac{T_A}{T_E}$	$-x\frac{T_A}{T_C}$	$+x\times\frac{T_A}{T_C}\times\frac{T_D}{T_F}$	$+x\times\frac{T_A}{T_C}\times\frac{T_D}{T_B}$
3	Add $+y$ revolutions to all elements	$+y$	$+y$	$+y$	$+y$	$+y$	$+y$
4	Total motion	$+y$	$x+y$	$x\frac{T_A}{T_E}+y$	$y-x\frac{T_A}{T_C}$	$y+x\times\frac{T_A}{T_C}\times\frac{T_D}{T_F}$	$+y+x\times\frac{T_A}{T_C}\times\frac{T_D}{T_B}$

Since the arm *G* makes 100 r.p.m. clockwise, therefore from the fourth row of the table

$$y = -100$$

Also, wheel *A* is fixed, therefore from the fourth row of the table,

$$x + y = 0 \quad \text{or} \quad x = -y = 100$$

$$\text{Speed of wheel B} = y + x \times \frac{T_A}{T_C} \times \frac{T_D}{T_B}$$

$$= -100 + 100 \times \frac{64}{28} \times \frac{26}{62}$$

$$= -100 + 95.8 \text{ r.p.m.} = -4.2 \text{ r.p.m}$$

$$\boxed{\text{Speed of wheel B} = 4.2 \text{ r.p.m (Clockwise)}}$$

4. Speed of wheel B when arm G makes 100 r.p.m. clockwise and wheel A makes 10 r.p.m. counter- clockwise

Since the arm *G* makes 100 r.p.m. clockwise, therefore from the fourth row of the table

$$y = -100 \qquad \ldots(3)$$

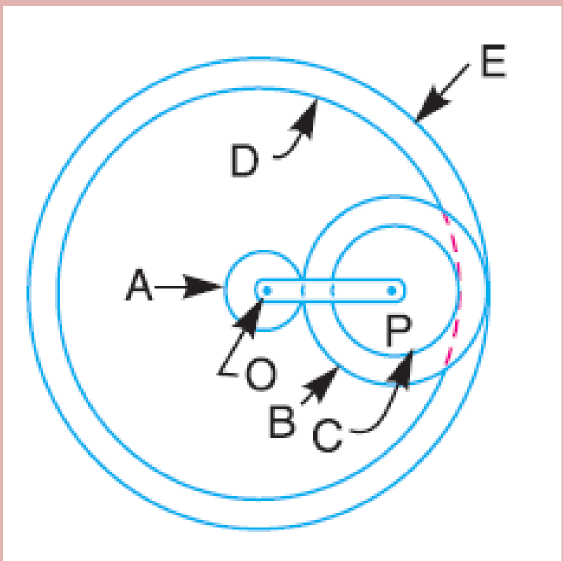
Also, wheel *A* makes 10 r.p.m. counter-clockwise, therefore from the fourth row of the table,

$$x + y = 10$$
$$\therefore x = 10 - y$$
$$\therefore x = 10 + 100$$
$$\therefore x = 110 \qquad \text{...(4)}$$

$$\therefore \text{Speed of wheel B} = +y + x \times \frac{T_A}{T_C} \times \frac{T_D}{T_B}$$
$$= -100 + 110 \times \frac{64}{28} \times \frac{26}{62}$$
$$= -100 + 105.4 \text{ r.p.m}$$
$$= +5.4 \text{ r.p.m}$$

$$\boxed{\therefore \text{Speed of wheel B} = 5.4 \text{r.p.m (Anticlockwise)}}$$

Ex. 7.33 **The figure shows diagrammatically a compound epicyclic gear train. Wheels A, D and E are free to rotate independently on spindle O, while B and C are compound and rotate together on spindle P, on the end of arm OP. All the teeth on different wheels have the same module. A has 12 teeth, B has 30 teeth and C has 14 teeth cut externally. Find the number of teeth on wheels D and E which are cut internally. If the wheel A is driven clockwise at 1 r.p.s. while D is driven counter-clockwise at 5 r.p.s., determine the**



Solution : Given: $T_A = 12;\ T_B = 30; T_C = 14; N_A = 1 \text{ r.p.s.}; N_D = 5 \text{r.p.s}$

Number of teeth on wheels D and E

Let T_D and T_E as the number of teeth on wheels D and E respectively. Let d_A, d_B, d_C, d_D, and d_E be the pitch circle diameters of wheels A, B, C, D, and E respectively. From the geometry of the figure,

$$d_E = d_A + 2d_B \qquad \text{and} \qquad d_D = d_E - (d_B - d_C)$$

Since the number of teeth is proportional to their pitch circle diameters for the same module, therefore

$$T_E = T_A + 2T_B \qquad\qquad T_D = T_E - (T_B - T_C)$$
$$\therefore T_E = 12 + 2 \times 30 \qquad\qquad \therefore T_D = 72 - (30 - 14)$$
$$\boxed{\therefore T_E = 72} \qquad\qquad \boxed{\therefore T_D = 56}$$

Magnitude and direction of angular velocities of arm OP and wheel

The table of motions is drawn as follows:

Sr. No.	Condition of motion	Revolutions of elements				
		Arm	Wheel A	Compound wheel B-C	Wheel D	Wheel E
1	Arm fixe, gear A rotates -1 revolution(clockwise)	0	-1	$+\frac{T_A}{T_B}$	$+\frac{T_A}{T_B}\times\frac{T_C}{T_D}$	$+\frac{T_A}{T_B}\times\frac{T_B}{T_E}=+\frac{T_A}{T_E}$
2	Arm fixed gear A rotates through - x revolutions	0	$-x$	$+x\frac{T_A}{T_B}$	$+x\frac{T_A}{T_B}\times\frac{T_C}{T_D}$	$+x\frac{T_A}{T_E}$
3	Add - y revolutions to all elements	$-y$	$-y$	$-y$	$-y$	$-y$
4	Total motion	$-y$	$-x-y$	$x\frac{T_A}{T_B}-y$	$x\frac{T_A}{T_B}\times\frac{T_C}{T_D}-y$	$x\frac{T_A}{T_E}-y$

Since the wheel A makes 1 r.p.s. clockwise, therefore from the fourth row of the table,

$$-x-y=-1$$

$$\therefore x+y=1 \qquad (1)$$

Also, the wheel D makes 5 r.p.s. counter-clockwise, therefore

$$x\frac{T_A}{T_B}\times\frac{T_C}{T_D}-y=5$$

$$\therefore x\frac{T_A}{T_B}\times\frac{T_C}{T_D}-y=5$$

$$\therefore x\frac{12}{30}\times\frac{14}{56}-y=5$$

$$\therefore 0.1x-y=5 \qquad (2)$$

From equations (1) and (2),

$$x = 5.45 \qquad \text{and} \qquad y=-4.45$$

Angular velocity of arm OP

$$=-y=-(-4.45)=4.45\ \text{r.p.s}$$

$$\boxed{=4.45\times 2\pi=27.964\,\text{rad}/\sec\ (\text{Anti clockwise})}$$

And the angular velocity of wheel E

$$=x\frac{T_A}{T_E}-y$$

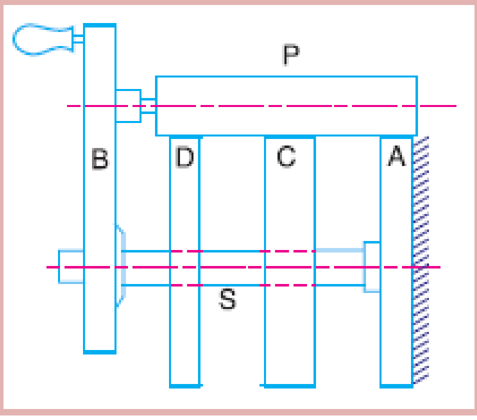
$$=5.45\times\frac{12}{72}-(-4.45)$$

$$=5.36\ \text{r.p.s}$$

$$=5.36\times 2\pi$$

$$\boxed{=33.68\ \text{rad}/\sec\ (\text{Anti clockwise})}$$

Ex. 7.34 **The figure shows an epicyclic gear train known as Ferguson's paradox. Gear A is fixed to the frame and is, therefore, stationary. The arm B and gears C and D are free to rotate on the shaft S. Gears A, C and D have 100, 101 and 99 teeth respectively. The planet gear has 20 teeth. The pitch circle diameters of all are the same so that the planet gear P meshes with all of them. Determine the revolutions of gears C and D for one revolution of**



Solution : Given : $T_A = 100$; $T_C = 101$; $T_D = 99$; $T_P = 20$

The table of motions is given below:

Sr. No.	Condition of motion	Revolutions of elements			
		Arm B	Gear A	Gear C	Gear D
1	Arm B fixe, gear A rotates +1 revolution(anticlockwise)	0	+1	$+\frac{T_A}{T_C}$	$+\frac{T_A}{T_C}\times\frac{T_C}{T_D}=\frac{T_A}{T_D}$
2	Arm B fixed gear A rotates through + *x* revolutions	0	$+x$	$+x\frac{T_A}{T_C}$	$x\frac{T_A}{T_D}$
3	Add + *y* revolutions to all elements	$+y$	$+y$	$+y$	$+y$
4	Total motion	$+y$	$x+y$	$y+x\frac{T_A}{T_C}$	$y+x\frac{T_A}{T_D}$

The arm B makes one revolution, therefore

$$y = 1$$

Since the gear A is fixed, therefore from the fourth row of the table,

$$x + y = 0$$

$$\therefore x = -y = -1$$

Let N_C and N_D = Revolutions of gears *C* and *D* respectively.

From the fourth row of the table, the revolutions of gear *C*,

$$N_C = y + x\frac{T_A}{T_C}$$

$$= 1 - 1\times\frac{100}{101}$$

$$\boxed{\therefore N_C = +\frac{1}{101}}$$

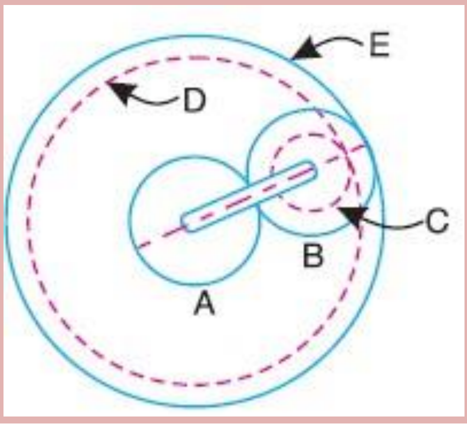
And the revolutions of gear *D*,

$$N_D = y + x\frac{T_A}{T_D} = 1 - \frac{100}{99}$$

$$\boxed{\therefore N_D = -\frac{1}{99}}$$

From above we see that for one revolution of the arm B, the gear C rotates through 1/101 Revolution in the same direction and the gear D rotates through 1/99 revolutions in the opposite direction.

Ex. 7.35 **The figure shows an epicyclic gear train. Pinion A has 15 teeth and is rigidly fixed to the motor shaft. The wheel B has 20 teeth and gears with A and also with the annular fixed wheel E. Pinion C has 15 teeth and is integral with B (B, C being a compound gear wheel). Gear C meshes with annular wheel D, which is keyed to the machine shaft. The arm rotates about the same shaft on which A is fixed and carries the compound wheel B, C. If the motor runs at 1000 r.p.m., find the speed of the machine shaft. Find the torque**



Solution : Given : $T_A = 15$; $T_B = 20$; $T_C = 15$; $N_A = 1000$ r.p.m.;

Torque developed by motor (or pinion A) = 100 N-m

1. Speed of the machine shaft

The table of motions is given below:

Sr. No.	Condition of motion	Revolution of element				
		Arm	Pinion A	Compound wheel D-C	Wheel D	Wheel E
1	Arm fixe, gear A rotates +1 revolution(anticlockwise)	0	+1	$-\frac{T_A}{T_B}$	$-\frac{T_A}{T_B}\times\frac{T_C}{T_D}$	$-\frac{T_A}{T_B}\times\frac{T_B}{T_E} = -\frac{T_A}{T_E}$
2	Arm fixed gear A rotates through + *x* revolutions	0	$+x$	$-x\frac{T_A}{T_B}$	$-x\frac{T_A}{T_B}\times\frac{T_C}{T_D}$	$-x\frac{T_A}{T_E}$
3	Add + *y* revolutions to all elements	$+y$	$+y$	$+y$	$+y$	$+y$
4	Total motion	$+y$	$x+y$	$y-x\frac{T_A}{T_B}$	$y-x\frac{T_A}{T_B}\times\frac{T_C}{T_D}$	$y-x\frac{T_A}{T_E}$

First of all, let us find the number of teeth on wheels D and E. Let T_D and T_E as the number of teeth on wheels D and E respectively. Let d_A, d_B, d_C, d_D, and d_E be the pitch circle diameters of wheels A, B, C, D, and E respectively. From the geometry of the figure,

$$d_E = d_A + 2\,d_B \qquad \text{and} \qquad d_D = d_E - (d_B - d_C)$$

Since the number of teeth is proportional to their pitch circle diameters, therefore,

$$T_E = T_A + 2\,T_B = 15 + 2\times 20 = 55$$
$$T_D = T_E - (T_B - T_C) = 55 - (20 - 15) = 50$$

We know that the speed of the motor or the speed of the pinion *A* is 1000 r.p.m.

Therefore

$$x + y = 1000 \qquad \text{...(1)}$$

Also, the annular wheel *E* is fixed, therefore

$$y - x\frac{T_A}{T_E} = 0$$
$$\therefore y = x\frac{T_A}{T_E}$$
$$\therefore y = x\frac{15}{55}$$
$$\therefore y = 0.273x \qquad \text{...(2)}$$

From equations (1) and (2),

$$x = 786 \qquad \text{and} \qquad y = 214$$

$\therefore$ Speed of machine shaft $=$ Speed of wheel D

$$N_D = y - x\frac{T_A}{T_B}\times\frac{T_C}{T_D}$$
$$= 214 - 786\times\frac{15}{20}\times\frac{15}{50}$$
$$= +37.15 \text{ r.p.m.}$$

$$\boxed{\therefore N_D = 37.15 \text{ (Anticlockwise)}}$$

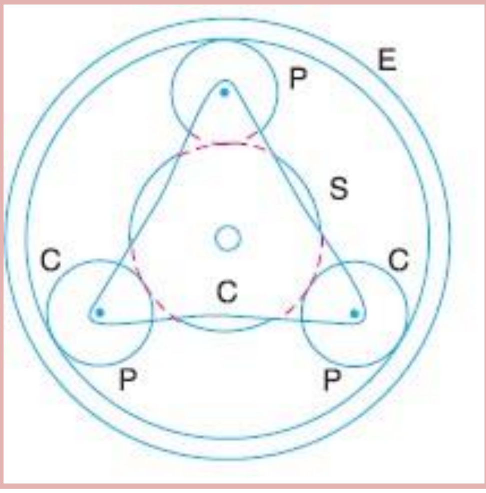
The torque exerted on the machine shaft

We know that

$$\text{Torque developed by motor} \times \text{Angular speed of motor}$$
$$= \text{Torque exerted on machine shaft} \times \text{Angular speed of machine shaft}$$
$$\therefore 100 \times \omega_A = \text{Torque exerted on machine shaft} \times \omega_D$$
$$\therefore \text{Torque exerted on machine shaft} = 100 \times \frac{\omega_A}{\omega_D}$$
$$= 100 \times \frac{N_A}{N_D} = 100 \times \frac{1000}{37.5}$$

$$\boxed{\therefore \text{Torque exerted on machine shaft} = 2692 \text{ N}\cdot\text{m}}$$

Ex. 7.36 **An epicyclic gear train consists of a sun wheel S, a stationary internal gear E and three identical planet wheels P carried on a star-shaped planet carrier C. The sizes of different toothed wheels are such that the planet carrier C rotates at 1/5th of the speed of the sun wheel S. The minimum number of teeth on any wheel is 16. The driving torque on the sun wheel is 100 N-m. Determine 1. Number of teeth on different wheels of the train, and 2.**



Solution : Given $N_C = \frac{N_S}{5}$

1. Number of teeth on different wheels

The arrangement of the epicyclic gear train is shown in the figure. Let T_S and T_E as the number of teeth on the sun wheel S and the internal gear E respectively. The table of motions is given below:

Sr. No.	Conditions of motion	Revolutions of elements			
		Plant carrier C	Sun wheel S	Planet Wheel P	Internal Gear E
1	Planet carrier *C* fixed, sun wheel *S* rotates through + 1 revolution (anticlockwise)	0	+1	$-\frac{T_S}{T_P}$	$-\frac{T_S}{T_P} \times \frac{T_P}{T_E} = -\frac{T_S}{T_E}$
2	Planet carrier *C* fixed, sun wheel *S* rotates through + *x* revolutions	0	$+x$	$-x \cdot \frac{T_S}{T_P}$	$-x \cdot \frac{T_S}{T_E}$
3	Add + *y* revolutions to all elements	$+y$	$+y$	$+y$	$+y$
4	Total motion	$+y$	$x+y$	$y - x\frac{T_S}{T_P}$	$y - x\frac{T_S}{T_E}$

We know that when the sun wheel *S* makes 5 revolutions, the planet carrier *C* makes 1 revolution. Therefore from the fourth row of the table,

$$y = 1, \quad \text{and} \quad x + y = 5$$

$$\therefore x = 4$$

Since the gear E is stationary, therefore from the fourth row of the table,

$$y - x\frac{T_S}{T_E} = 0$$

$$\therefore 1 - 4\frac{T_S}{T_E} = 0$$

$$\therefore T_E = 4T_S$$

Since the minimum number of teeth on any wheel is 16, therefore let us take the number of teeth on the sun wheel,

$$T_S = 16$$

$$\boxed{\therefore T_E = 4 \times 16 = 64}$$

Let d_S, d_P, and d_E be the pitch circle diameters of wheels S, P and E respectively. Now from the geometry of Fig

$$d_S + 2\,d_P = d_E$$

Assuming the module of all the gears to be the same, the number of teeth is proportional to their pitch circle diameters.

$$T_S + 2\,T_P = T_E$$

$$\therefore 16 + 2T_P = 64$$

$$\boxed{\therefore T_P = 24}$$

2. Torque necessary to keep the internal gear stationary

We know that

$$\text{Torque on S} \times \text{Angular speed of S} = \text{Torque on C} \times \text{Angular speed of C}$$

$$100 \times \omega_S = \text{Torque on C} \times \omega_C$$

$$\therefore \text{Torque on C} = 100 \times \frac{\omega_S}{\omega_C}$$

$$= 100 \times \frac{N_S}{N_C}$$

$$= 100 \times 5$$

$$\therefore \text{Torque on C} = 500 \text{ N·m}$$

$\therefore$ Torque necessary to keep the internal gear stationary

$$= 500 - 100$$

$$\boxed{= 400 \text{ N·m}}$$

References:

1. Theory of Machines, Rattan S S, Tata McGraw-Hill
2. Theory of Machines, Khurmi R. S., Gupta J. K., S. Chand Publication
3. Theory of machines and mechanisms, Ballaney P. L., Khanna Publishers